

MICROECONOMICS 2009-2010

PRODUCTION THEORY

1. - Consider that a firm in order to produce a unit of good q knows just a few productive processes (summarized in the following table):

Process	Input 1	Input 1	Output
A	4	3	1
B	5	2	1
C	3	4	1
D	1	8	1
E	3	3	1

- (a) Which of these processes are technically efficient and which are inefficient?
 (b) Combine different processes in order to obtain the unitary isoquant. Interpret the shape of the isoquant. Which processes and combinations of processes are not relevant to determine the isoquant.

2. Suppose there is a firm that produces a consumption good which faces the technology $q = f(K,L) = 5L^{1/2}K^{1/3}$. Let us fix the capital at $K = 8$ to do a short run analysis.

- a) Derive and draw the production function
 b) Derive and draw the Marginal product and the Average product functions. Does any relationship exist between the shape of these curves and the production function?
 c) Get the "conditional factor demand" of labor and the cost function.

Definition: The conditional factor demand for L is the cost minimizing amount of L purchased by a firm, q and K held constant.

- d) Compute and draw the variable cost, average cost, average variable cost, average fixed cost and marginal cost functions.
 e) Check that the expected relationship between cost functions hold.
 f) How is the production function related to the cost-functions?

3.- Consider the following Cobb-Douglas utility function $q = f(L,K) = 5L^a K^b$ being $a, b > 0$. Analyze the returns to scale depending on the values a and b . Is there any relationship between the returns to scale of the production function and the shape of the long run cost curve and the long run average cost curve?

4.- Firm's production is given by the following production's function:

Where q is the level of production and (l,k) represent units of labor and capital, respectively.

- (a) Interpret technology
 (b) Analyze Returns to scale
 (c) Determine the combination of factors that minimize the cost for any level of production q . Show that the cost of production is proportional to the level of production q , state why this is so.
 (d) Using the previous analysis, if the firm has to produce $q=1000$, and prices for inputs are $w=r=1$, find the demand of inputs and the cost of production. What if $r=3 > 1=w$?

5.- If prices for inputs (x_1, x_2) are $(4, 1)$ respectively, compute the minimum cost of producing one unit of output if

- (a) $f(x_1, x_2) = ax_1 + bx_2$ being a and $b > 0$.
 (b) $f(x_1, x_2) = \min\{x_1/a, x_2/b\}$, being a y $b > 0$.

6. You manage a plant that mass produces engines by teams of workers using assembly machines. The technology is summarized by the production function $q = f(x_1, x_2) = x_1x_2$, where q is the number of engines per week, x_1 is the number of assembly machines, and x_2 is the number of labor teams. Each assembly machine rent for 25€ per week and each team costs $w=100€$ / month. The weekly production-plan forecast a production of 100 engines per week.

- a) Your boss asked you to revise the proposal of signing a contract that he intends to present to the CEO (Chief Executive Officer). This proposal states the necessity of hiring 20 teams per week. Bearing in mind that the plant has a fixed stock of 10 machines already installed - and that the target of the firm is to minimize costs-, do you agree with your boss' proposal? Justify.
- b) Your boss also thinks that, should the provision of all production factors be variable, the minimum cost of production does not change. He asked you to write a report analyzing his own idea. Which would be the main results of your report? Justify
- c) The firm CEO plans to increase the weekly production plan. Assuming that both inputs are variable, how would the average cost evolve?

7.- Assume the following Cobb-Douglas production function:

$$f(x_1, x_2) = x_1^{1/3} x_2^{3/2}$$

- (a) Find the marginal product of x_1
 (b) For small increases in x_1 , keeping fixed the second input, is the marginal product increasing, decreasing or constant?
 (c) For small increases in x_2 , the marginal product of the second input is _____, and keeping fixed the first input, is the marginal product increasing, decreasing or constant?
 (d) Given an increase in the quantity of the second input, which is the effect on the marginal product of the first input?
 (e) Find the MRTS
 (f) Does this technology exhibit a decreasing MRTS?
 (g) Find the RTS of this technology

8. The following production function describes how billiard boards are produced:

$$f(K, T) = T/2 + K^{1/2}, \text{ where}$$

T represents labor and K the level of capital employed

- (a) Analyze RTS
 (b) The marginal productivity of labor is _____ (increasing, decreasing or constant)?
 (c) In the short run, the level of capital is 4 units, whereas the labor is variable. Represent the production function against labor in the short run. Do the same for the marginal productivity of labor.
 (d) Represent graphically the short-run average product against labor employed.

9. - The firm Bender PLC owns two different plants in order to produce ukeleles (most famous product). The first plant is located in Jamaica and the other in Ghana. The production functions differ depending on the country, that is to say, $f_M(x_1, x_2) = \min\{x_1, 2x_2\}$ for the Jamaican plant, and $f_G(x_1, x_2) = \min\{2x_1, x_2\}$, for the Ghana plant. x_1, x_2 are the quantities used of input 1 and input 2.

- (a) Draw the isoquants associated to produce 4000 ukeleles in Jamaica and in Ghana.
- (b) If the firm decides to produce 2000 ukeleles in each factory, which is the quantity of inputs needed in Jamaica? And in Ghana?
- (c) Show graphically the quantity of both inputs needed to produce 4000 ukeleles.
- (d) Represent graphically the isoquant to produce 4000 ukeleles and distribute the production between both factories in the most profitable way. Is the firm's technology convex?

10.- A firm is producing a good using two inputs through the following technology $q = f(L, K) = L^{1/6} K^{1/3}$. Assume that both inputs are variable and prices are $w=r=1$, respectively.

- (a) Compute the expansion path, the input demand of both inputs and the cost function. Show it graphically
- (b) Obtain and show graphically both the average and the marginal cost functions

11.- A firm is producing a good using two inputs through the following technology $q = f(L, K) = \min\{L, 2K\}$. Assume that both inputs are variable and prices are $w=1 < r=2$, respectively.

- (a) Compute the expansion path, the input demand of both inputs and the cost function. Show it graphically
- (b) Obtain and show graphically both the average and the marginal cost functions