Topic 2

Production and costs

Topics to be Discussed

- The Technology of Production
- Production with One Variable Input (Labor)
- Isoquants
- Production with Two Variable Inputs
- Returns to Scale

Topics to be Discussed

- Costs
- Basic concepts
- Short-run Costs
- Long-run Costs

The Technology of Production

- Production Function:
  - Indicates the highest output (*q*) that a firm can produce for every specified combination of inputs
  - For simplicity, we will consider only labor (L) and capital (K)
  - Shows what is technically feasible when the firm operates efficiently

The Technology of Production

- The production function for two inputs:

$$q = F(K,L)$$

  - Output (q) is a function of capital (K) and labor (L)
  - The production function is true for a given technology
    - If technology increases, more output can be produced for a given level of inputs

The Technology of Production

- Short Run versus Long Run
  - It takes time for a firm to adjust production from one set of inputs to another
  - Firms must consider not only what inputs can be varied but over what period of time that can occur
  - We must distinguish between long run and short run

The Technology of Production

- Short Run
  - Period of time in which quantities of one or more production factors cannot be changed
  - These inputs are called fixed inputs
- Long Run
  - Amount of time needed to make all production inputs variable
- Short run and long run are not *per se* time specific

Production: One Variable Input

- We will begin looking at the short run when only one input can be varied
- We assume capital is fixed and labor is variable
  - Output can only be increased by increasing labor
  - Must know how output changes as the amount of labor is changed (Table 6.1)

Production: One Variable Input

Amount of Labor (L)	Amount of Capital (K)	Total Output (q)
0	10	0
1	10	10
2	10	30
3	10	60
4	10	80
5	10	95
6	10	108
7	10	112
8	10	112
9	10	108
10	10	100

Production: One Variable Input

- Firms make decisions based on the benefits and costs of production
- Sometimes useful to look at benefits and costs on an *incremental basis*
  - How much more can be produced when at incremental units of an input?
- Sometimes useful to make comparison on an *average basis*

Production: One Variable Input

- Average product of Labor - Output per unit of a particular product
- Measures the productivity of a firm's labor in terms of how much, on average, each worker can produce

$$AP_L = \frac{Output}{Labor \quad Input} = \frac{\boldsymbol{q}}{\boldsymbol{L}}$$

Production: One Variable Input

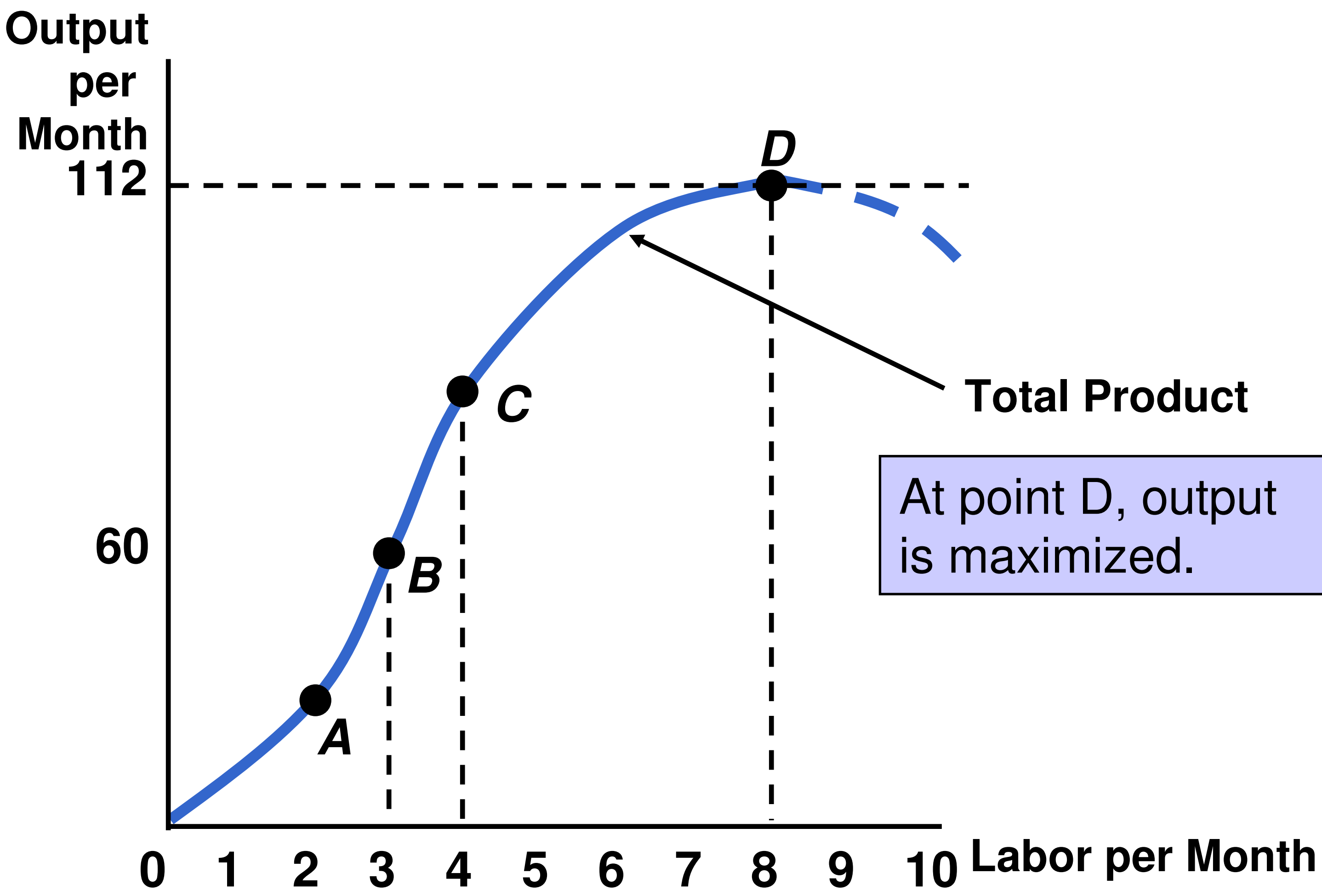
- Marginal Product of Labor – additional output produced when labor increases by one unit
- Change in output divided by the change in labor

$$MP_L = \frac{\Delta Output}{\Delta Labor\ Input} = \frac{\Delta \boldsymbol{q}}{\Delta \boldsymbol{L}}$$

Production: One Variable Input

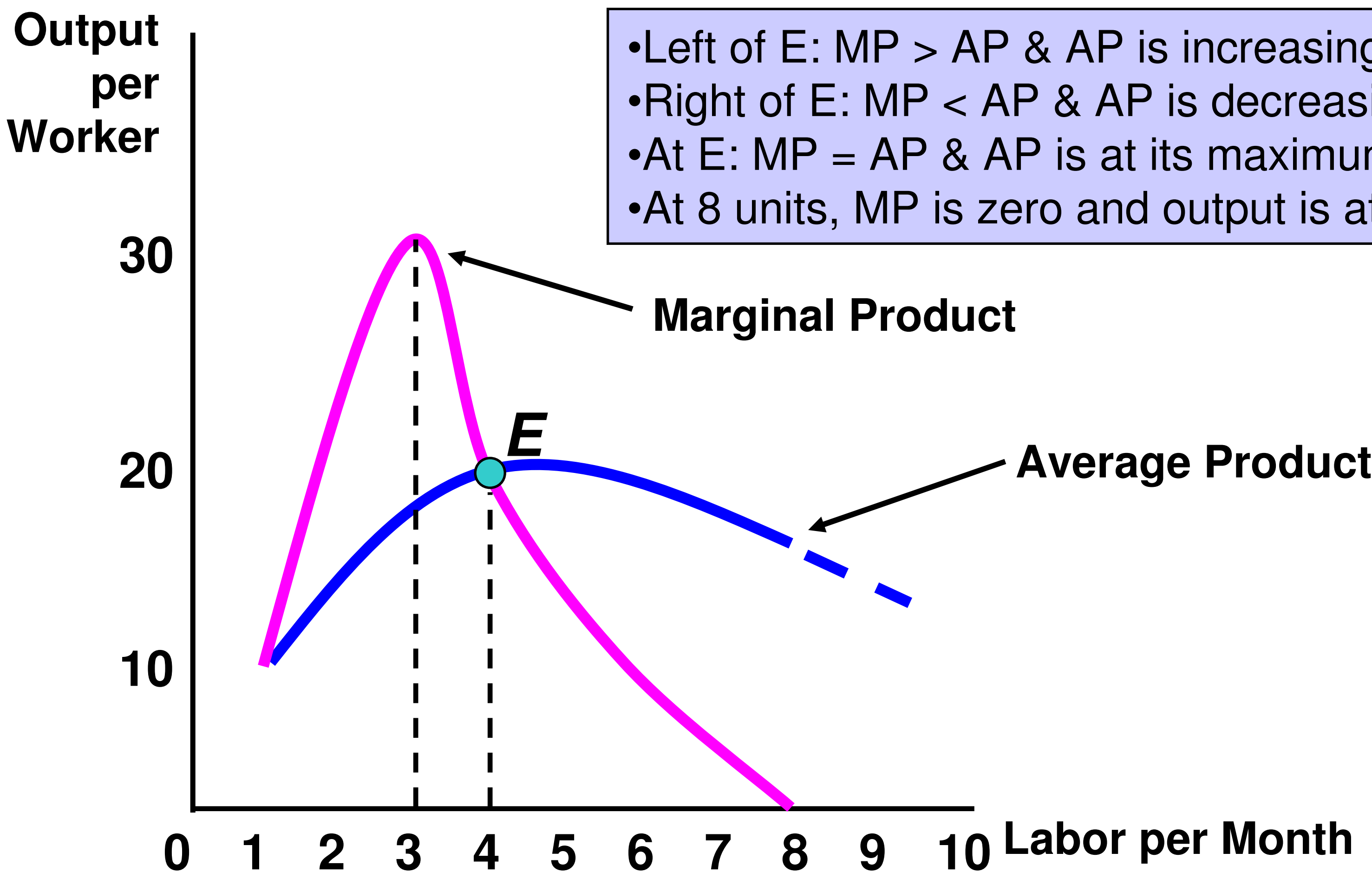
Amount of Labor (L)	Amount of Capital (K)	Total Output (q)	Average Product (q/L)	Marginal Product (Δq/ΔL)
0	10	0	—	—
1	10	10	10	10
2	10	30	15	20
3	10	60	20	30
4	10	80	20	20
5	10	95	19	15
6	10	108	18	13
7	10	112	16	4
8	10	112	14	0
9	10	108	12	–4
10	10	100	10	–8

Production: One Variable Input



At point D, output is maximized.

Production: One Variable Input



- Left of E: MP > AP & AP is increasing
- Right of E: MP < AP & AP is decreasing
- At E: MP = AP & AP is at its maximum
- At 8 units, MP is zero and output is at max

Marginal and Average Product

- When marginal product is greater than the average product, the average product is increasing
- When marginal product is less than the average product, the average product is decreasing
- When marginal product is zero, total product (output) is at its maximum
- Marginal product crosses average product at its maximum

Production: One Variable Input

- From the previous example, we can see that as we increase labor the additional output produced declines
- **Law of Diminishing Marginal Returns**: As the use of an input increases with other inputs fixed, the resulting additions to output will eventually decrease

Production: Two Variable Inputs

- Firm can produce output by combining different amounts of labor and capital

$$Q = F(K,L)$$

- In the long run, capital and labor are both variable
- We can look at the output we can achieve with different combinations of capital and labor

Production: Two Variable Inputs

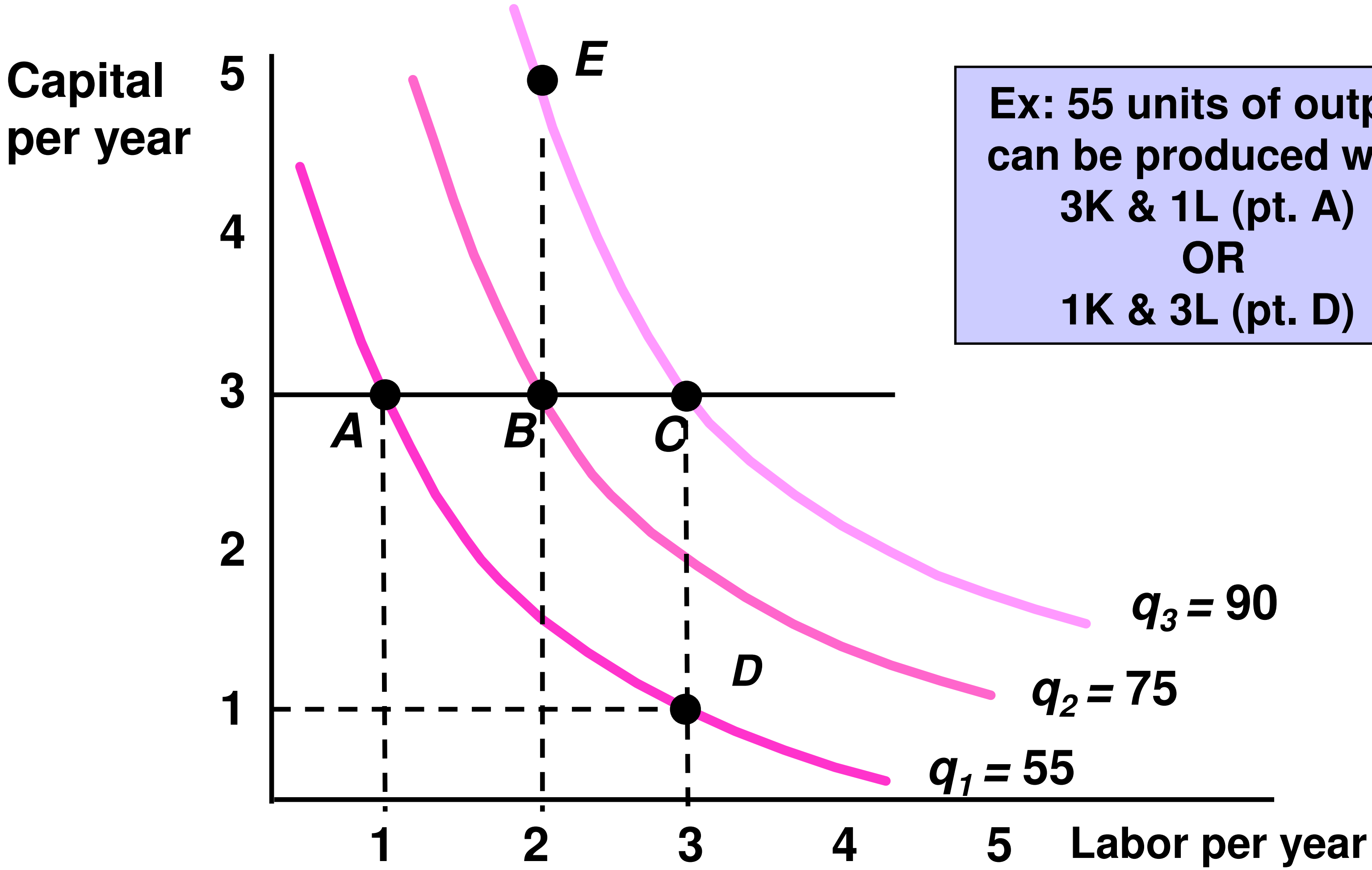
Capital Input	Labor Input 1	 2	 3	 4	 5
1	20	40	55	65	75
2	40	60	75	85	90
3	55	75	90	100	105
4	65	85	100	110	115
5	75	90	105	115	120

Different combinations of factors might produce the same level of production.

Production: Two Variable Inputs

- The information can be represented graphically using **isoquants**
  - Curves showing all possible combinations of inputs that yield the same output

Isoquant Map



Ex: 55 units of output can be produced with
3K & 1L (pt. A)
OR
1K & 3L (pt. D)

Assumptions on technology

Monotonicity → Increasing the quantity of at least one factor allows to obtain at least the same volume of production.

Convexity → If there exists at least two different ways to produce "y" units, its convex combination will allow to achieve at least "y" units.

Production: Two Variable Inputs

- Substituting Among Inputs
  - Companies must decide what combination of inputs to use to produce a certain quantity of output
  - There is a trade-off between inputs, allowing them to use more of one input and less of another for the same level of output

Production: Two Variable Inputs

- Substituting Among Inputs
  - Slope of the isoquant shows how one input can be substituted for the other and keep the level of output the same
  - The negative of the slope is the **marginal rate of technical substitution (MRTS)**
    - Amount by which the quantity of one input can be reduced when one extra unit of another input is used, so that output remains constant

Production: Two Variable Inputs

- The marginal rate of technical substitution equals:

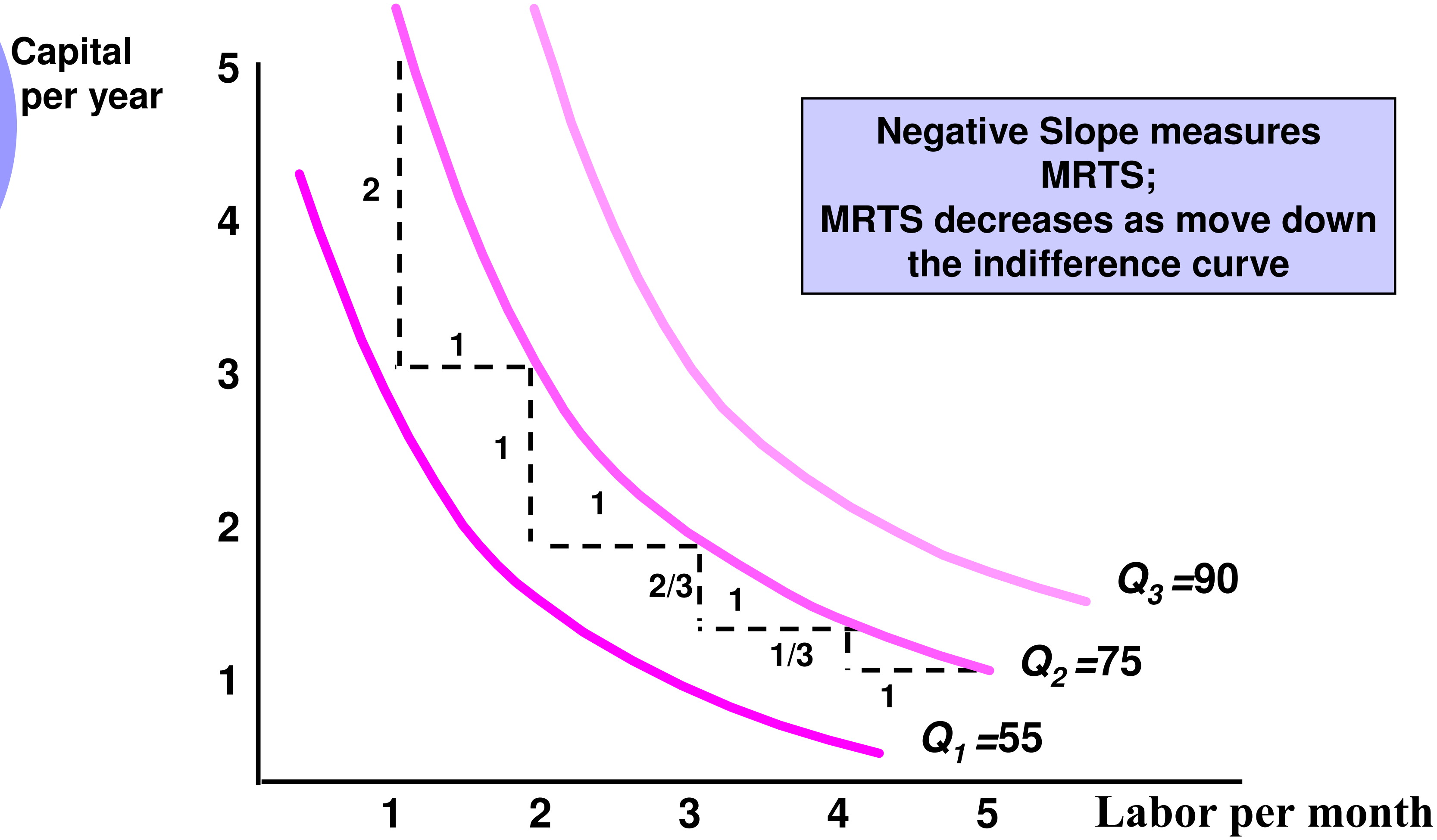
$$MRTS = -\frac{Change\ in\ Capital\ Input}{Change\ in\ Labor\ Input}$$

$$MRTS = -\Delta K / \Delta L \text{ (for a fixed level of } q\text{)}$$

Production: Two Variable Inputs

- As labor increases to replace capital
  - Labor becomes relatively less productive
  - Capital becomes relatively more productive
  - Need less capital to keep output constant
  - Isoquant becomes flatter

Marginal Rate of Technical Substitution



MRTS and Isoquants

- We assume there is diminishing MRTS
  - Increasing labor in one unit increments from 1 to 5 results in a decreasing MRTS from 1 to 1/2
  - Productivity of any one input is limited
- Diminishing MRTS occurs because of diminishing returns and implies isoquants are convex
- There is a relationship between MRTS and marginal products of inputs

MRTS and Marginal Products

- If we increase labor and decrease capital to keep output constant, we can see how much the increase in output is due to the increased labor
  - Amount of labor increased times the marginal productivity of labor

$$= (MP_L)(\Delta L)$$

MRTS and Marginal Products

- Similarly, the decrease in output from the decrease in capital can be calculated
  - Decrease in output from reduction of capital times the marginal produce of capital

$$= (MP_K)(\Delta K)$$

MRTS and Marginal Products

- If we are holding output constant, the net effect of increasing labor and decreasing capital must be zero
- Using changes in output from capital and labor we can see

$$(MP_L)(\Delta L) + (MP_K)(\Delta K) = 0$$

MRTS and Marginal Products

- Rearranging equation, we can see the relationship between MRTS and MPs

$$(MP_L)(\Delta L) + (MP_K)(\Delta K) = 0$$

$$(MP_L)(\Delta L) = -(MP_K)(\Delta K)$$

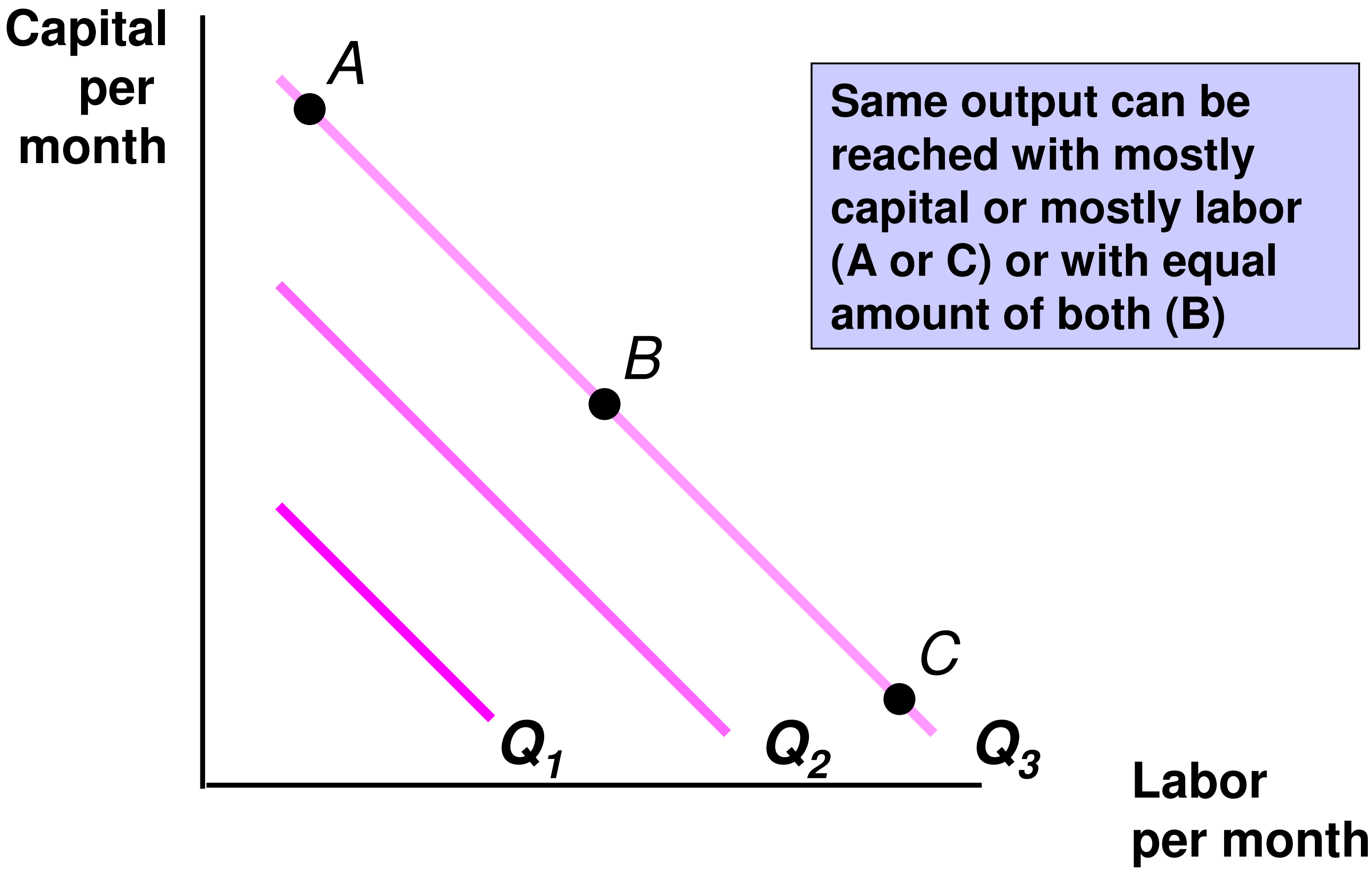
$$\frac{(MP_L)}{(MP_K)} = -\frac{\Delta L}{\Delta K} = MRTS$$

Isoquants: Special Cases

- Two extreme cases show the possible range of input substitution in production

1. Perfect substitutes
   - MRTS is constant at all points on isoquant
   - Same output can be produced with a lot of capital or a lot of labor or a balanced mix

Perfect Substitutes

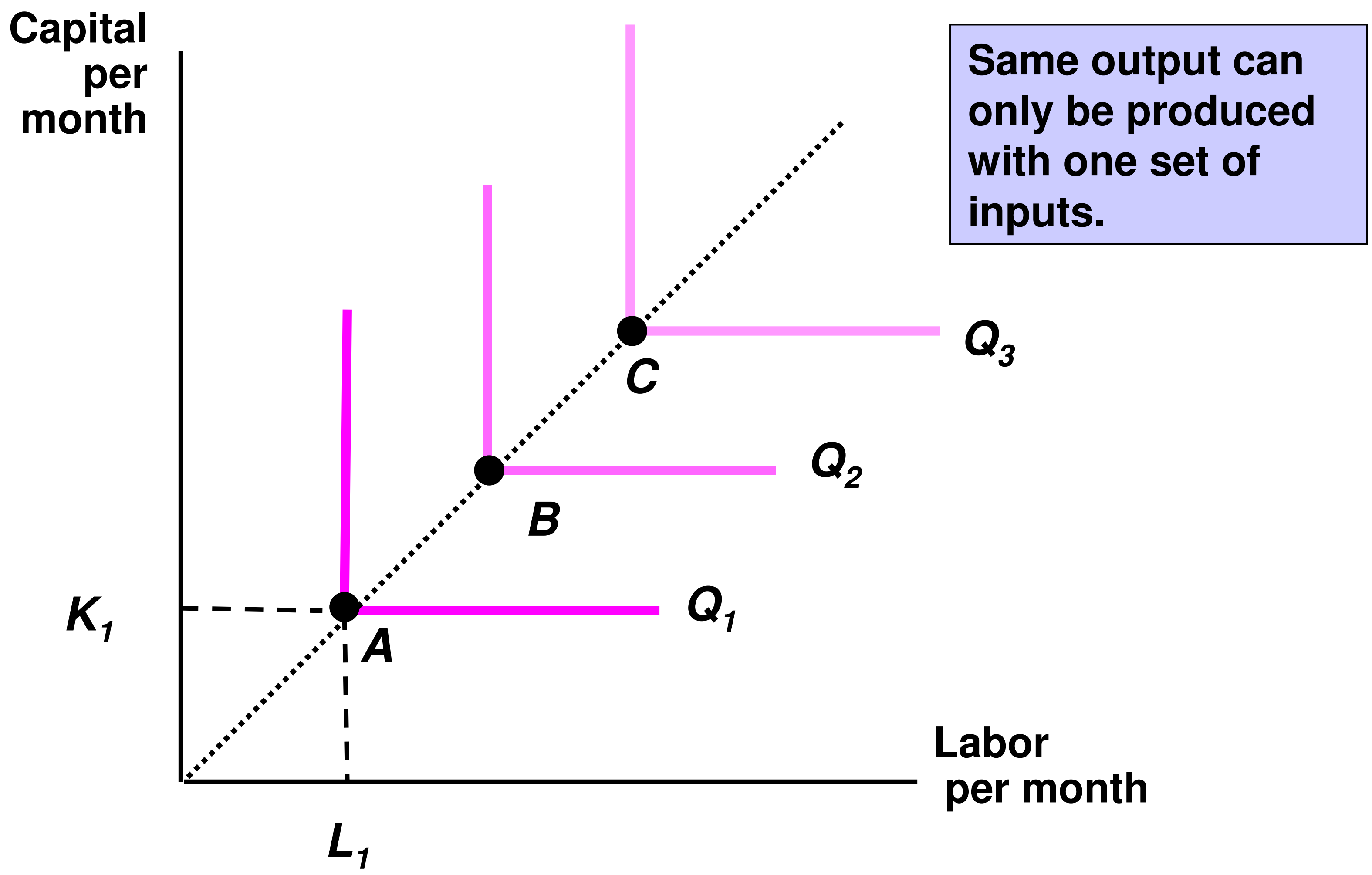


Same output can be reached with mostly capital or mostly labor (A or C) or with equal amount of both (B)

Isoquants: Special Cases

2. Perfect Complements
   - Fixed proportions production function
   - There is no substitution available between inputs
   - The output can be made with only a specific proportion of capital and labor
   - Cannot increase output unless increase both capital and labor in that specific proportion

Fixed-Proportions Production Function



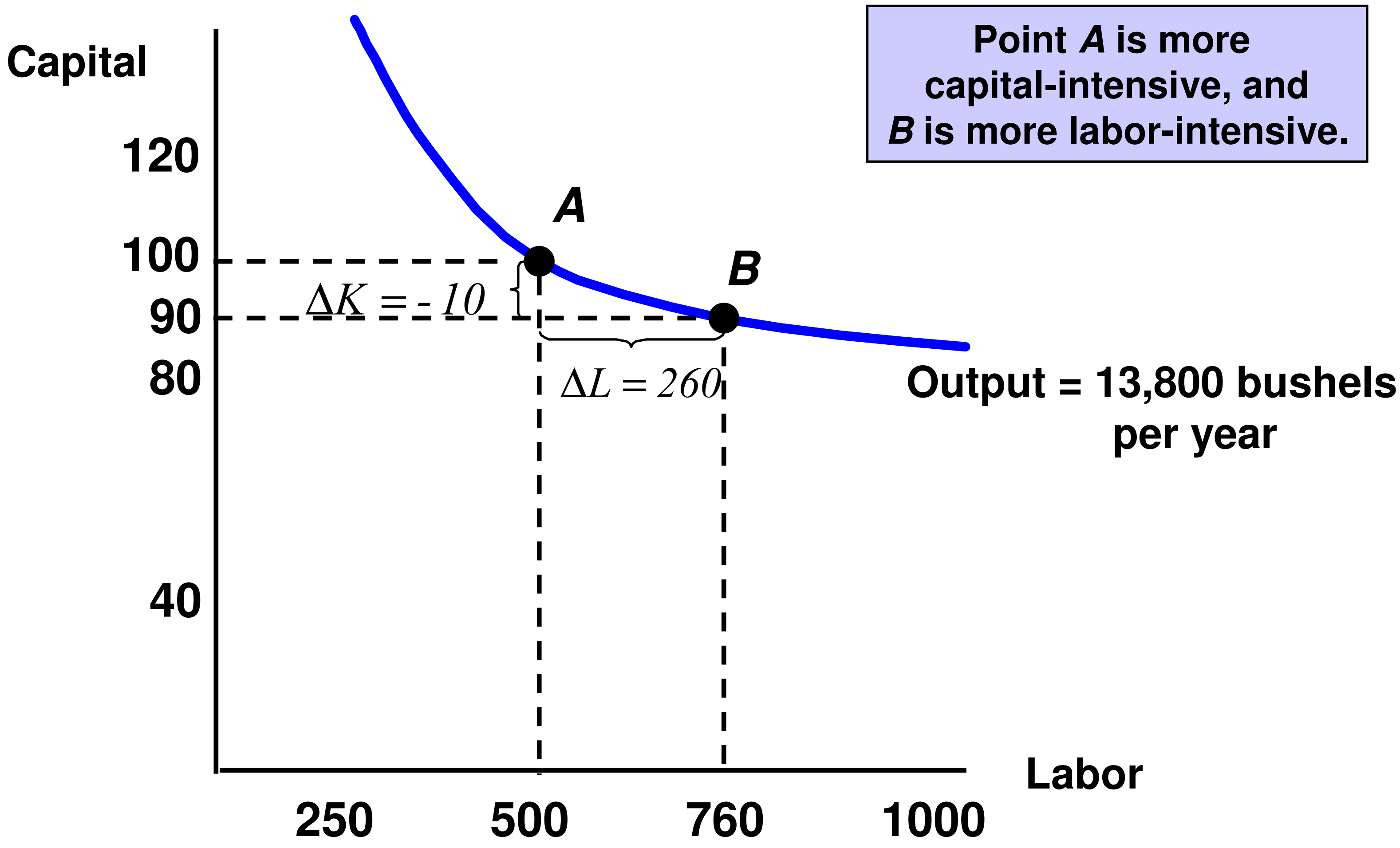
A Production Function for Wheat

- Farmers can produce crops with different combinations of capital and labor
  - Crops in US are typically grown with capital-intensive technology
  - Crops in developing countries grown with labor-intensive productions
- Can show the different options of crop production with isoquants

A Production Function for Wheat

- Manager of a farm can use the isoquant to decide what combination of labor and capital will maximize profits from crop production
  - A: 500 hours of labor, 100 units of capital
  - B: decreases unit of capital to 90, but must increase hours of labor by 260 to 760 hours
  - This experiment shows the farmer the shape of the isoquant

Isoquant Describing the Production of Wheat



A Production Function for Wheat

- Increase L to 760 and decrease K to 90 the MRTS =0.04 < 1

$$MRTS = -\Delta K / \Delta L = -(-10/260) = 0.04$$

  - When wage is equal to cost of running a machine, more capital should be used
  - Unless labor is much less expensive than capital, production should be capital intensive

Returns to Scale

- In addition to discussing the tradeoff between inputs to keep production the same
- How does a firm decide, in the long run, the best way to increase output?
  - Can change the scale of production by increasing all inputs in proportion
  - If double inputs, output will most likely increase but by how much?

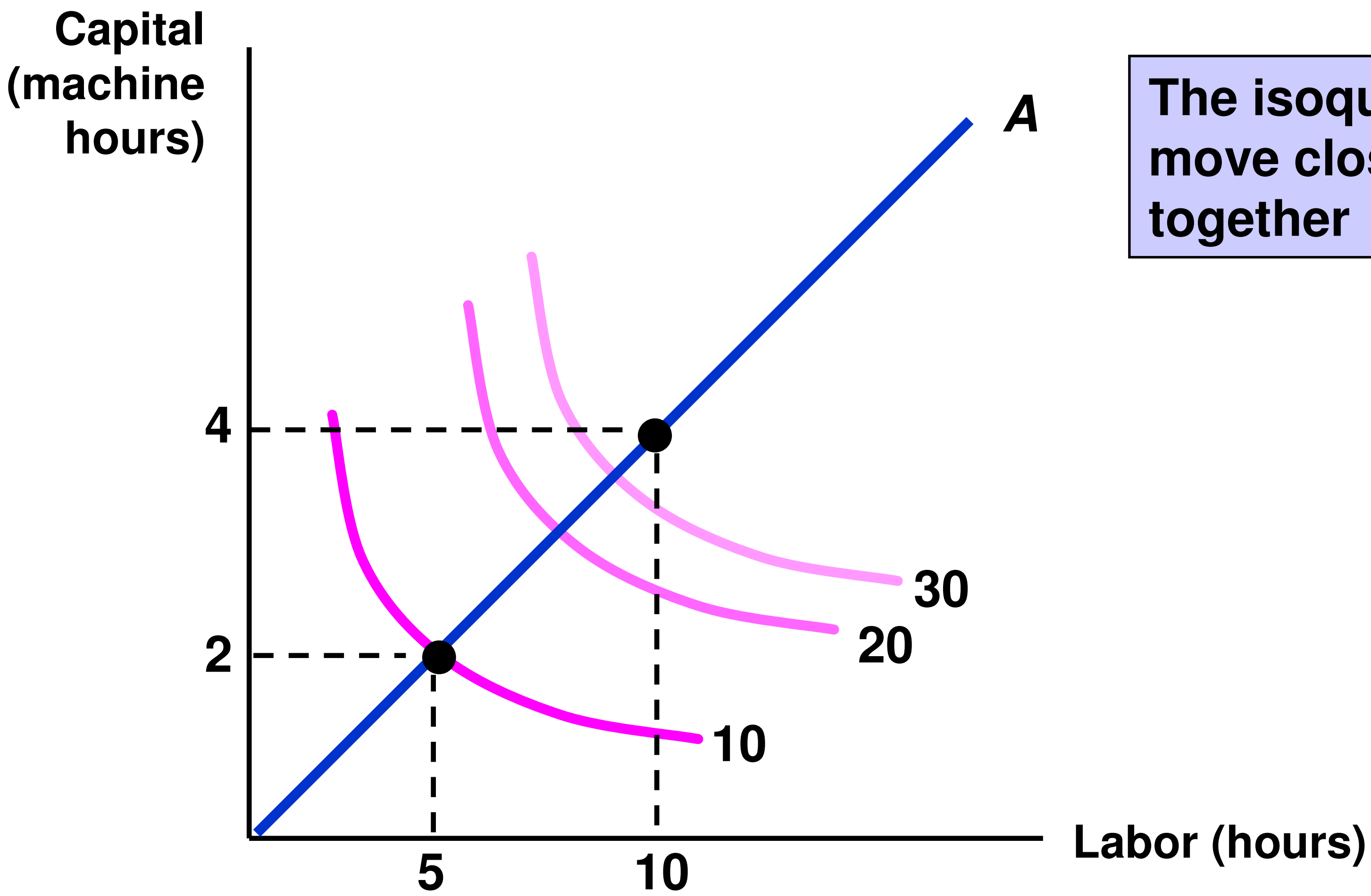
Returns to Scale

- Rate at which output increases as inputs are increased proportionately
  - Increasing returns to scale
  - Constant returns to scale
  - Decreasing returns to scale

Returns to Scale

- **Increasing returns to scale**: output more than doubles when all inputs are doubled
  - Larger output associated with lower cost (cars)
  - One firm is more efficient than many (utilities)
  - The isoquants get closer together

Increasing Returns to Scale

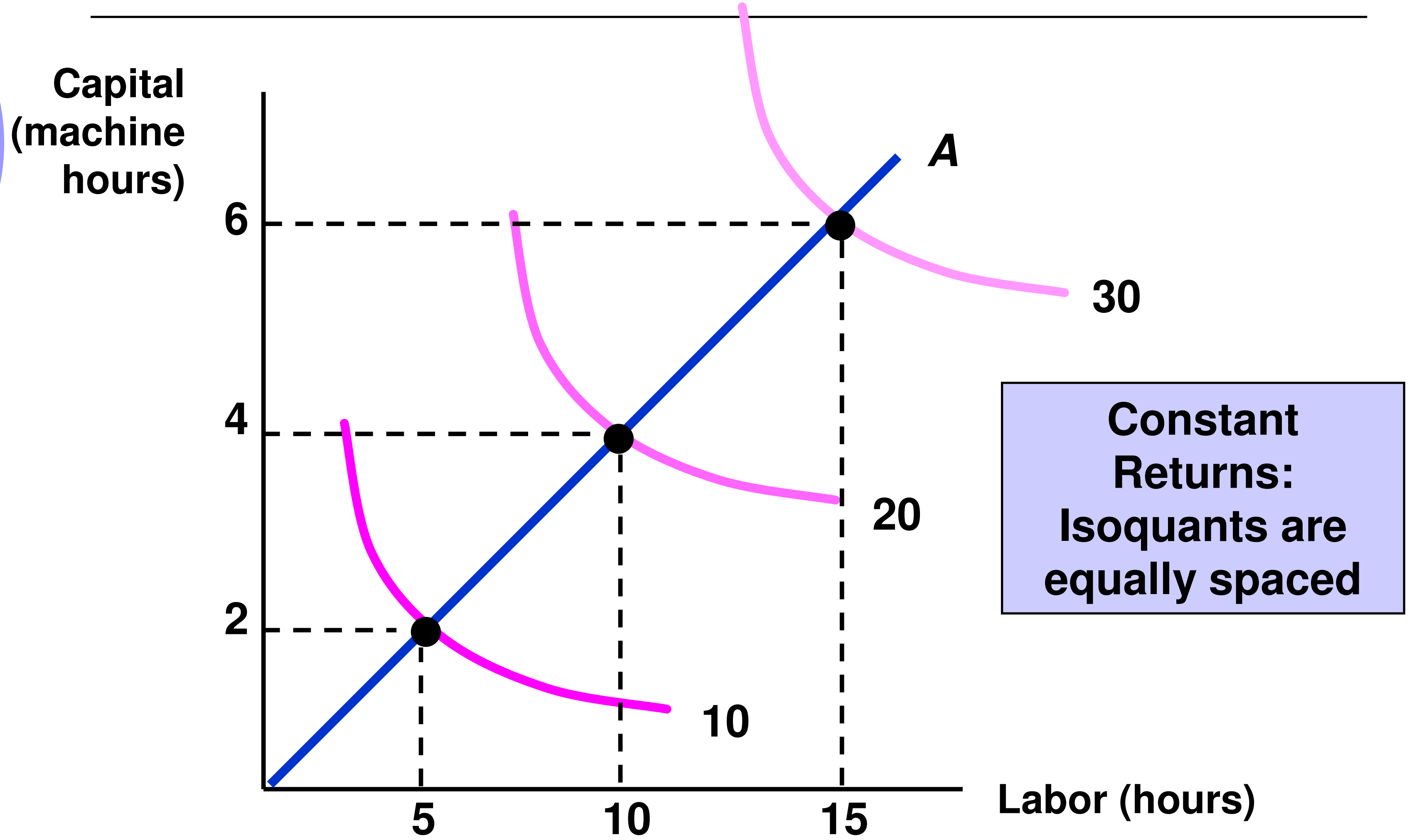


The isoquants move closer together

Returns to Scale

- **Constant returns to scale**: output doubles when all inputs are doubled
  - Size does not affect productivity
  - May have a large number of producers
  - Isoquants are equidistant apart

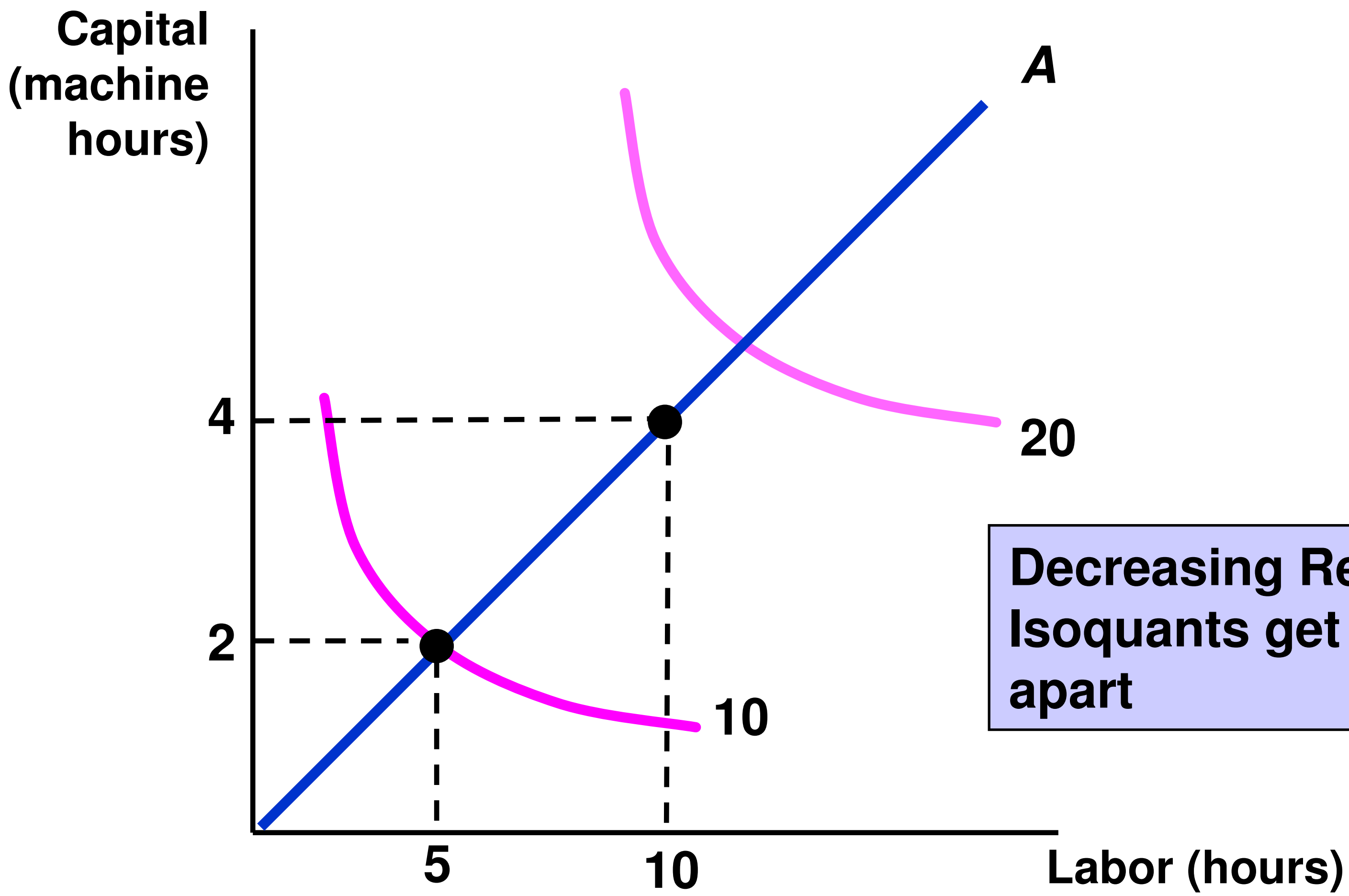
Constant Returns to Scale



Returns to Scale

- **Decreasing returns to scale**: output less than doubles when all inputs are doubled
  - Decreasing efficiency with large size
  - Reduction of entrepreneurial abilities
  - Isoquants become farther apart

Decreasing Returns to Scale



Decreasing Returns: Isoquants get further apart

Chapter 7

The Cost of Production

Introduction

- Production technology measures the relationship between input and output
- Production technology, together with prices of factor inputs, determine the firm's cost of production
- Given the production technology, managers must choose how to produce

Introduction

- The optimal, cost minimizing, level of inputs can be determined
- A firm's costs depend on the rate of output and we will show how these costs are likely to change over time
- The characteristics of the firm's production technology can affect costs in the long run and short run

Measuring Cost: Which Costs Matter?

- For a firm to minimize costs, we must clarify what is meant by *costs* and how to measure them
  - It is clear that if a firm has to rent equipment or buildings, the rent they pay is a cost
  - What if a firm owns its own equipment or building?
    - How are costs calculated here?

Measuring Cost: Which Costs Matter?

- Accountants tend to take a retrospective view of firms' costs, whereas economists tend to take a forward-looking view
- Accounting Cost
  - Actual expenses plus depreciation charges for capital equipment
- Economic Cost
  - Cost to a firm of utilizing economic resources in production, including opportunity cost

Measuring Cost: Which Costs Matter?

- Economic costs distinguish between costs the firm can control and those it cannot
  - Concept of opportunity cost plays an important role
- **Opportunity cost**
  - Cost associated with opportunities that are foregone when a firm's resources are not put to their highest-value use

Opportunity Cost

- An Example
  - A firm owns its own building and pays no rent for office space
  - Does this mean the cost of office space is zero?
  - The building could have been rented instead
  - Foregone rent is the opportunity cost of using the building for production and should be included in the economic costs of doing business

Measuring Cost: Which Costs Matter?

- Some costs vary with output, while some remain the same no matter the amount of output
- Total cost can be divided into:

1. Fixed Cost
   - Does not vary with the level of output
2. Variable Cost
   - Cost that varies as output varies

Fixed and Variable Costs

- Total output is a function of variable inputs and fixed inputs
- Therefore, the total cost of production equals the fixed cost (the cost of the fixed inputs) plus the variable cost (the cost of the variable inputs), or…

$$TC = FC + VC$$

Fixed and Variable Costs

- Which costs are variable and which are fixed depends on the time horizon
- Short time horizon – most costs are fixed
- Long time horizon – many costs become variable
- In determining how changes in production will affect costs, must consider if fixed or variable costs are affected.

Measuring Cost: Which Costs Matter?

- Personal Computers
  - Most costs are variable
  - Largest component: labor
- Software
  - Most costs are sunk
  - Initial cost of developing the software

Marginal and Average Cost

- In completing a discussion of costs, must also distinguish between
  - Average Cost
  - Marginal Cost
- After definition of costs is complete, one can consider the analysis between short-run and long-run costs

Measuring Costs

- Marginal Cost (MC):
  - The cost of expanding output by one unit
  - Fixed costs have no impact on marginal cost, so it can be written as:

$$MC = \frac{\Delta VC}{\Delta q} = \frac{\Delta TC}{\Delta q}$$

Measuring Costs

- Average Total Cost (ATC)
  - Cost per unit of output
  - Also equals average fixed cost (AFC) plus average variable cost (AVC)

$$ATC = \frac{TC}{q} = AFC + AVC$$

$$ATC = \frac{TC}{q} = \frac{TFC}{q} + \frac{TVC}{q}$$

Measuring Costs

- All the types of costs relevant to production have now been discussed
- Can now discuss how they differ in the long and short run
- Costs that are fixed in the short run may not be fixed in the long run
- Typically in the long run, most if not all costs are variable

A Firm's Short Run Costs

Rate of Output (Units per Year)	Fixed Cost (Dollars per Year)	Variable Cost (Dollars per Year)	Total Cost (Dollars per Year)	Marginal Cost (Dollars per Unit)	Average Fixed Cost (Dollars per Unit)	Average Variable Cost (Dollars per Unit)	Average Total Cost (Dollars per Unit)
	(FC) (1)	(VC) (2)	(TC) (3)	(MC) (4)	(AFC) (5)	(AVC) (6)	(ATC) (7)
0	50	0	50	—	—	—	—
1	50	50	100	50	50	50	100
2	50	78	128	28	25	39	64
3	50	98	148	20	16.7	32.7	49.3
4	50	112	162	14	12.5	28	40.5
5	50	130	180	18	10	26	36
6	50	150	200	20	8.3	25	33.3
7	50	175	225	25	7.1	25	32.1
8	50	204	254	29	6.3	25.5	31.8
9	50	242	292	38	5.6	26.9	32.4
10	50	300	350	58	5	30	35
11	50	385	435	85	4.5	35	39.5

Determinants of Short Run Costs

- The rate at which these costs increase depends on the nature of the production process
  - The extent to which production involves diminishing returns to variable factors
- Diminishing returns to labor
  - When marginal product of labor is decreasing

Determinants of Short Run Costs

- If marginal product of labor decreases significantly as more labor is hired
  - Costs of production increase rapidly
  - Greater and greater expenditures must be made to produce more output
- If marginal product of labor decreases only slightly as increase labor
  - Costs will not rise very fast when output is increased

Determinants of Short Run Costs – An Example

- Assume the wage rate (*w*) is fixed relative to the number of workers hired
- Variable costs is the per unit cost of extra labor times the amount of extra labor: *w*L

$$\text{MC} = \frac{\Delta\text{VC}}{\Delta\text{q}} = \frac{w\Delta L}{\Delta q}$$

Determinants of Short Run Costs – An Example

- Remembering that

$$\Delta MP_L = \frac{\Delta Q}{\Delta L}$$

- And rearranging

$$\Delta L \text{ for a 1 unit } \Delta Q = \frac{\Delta L}{\Delta Q} = \frac{1}{\Delta MP_L}$$

Determinants of Short Run Costs – An Example

- We can conclude:

$$MC = \frac{w}{MP_L}$$

- …and a low marginal product (MP_L) leads to a high marginal cost (MC) and vice versa

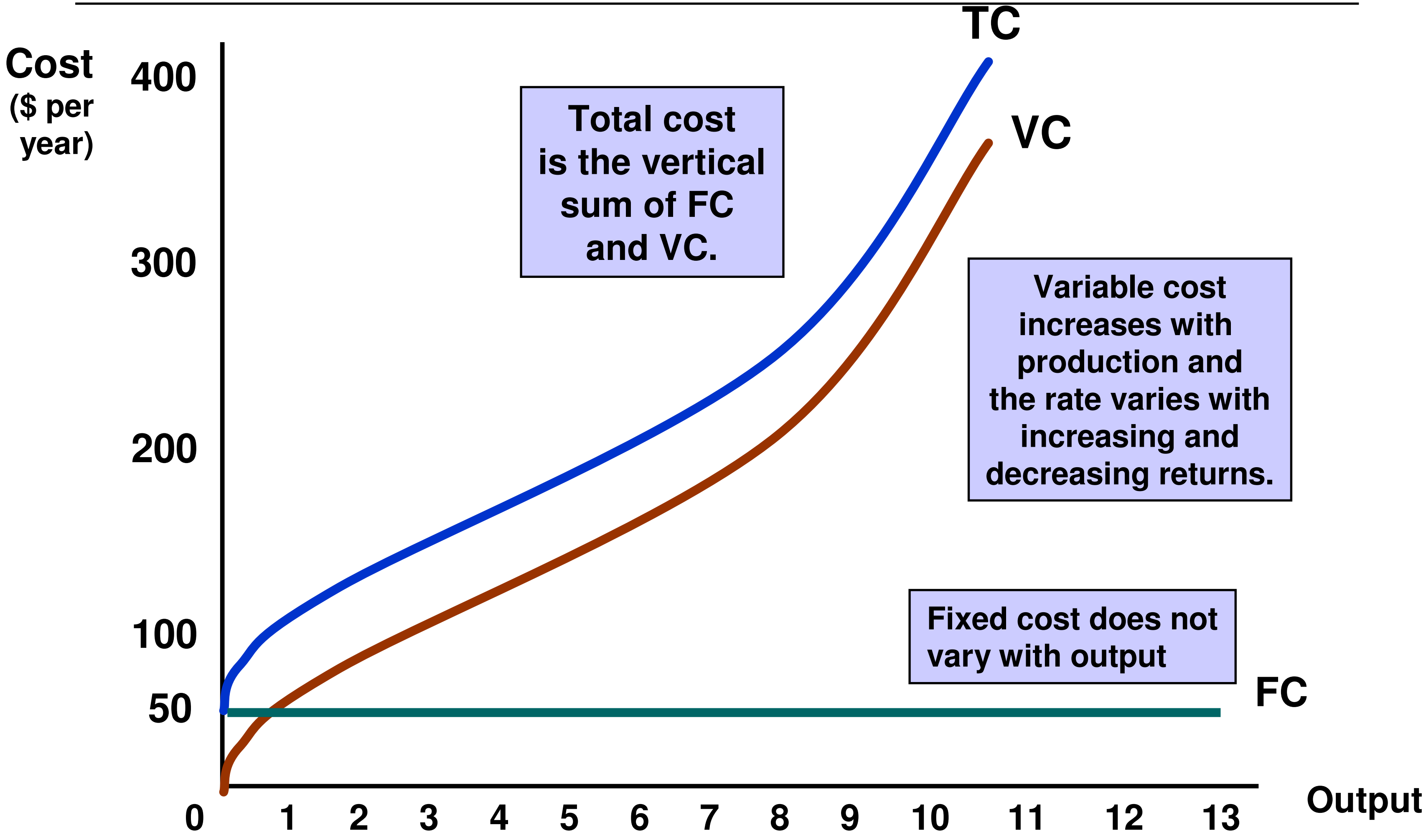
Determinants of Short Run Costs

- Consequently (from the table):
  - MC decreases initially with increasing returns
    - 0 through 4 units of output
  - MC increases with decreasing returns
    - 5 through 11 units of output

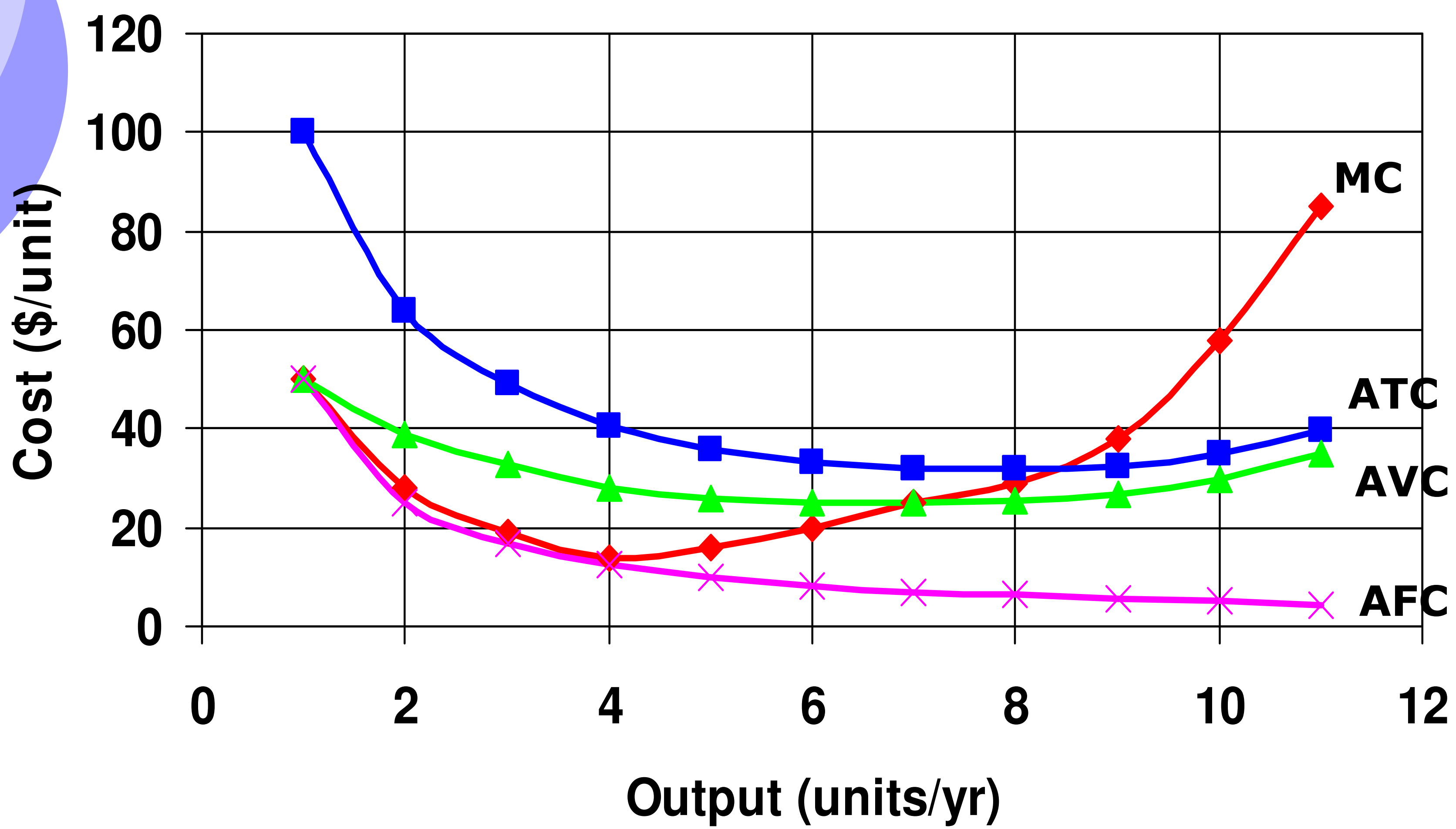
Cost Curves

- The following figures illustrate how various cost measures change as outputs change
- Curves based on the information in table 7.1 discussed earlier

Cost Curves for a Firm



Cost Curves



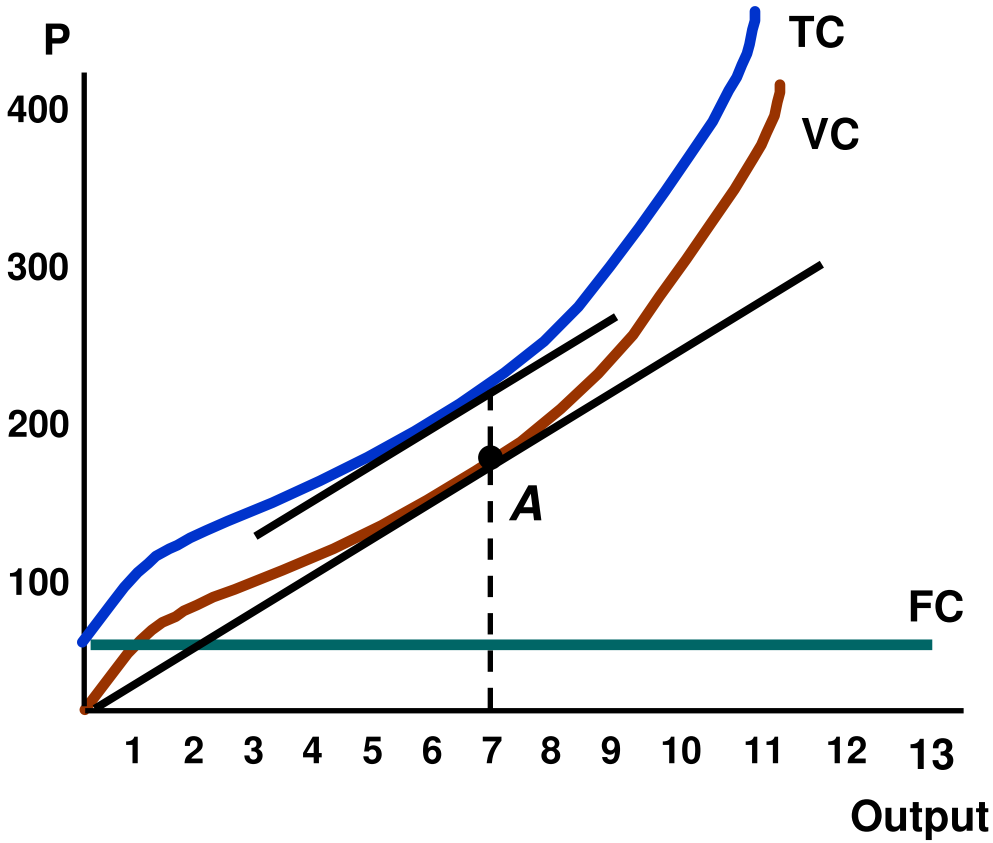
120
100
80
60
40
20
0
Cost ($/unit)
0
2
4
6
8
10
12
Output (units/yr)
MC
ATC
AVC
AFC

Cost Curves

- When MC is below AVC, AVC is falling
- When MC is above AVC, AVC is rising
- When MC is below ATC, ATC is falling
- When MC is above ATC, ATC is rising
- Therefore, MC crosses AVC and ATC at the minimums
  - The Average – Marginal relationship

Cost Curves for a Firm

- The line drawn from the origin to the variable cost curve:
  - Its slope equals AVC
  - The slope of a point on VC or TC equals MC
  - Therefore, MC = AVC at 7 units of output (point A)



Cost in the Long Run

- In the long run a firm can change all of its inputs
- In making cost minimizing choices, must look at the cost of using capital and labor in production decisions

Cost in the Long Run

- Capital is either rented/leased or purchased
  - We will consider capital rented as if it were purchased
- Assume Delta is considering purchasing an airplane for $150 million
  - Plane lasts for 30 years
  - $5 million per year – economic depreciation for the plane

Cost in the Long Run

- Delta needs to compare its revenues and costs on an annual basis
- If the firm had not purchased the plane, it would have earned interest on the $150 million
- Forgone interest is an opportunity cost that must be considered

User Cost of Capital

- The user cost of capital must be considered
  - The annual cost of owning and using the airplane instead of selling or never buying it
  - Sum of the economic depreciation and the interest (the financial return) that could have been earned had the money been invested elsewhere

Cost in the Long Run

- User Cost of Capital = Economic Depreciation + (Interest Rate)*(Value of Capital)
- = $5 mil + (.10)($150 mil – depreciation)
  - Year 1 = $5 million + (.10)($150 million) = $20 million
  - Year 10 = $5 million +(.10)($100 million) = $15 million

Cost in the Long Run

- User cost can also be described as:
  - Rate per dollar of capital, r
  - r = Depreciation Rate + Interest Rate
- In our example, depreciation rate was 3.33% and interest was 10%, so
  - r = 3.33% + 10% = 13.33%

Cost Minimizing Input Choice

- How do we put all this together to select inputs to produce a given output at minimum cost?
- Assumptions
  - Two Inputs: Labor (L) and capital (K)
  - Price of labor: wage rate (w)
  - The price of capital
    - r = depreciation rate + interest rate
    - Or rental rate if not purchasing
    - These are equal in a competitive capital market

Cost in the Long Run

- The Isocost Line
  - A line showing all combinations of L & K that can be purchased for the same cost
  - Total cost of production is sum of firm's labor cost, wL, and its capital cost, rK:

$$C = wL + rK$$

  - For each different level of cost, the equation shows another isocost line

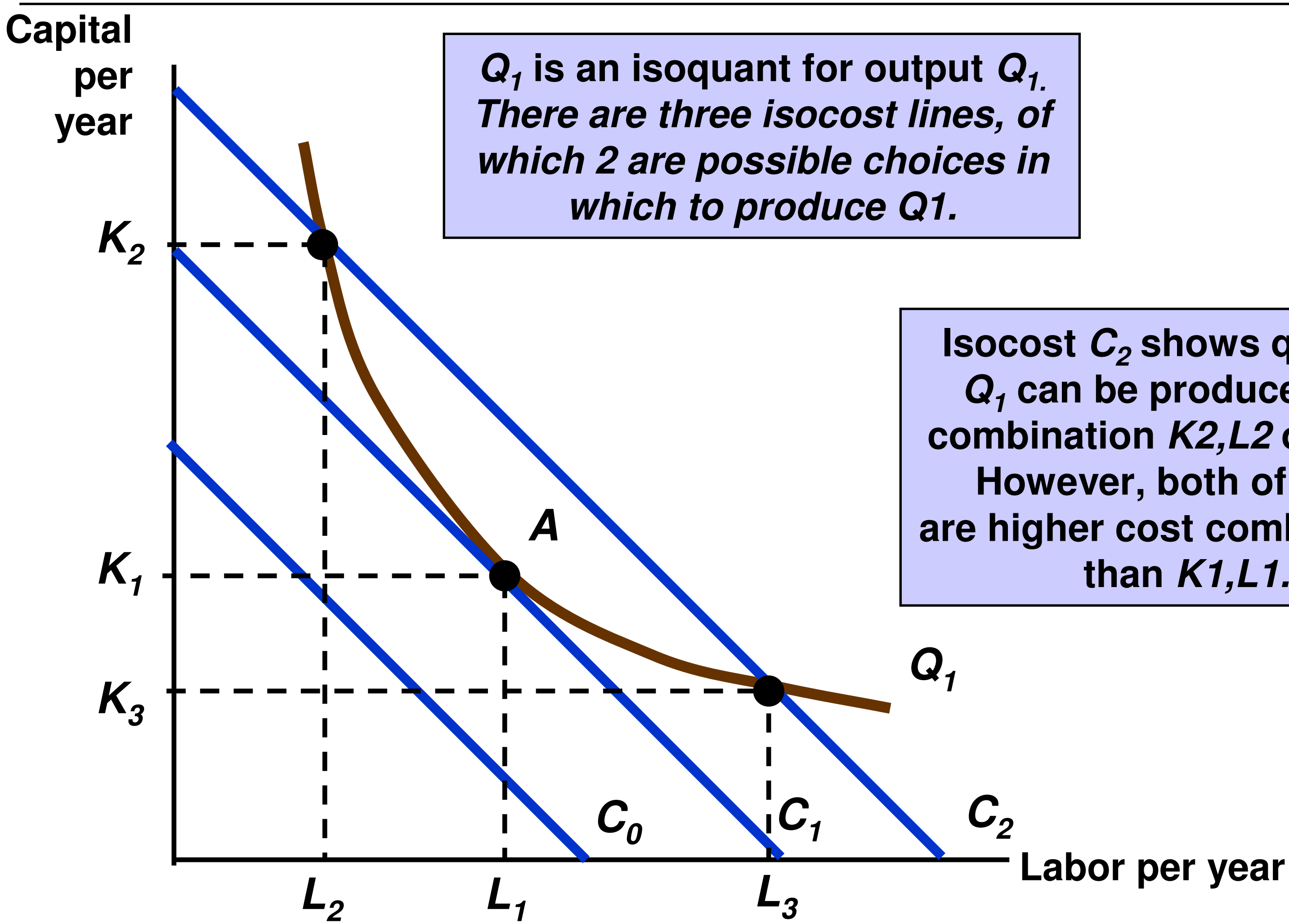
Cost in the Long Run

- Rewriting C as an equation for a straight line:
  - $K = C/r - (w/r)L$
  - Slope of the isocost: $\Delta K / \Delta L = -(w/r)$
    - -(w/r) is the ratio of the wage rate to rental cost of capital.
    - This shows the rate at which capital can be substituted for labor with no change in cost

Choosing Inputs

- We will address how to minimize cost for a given level of output by combining isocosts with isoquants
- We choose the output we wish to produce and then determine how to do that at minimum cost
  - Isoquant is the quantity we wish to produce
  - Isocost is the combination of K and L that gives a set cost

Producing a Given Output at Minimum Cost



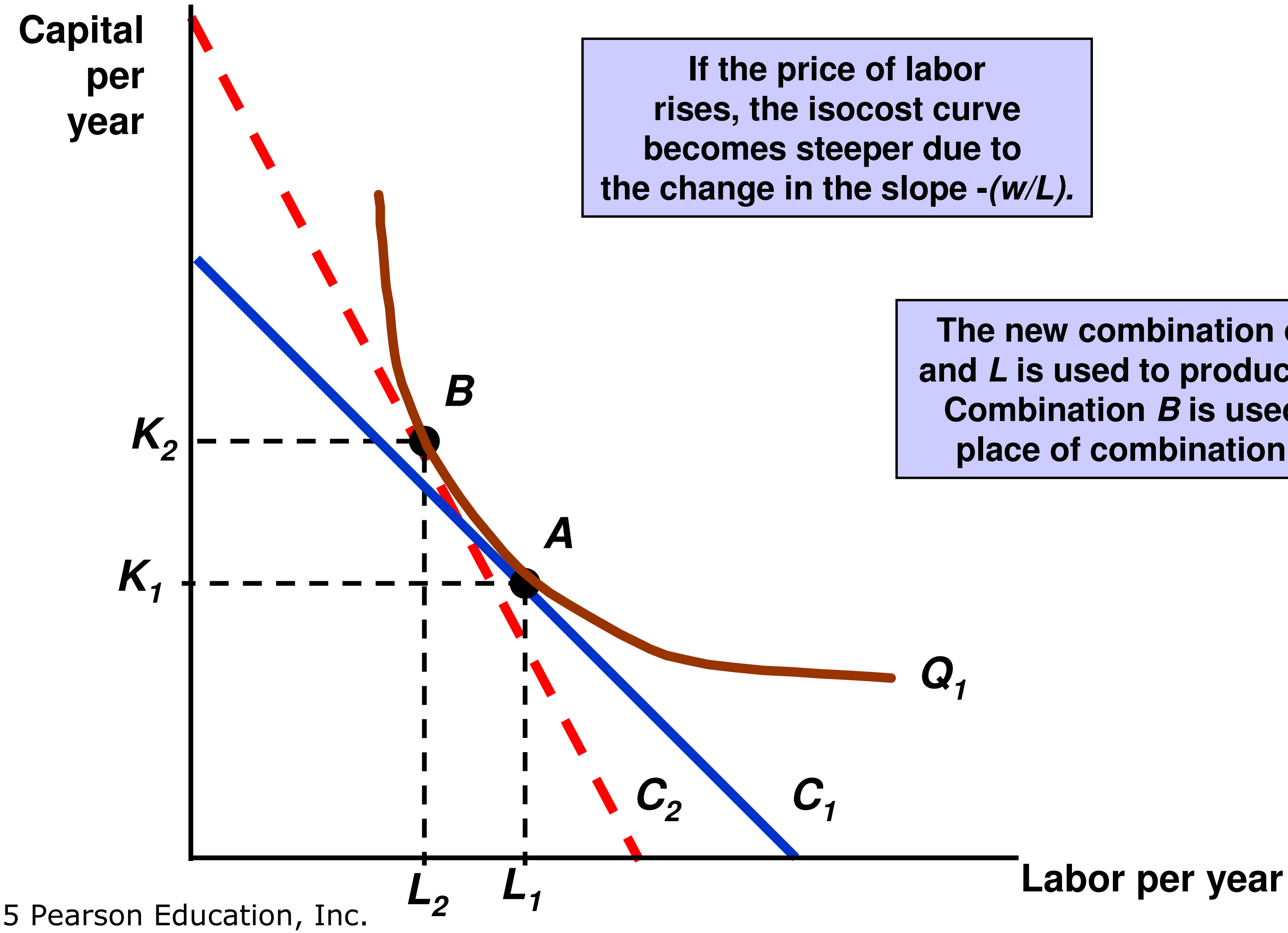
Q_1 is an isoquant for output Q_1. There are three isocost lines, of which 2 are possible choices in which to produce Q1.

Isocost C_2 shows quantity Q_1 can be produced with combination *K2,L2* or *K3,L3.* However, both of these are higher cost combinations than *K1,L1.*

Input Substitution When an Input Price Change

- If the price of labor changes, then the slope of the isocost line changes, $-(w/r)$
- It now takes a new quantity of labor and capital to produce the output
- If price of labor increases relative to price of capital, and capital is substituted for labor

Input Substitution When an Input Price Change



If the price of labor rises, the isocost curve becomes steeper due to the change in the slope -*(w/L).*

The new combination of *K* and *L* is used to produce Q_1. Combination *B* is used in place of combination *A.*

Cost in the Long Run

- How does the isocost line relate to the firm's production process?

$$\text{MRTS} = -\Delta K/\Delta L = -\text{MP}_\text{L}/\text{MP}_\text{K}$$

$$\text{Slope of isocost line} = \Delta K/\Delta L = -w/r$$

$$MP_L/MP_K = w/r \quad \text{when firm minimizes cost}$$

Cost in the Long Run

- The minimum cost combination can then be written as:

$$MP_L / w = MP_K / r$$

  - Minimum cost for a given output will occur when each dollar of input added to the production process will add an equivalent amount of output.

Cost in the Long Run

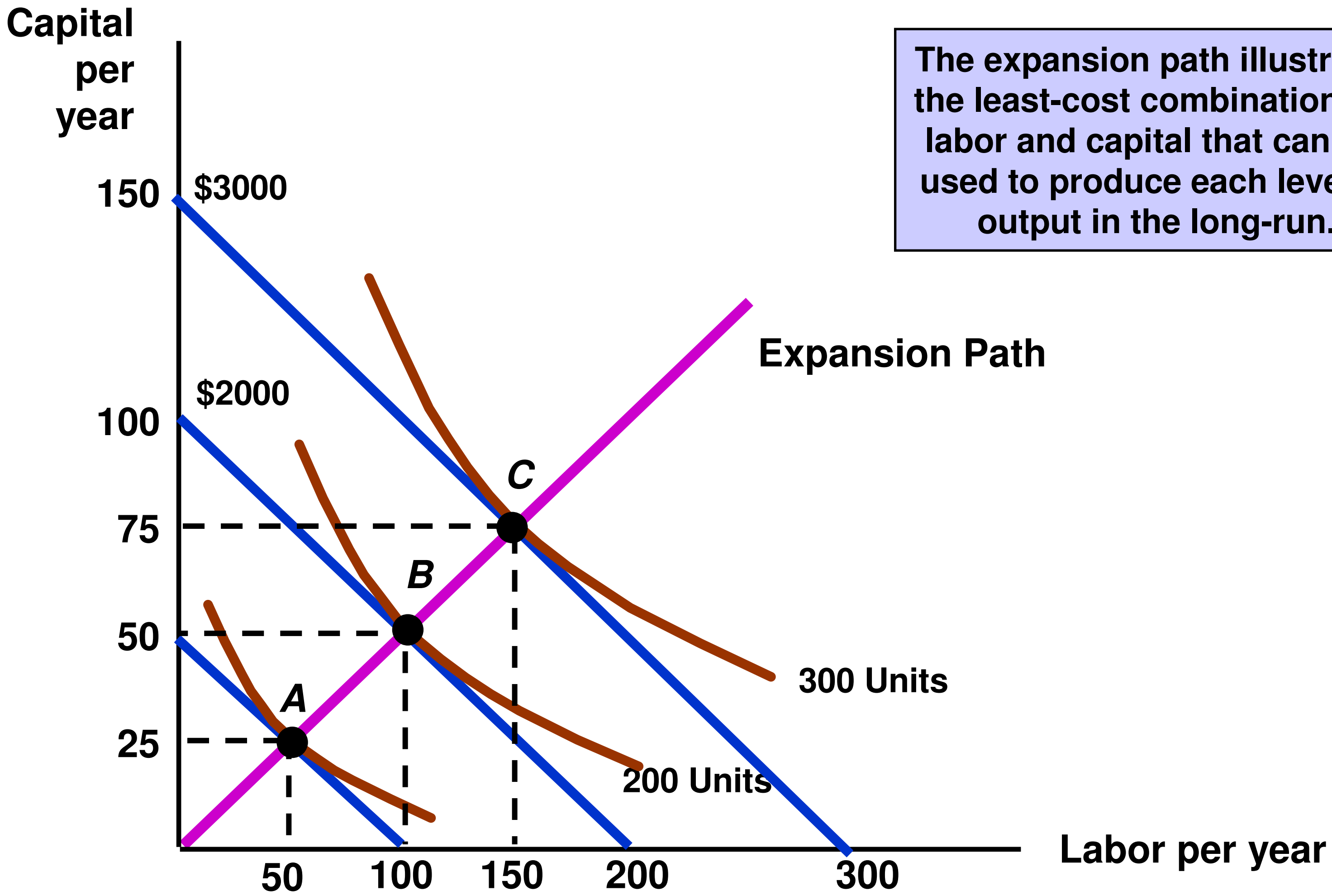
- If w = \$10, r = \$2, and $MP_L = MP_K$, which input would the producer use more of?
  - Labor because it is cheaper
  - Increasing labor lowers MP_L
  - Decreasing capital raises MP_K
  - Substitute labor for capital until

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Cost in the Long Run

- Cost minimization with Varying Output Levels
  - For each level of output, there is an isocost curve showing minimum cost for that output level
  - A firm's **expansion path** shows the minimum cost combinations of labor and capital at each level of output
  - Slope equals $\Delta K/\Delta L$

A Firm's Expansion Path

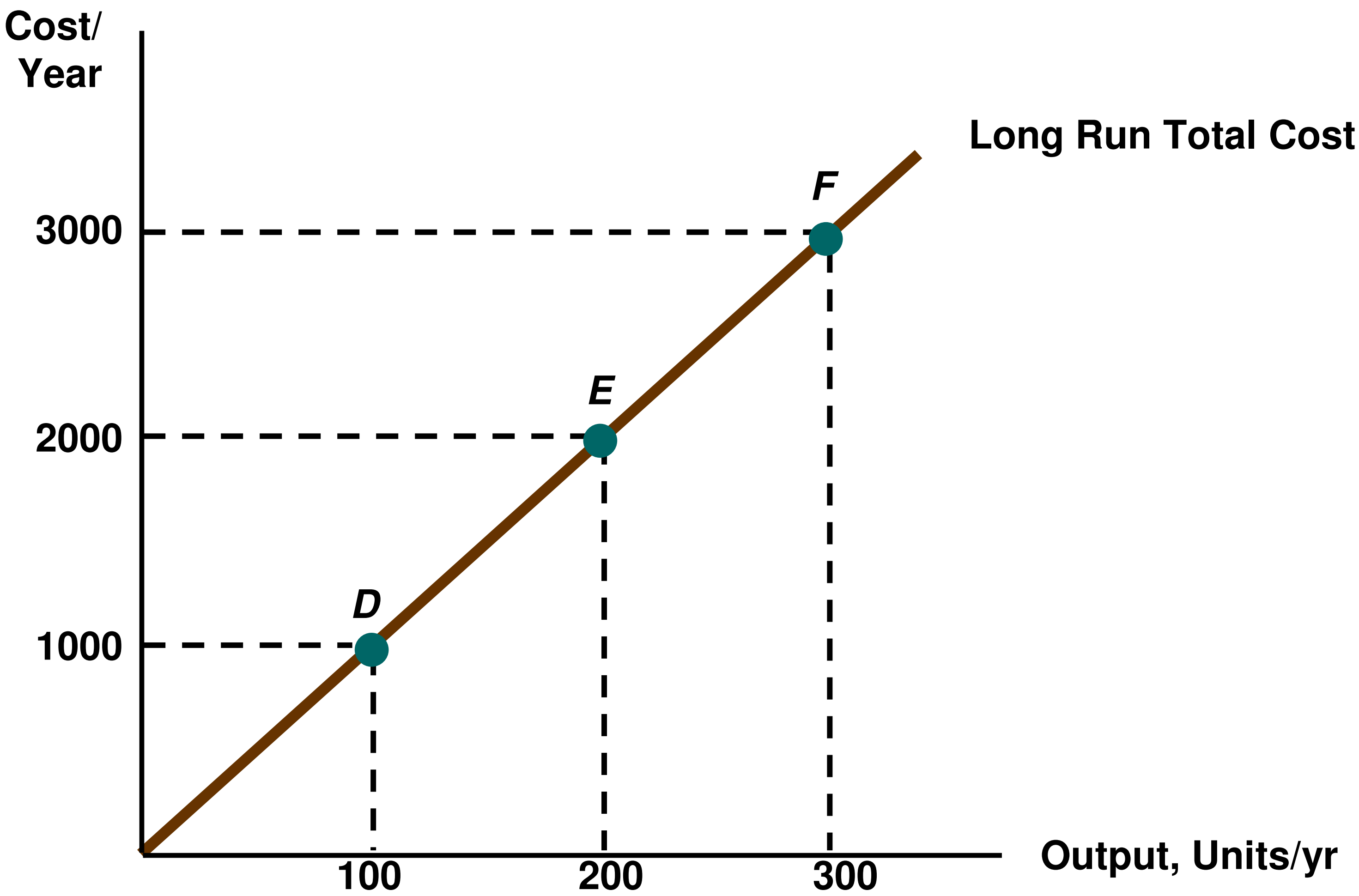


The expansion path illustrates the least-cost combinations of labor and capital that can be used to produce each level of output in the long-run.

Expansion Path and Long Run Costs

- Firm's expansion path has same information as long-run total cost curve
- To move from expansion path to LR cost curve
  - Find tangency with isoquant and isocost
  - Determine min cost of producing the output level selected
  - Graph output-cost combination

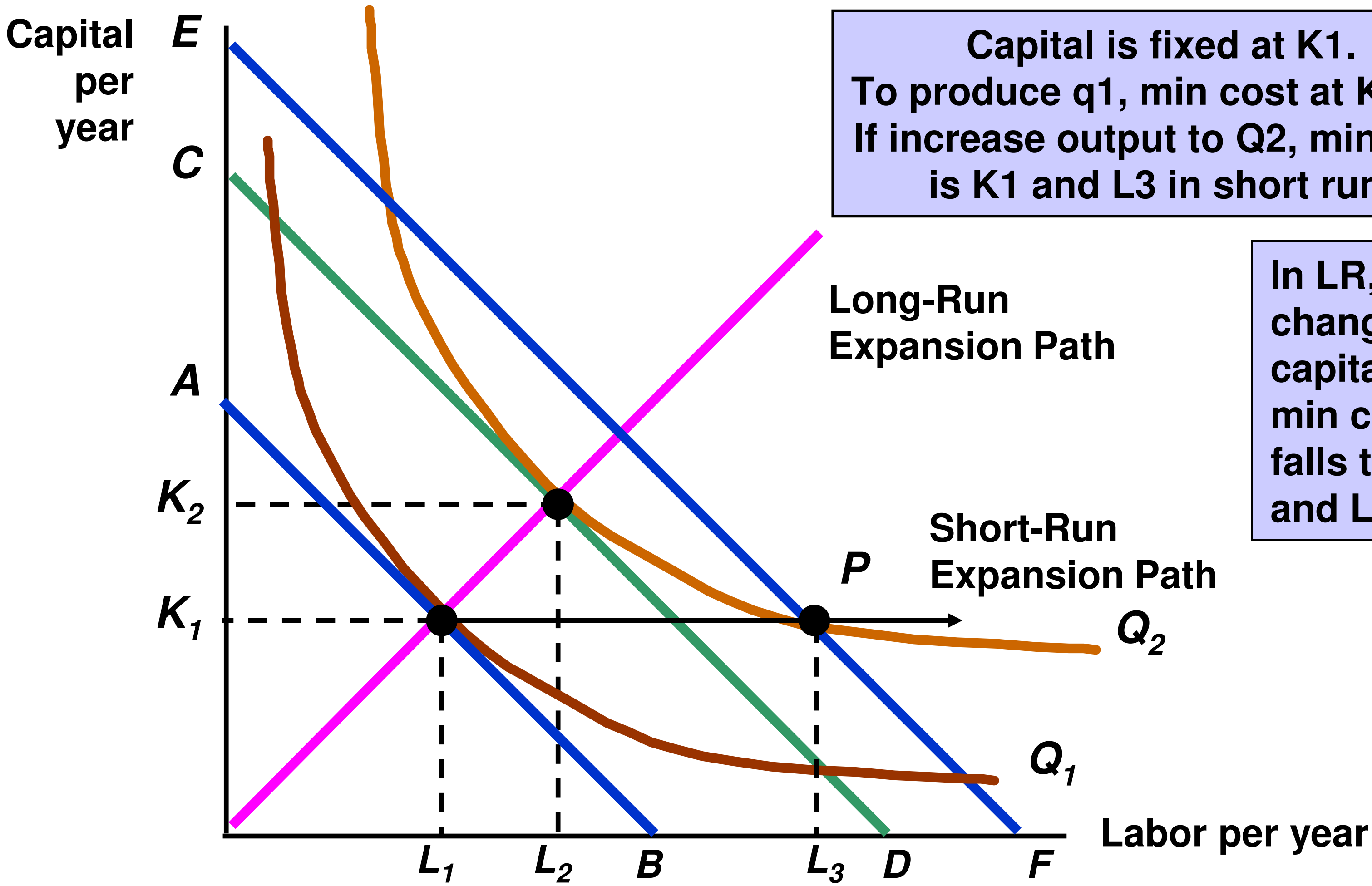
A Firm's Long Run Total Cost Curve



Long Run Versus Short Run Cost Curves

- In the short run, some costs are fixed
- In the long run, firm can change anything including plant size
  - Can produce at a lower average cost in long run than in short run
  - Capital and labor are both flexible
- We can show this by holding capital fixed in the short run and flexible in long run

The Inflexibility of Short Run Production



Capital is fixed at K1.
To produce q1, min cost at K1,L1.
If increase output to Q2, min cost is K1 and L3 in short run.

In LR, can change capital and min costs falls to K2 and L2.

Long Run Versus Short Run Cost Curves

- Long-Run Average Cost (LAC)
  - Most important determinant of the shape of the LR AC and MC curves is relationship between scale of the firm's operation and inputs required to minimize cost

1. Constant Returns to Scale
   - If input is doubled, output will double
   - AC cost is constant at all levels of output

Long Run Versus Short Run Cost Curves

2. Increasing Returns to Scale
   - If input is doubled, output will more than double
   - AC decreases at all levels of output
3. Decreasing Returns to Scale
   - If input is doubled, output will less than double
   - AC increases at all levels of output

Long Run Versus Short Run Cost Curves

- In the long run:
  - Firms experience increasing and decreasing returns to scale and therefore long-run average cost is "U" shaped.
  - Source of U-shape is due to returns to scale instead of decreasing returns to scale like the short-run curve
  - Long-run marginal cost curve measures the change in long-run total costs as output is increased by 1 unit

Long Run Versus Short Run Cost Curves

- Long-run marginal cost leads long-run average cost:
  - If LMC < LAC, LAC will fall
  - If LMC > LAC, LAC will rise
  - Therefore, LMC = LAC at the minimum of LAC
- In special case where LAC is constant, LAC and LMC are equal

Long Run Average and Marginal Cost