Chapter 1: LABOUR SUPPLY

J. Ignacio García Pérez

Universidad Pablo de Olavide - Department of Economics

BASIC REFERENCE: Cahuc & Zylberberg (2004), Chapter 1
In this chapter we will see:

- How people make choices between consumption and leisure.
- What the reservation wage is.
- How the shape of the labour supply results from the combination of substitution and income effects.
- What are the wage elasticities of labour supply.
- Examples of natural experiments to evaluate labour supply policies.
- The principles guiding labour supply econometrics.
INTRODUCTION

In 1900, in the US prime-age men used to work 50 hours per week, while prime-age women worked only 8 hours (many were not employed at all) but did 50 hours of unpaid home work compared with only 4 hours for men.

In 2005, the situation changed dramatically: prime-age men worked on average 37 hours and did more at home (17 hours per week), while women were employed for 26 hours and worked at home for 31 hours.

This change is the result of the choices made by every single working-age person regarding work hours, home duties and leisure.
To hold a paid job, you must first have decided to do so. This is the starting point of the so-called “neo-classical” theory of the labour supply.

It posits that each individual disposes of a limited amount of time and which chooses to allocate between paid work and leisure.

Evidently the wage an individual can demand constitutes an important factor in the choice but income derived from sources outside the labour market, and even the familial environment, also play a decisive role.
But the counterpart of paid work is not simply leisure, for much of it consists of time devoted to “household production”.

The supply of wage labour takes into account the costs and benefits of this household production, and that most often it is the result of planning, and even actual negotiation, within the family.

The family situation, the number of children, the income a person enjoys apart from any wage labour (personal wealth, illegal work, spousal income, etc.), all weigh heavily in this choice.
Empirical studies on labour supply have multiplied in the course of the last thirty years.

Their development has benefited from advances made in the application of econometric methods to individual data.

It has also been driven by a need to evaluate public policies that attempt to influence labour supply directly, such as tax and benefit systems.

Another motivation is the need to analyse fluctuations of employment over the economic cycle, which depends on how labour supply reacts to changes in wages.
Some facts about labour supply

Some basic definitions

- The **labour force** is made up of all persons who are either employed (whatever the duration or type of work) or looking for a job (i.e. the unemployed).

- To be considered **unemployed**, according to the ILO definition, people must be (i) not in paid employment or self-employment, (ii) currently available for work, and (iii) seeking work.

- The **participation rate (or activity rate)** is the ratio of the labour force to a reference population (the working-age (15-64) population).

- The **employment rate** is the ratio of the number of employed people to the working-age population.
The trend in the amount of time worked

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The evolution of participation rates (MALES)

The participation rate of men has markedly diminished since the beginning of the mid-1950s.
The evolution of participation rates (FEMALES)

The participation rate of women has not stopped growing over the same period.
On the contrary, females are increasing the amount of time they spend at work.
The effect of taxes on annual hours

The correlation between annual hours worked and the rate of taxation on income earned per year of work is -0.6.
1. PREFERENCES

This theory is grounded on the model of a consumer making a choice between consuming more goods and consuming more leisure.

We define **leisure** as time not spent at work, $L$.

Each individual disposes of a total amount of time, $L_0$, so the length of time worked is given by $h = L_0 - L$.

**PREFERENCES:** The tradeoff between consumption and leisure is shown with the help of a **utility function** proper to each individual: $U(C, L)$.

We will assume that an individual desires to consume the greatest possible quantity of goods and leisure.
1. PREFERENCES

The set of pairs $(C, L)$ by which the consumer obtains the same level of utility $\overline{U}$, i.e. such that $U(C, L) = \overline{U}$, is called an indifference curve.
2. PROPERTIES OF THE INDIFFERENCE CURVE

1. Each indifference curve corresponds to a higher level of utility, the farther out the curve is from the origin.

2. Indifference curves do not intersect.

3. Indifference curves are negatively sloped: their slope defines the marginal rate of substitution between consumption and leisure.

4. Indifference curves are convex.
3. CHOICES

An individual’s income derives from his activity as wage-earner, and from his activity (or inactivity) outside the labour market.

If we designate the real hourly wage by $w$, the income from wages totals $wh$.

Investment income, transfer income, etc. will be expressed in real terms by the single scalar $R$.

Thus the budget constraint of the agent takes the form:

\[ C \leq wh + R \]
3. CHOICES

This constraint is also expressed in the following manner:

\[ C + wL \leq R_0 \equiv wL_0 + R \]  

We can see here that the wage represents the \textit{opportunity cost} of leisure.

Thus, the program of the consumer is expressed:

\[ \max \{C,L\} \quad U(C,L) \quad \text{s.t.} \quad C + wL \leq R_0 \]

We begin by studying the so-called “interior” solutions, such as \( 0 < L < L_0 \) and \( C > 0 \).
3. CHOICES

Using $\mu \geq 0$ to denote the Lagrange multiplier associated with the budget constraint, the Lagrangian of this program is

$$(4) \quad \mathcal{L}(C, L, \mu) = U(C, L) + \mu (R_0 - C - wL)$$

Designating the partial derivatives of the function $U$ by $U_L$ and $U_C$, the first-order conditions are expressed as:

$$(5) \quad U_C(C, L) - \mu = 0 \quad \text{and} \quad U_L(C, L) - \mu w = 0$$

The solution is situated on the budget line of equation $C + wL = R_0$. 
3. CHOICES

It verifies that:

\[
\frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w \quad \text{and} \quad C^* + wL^* = R_0
\]
4. THE RESERVATION WAGE

- An agent offers a strictly positive quantity of hours of work if and only if the following condition is met:

\[
\left( \frac{U_L}{U_C} \right)_A < w
\]  

(7)

- The marginal rate of substitution at point \( A \) is called the *reservation wage*. It is thus defined by:

\[
w_A = \frac{U_L(R, L_0)}{U_C(R, L_0)}
\]  

(8)
4. THE RESERVATION WAGE

- The reservation wage depends only on the form of the function $U$ at point $A$ and on the value $R$ of non-earned income.

- It determines the conditions of *participation* in the labour market.

- It rises with $R$ if, and only if, leisure is a *normal* good.

- An increase in non-earned income increases the reservation wage, and thus has a *disincentive* effect on entry into the labour market.
5. LABOUR SUPPLY PROPERTIES

- These properties result from the combination of a substitution effect and two income effects.

- The combination of these effects seemingly leads to a non-monotonic relation between wages and the individual supply of labour.

- For an interior solution, the demand for leisure $L^*$ is implicitly defined by relations (6) and can be written in the form $L^* = \Lambda(w, R_0)$.

- The corresponding labour supply, i.e. $h^* = L_0 - L^*$, is often called the “Marshallian” or “uncompensated” labour supply.
5. LABOUR SUPPLY PROPERTIES: INCOME EFFECT

- The impact on leisure of an increase in non-earned income $R$ is given by the partial derivative of the function $\Lambda(w, R_0)$ with respect to its second argument, i.e. $\Lambda_2(w, R_0)$.

- If leisure is a *normal good*, this derivative is positive.

- The consequences of this increase are represented in the previous figure by the shift from point $E$ to point $E'$. 
5. LABOUR SUPPLY PROPERTIES: SUBSTITUTION EFFECT

- The impact of a variation in wages is obtained by differentiating function $\Lambda(w, R_0)$ with respect to $w$.

- Taking account the fact that $R_0 = wL_0 + R$, we arrive at:

\[
\frac{dL^*}{dw} = \Lambda_1 + \Lambda_2 \frac{\partial R_0}{\partial w} \quad \text{with} \quad \frac{\partial R_0}{\partial w} = L_0 > 0
\]
5. LABOUR SUPPLY PROPERTIES: SUBSTITUTION EFFECT
5. LABOUR SUPPLY PROPERTIES: SUBSTITUTION EFFECT

- The partial derivative of the function $\Lambda$ with respect to $w$ corresponds to the usual compound of substitution and income effects in the theory of the consumer.

- To learn the sign of this derivative, it is best to reason in two stages.

- In the first stage, we suppose that the potential income $R_0$ does not change: the consumer then faces a new budget line $A_1R_0$.

- In the second stage, we assume that the potential income grows from $R_0$ to $R_1 = R + w_1L_0$. 
5. LABOUR SUPPLY PROPERTIES: SUBSTITUTION EFFECT

In sum, a wage increase has an ambivalent effect on labour supply.

For convenience, we can aggregate the two income effects by retaining only the shift from $E'$ to $E_1$: the global income effect.

This allows us to analyze a rise in the hourly wage with the help of only two effects.

1. There is an incentive to increase labour supply, since this factor is better remunerated (the substitution effect).

2. And there is an opportunity to consume the same quantity of goods while working less, which motivates a decrease of labour supply (the global income effect).
6. COMPENSATED AND NON-COMPENSATED ELASTICITIES

Along with the Marshallian supply of labour $h^*$ considered to this point, we can also make use of the Hicksian supply of labour.

This is obtained by minimizing the consumer’s expenditure, given an exogenous minimal level of utility $\bar{U}$:

$$\text{Min}_{(L,C)} C + wL \quad \text{s.t.} \quad U(C, L) \geq \bar{U}$$
6. COMPENSATED AND NON-COMPENSATED ELASTICITIES

The **Hicksian elasticity** of labour supply, defined by
\[
\eta_H = \left( \frac{w}{\hat{h}} \right) \left( \frac{d\hat{h}}{dw} \right),
\]
represents the percentage of variation of the Hicksian supply of labour that follows from a 1% rise in wage.

It is called “compensated” elasticity because it posits that the income of the consumer varies in order for him to stay on the same indifference curve.
6. COMPENSATED AND NON-COMPENSATED ELASTICITIES

The **Marshallian elasticity** of labour supply defined by

\[
\eta_M = \left( \frac{w}{h^*} \right) \frac{dh^*}{dw},
\]

represents the percentage of variation of the Marshallian supply of labour that follows from a 1% rise in wage.

It is also called **non-compensated elasticity** because it takes into account the real variation in income resulting from the variation in wages.
6. COMPENSATED AND NON-COMPENSATED ELASTICITIES

Marshallian and Hicksian elasticities are linked by the Slutsky equation, which is written as:

\[ \eta_M = \eta_H + \frac{wh^*}{R_0} \eta R_0 \]  

The Slutsky equation shows that Marshallian elasticity is to be interpreted as the sum of two effects.

1. The substitution effect, represented by the Hicksian elasticity \( \eta_H \), which is necessarily negative.
2. The (global) income effect, represented by the term \( \frac{wh^*}{R_0} \eta R_0 \), which is positive if leisure is a normal good.
7. THE SHAPE OF THE LABOUR SUPPLY CURVE

- We can now offer a plausible graph of labour supply.
- When the hourly wage rises just above the reservation wage, the substitution effect prevails over income effects, and labour supply grows.
- But the global income effect swells with the wage, and it is reasonable to believe that when the latter reaches a certain level it will dominate the substitution effect.
- The supply of labour then begins to shrink.
7. THE SHAPE OF THE LABOUR SUPPLY CURVE
8. SUPPLEMENTARY CONSTRAINTS

- The budget constraint is actually piecewise linear, since on the one hand overtime hours are not remunerated at the same rate as normal ones, and on the other hand income tax is progressive.

- This constraint may even present non-convexities related to the ceilings on various social security contributions.

- All these elements pose serious problems for empirical assessment.
8. SUPPLEMENTARY CONSTRAINTS

Another element that may alter the foregoing analysis comes from the relative absence of freedom of choice in the number of hours worked.

There are full-time or part-time jobs but the reality is always far from the complete flexibility in hours assumed here.

The agent may well work more or less than she would have wished to.

We can see also a situation of “involuntary non-participation,” when the worker wishes to supply a certain quantity of work at the current wage but faces constraints that keep her from supplying them.
8. SUPPLEMENTARY CONSTRAINTS
9. EXTENSIVE & INTENSIVE MARGINS: AGGREGATE LABOUR SUPPLY

We arrive at the aggregate labour supply, for a wage level of $w$, by adding up the total number of hours supplied by each individual.

The wage exerts two distinct effects on labour supply.

1. It influences the decision to work or not; this is called the **extensive margin**.
2. It determines the number of hours supplied; this is called the **intensive margin**.
9. EXTENSIVE & INTENSIVE MARGINS: AGGREGATE LABOUR SUPPLY

- Let us suppose that leisure is a normal good, such that the supply of labour, denoted \( h(w, R) \), is a decreasing function of non-earned income.

- Let us suppose also that individuals have different non-earned incomes \( R \in [0, +\infty[ \), with a cumulative distribution function \( \Phi(\cdot) \).

- For every wage level \( w \), there then exists a positive value of \( R \), denoted \( \bar{R} \) and defined by \( h(w, \bar{R}) = 0 \), such that only individuals whose non-earned income is inferior to \( \bar{R} \) work.

- The others do not work, since their reservation wage is superior to \( w \).
The neo-classical theory of labour supply

9. EXTENSIVE & INTENSIVE MARGINS: AGGREGATE LABOUR SUPPLY

- If the size of the total population is normalized to unity, the aggregate labour supply is:

\[ L_A(w) = \int_0^\bar{R} h(w, R) d\Phi(R). \] (12)

- The derivative of the aggregate labour supply with respect to the wage \( w \) is

\[ \int_0^\bar{R} \frac{\partial h(w, R)}{\partial w} d\Phi(R) + h(w, \bar{R}) \Phi'(\bar{R}) \frac{d\bar{R}}{dw}. \] (13)
The neo-classical theory of labour supply

9. EXTENSIVE & INTENSIVE MARGINS: AGGREGATE LABOUR SUPPLY

- The first term represents the changes in the intensive margin, which can be either positive or negative, depending on the relative importance of income and substitution effects.

- The second term, which represents the changes in the extensive margin, is necessarily equal to zero to the extent that, by definition, \( h(w, \bar{R}) = 0 \).

- This would mean that small variations in wages have an impact on the aggregate supply of labour, an impact felt solely through changes at the intensive margin.

- In reality there exist, as noted above, indivisibilities in the supply of hours of work due to technological and institutional constraints.

- If persons who decide to work must supply a minimal duration of \( h_0 \) hours, then for the right-hand side of equation (13) we must substitute the term \( h_0 \Phi'(\bar{R}) \frac{d\bar{R}}{dw} \) where \( \bar{R} \) is defined by the relation \( U(\bar{R} + wh_0, L_0 - h_0) = U(\bar{R}, L_0) \).
The static models do not explain how agents substitute for their consumption of leisure over time.

In a dynamic framework, income is subject to transitory or permanent shocks.

Taking into explicit account a succession of periods does not markedly alter the conclusions of the static model,

but it does provide an adequate framework to analyze dynamic behaviors,

which is useful to understand how labor supply changes over business cycles.

Let’s see the basics of a dynamic model of labor supply.
A Dynamic model of Labour Supply

- This model gives a central role to the possibility of substituting for the consumption of physical goods and leisure over time.

- We highlight this possibility using a dynamic model in discrete time.

- In a dynamic perspective, a consumer must make his or her choices over a “life cycle”

- This will be represented by a succession of periods that start with an initial date, 0, and end with an independent terminal date, $T$.

- The succession of periods is then given by the index $t = 0, 1, 2, .., T$. 
In a very general way, the preferences of the consumer must be represented by a utility function of the form 
\[ U(C_0, C_t, \ldots, C_T; L_0, \ldots, L_t, \ldots, L_T) \].

But this very general form does not permit us to obtain analytically simple and easily interpretable results.

That is why it is often assumed that the utility function of the consumer is temporally separable, in which case it is written 
\[ \sum_{t=0}^{T} U(C_t, L_t, t) \].

Under this hypothesis, the term \( U(C_t, L_t, t) \) represents simply the utility obtained by the consumer in period \( t \).

It is sometimes called the “instantaneous” utility of the period \( t \).
In this dynamic model, we shall assume that individuals have the opportunity to save,

and we will use $r_t$ to denote the real rate of interest between the periods $t - 1$ and $t$.

For each period, the endowment of time is an independent constant to which we shall give the value 1 in order to simplify the notation.

On this basis, the hours worked during a period $t$ are equal to $\left(1 - L_t\right)$. 
If we use $A_t$ to designate the consumer’s assets on date $t$, and $B_t$ to designate his or her income apart from wages and the yield on savings on the same date, the evolution of the assets of the consumer is described by:

\[(14) \quad A_t = (1 + r_t)A_{t-1} + B_t + w_t(1 - L_t) - C_t, \quad \forall t \geq 0\]

**INTERPRETATION:** at each period $t$, the increase in wealth $A_t - A_{t-1}$ is due to income $w_t(1 - L_t)$ from wage labor, to income $r_tA_{t-1}$ from savings, and to other income $B_t$. Consumption has to be deducted from these gains.
OPTIMAL SOLUTIONS

The consumer attempts to maximize his intertemporal utility subject to the budget constraint described, on each date, by the equation (22).

If we use \( \nu_t \) to denote the multiplier associated with this equation, the Langrangian of the consumer’s problem takes the form:

\[
\mathcal{L} = \sum_{t=0}^{T} U(C_t, L_t, t) - \sum_{t=0}^{T} \nu_t [A_t - (1 + r_t)A_{t-1} - B_t - w_t(1 - L_t) + C_t]
\]
OPTIMAL SOLUTIONS

- The first-order conditions are obtained by equating the derivatives of this Langrangian to zero with respect to variables $C_t$, $L_t$ and $A_t$.

- After a few simple calculation, we arrive at:

\begin{align*}
(16) \quad U_C(C_t, L_t, t) &= \nu_t \\
(17) \quad U_L(C_t, L_t, t) &= \nu_t w_t \\
(18) \quad \nu_t &= (1 + r_{t+1})\nu_{t+1}
\end{align*}

- The first two relations imply $U_L/U_C = w_t$. 

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LABOUR ECONOMICS

J. Ignacio Garcia-Perez
A Dynamic model of Labour Supply

OPTIMAL SOLUTIONS

Limiting ourselves to interior solutions, the optimal consumptions of physical goods and leisure are implicitly written in the following manner:

\[ C_t = C(w_t, \nu_t, t) \]  
\[ L_t = L(w_t, \nu_t, t) \]  

The supply of labor at date \( t \) is then defined by

\[ h(w_t, \nu_t, t) = 1 - L(w_t, \nu_t, t). \]

where \( \nu_t \), is the marginal utility of wealth.
OPTIMAL SOLUTIONS

- Successive iterations of the logarithms of the last FOC entail:

\[
\ln \nu_t = - \sum_{\tau=1}^{\tau=t} \ln (1 + r_{\tau}) + \ln \nu_0
\]

(21)

- This way of writing the law of motion of \( \nu_t \) proves extremely interesting from the empirical point of view.

- It shows that \( \nu_t \) can be broken down into a fixed individual effect \( \nu_0 \) and an age effect.

- This model is useful for distinguishing between temporary and permanent wage variations.
An Application to Retirement

- The economic analysis of the retirement decision fits well into the life-cycle model already studied.
- We need to consider the legal constraints and the flow of income specific to retirement within the model.
- This decision can be analyzed with the help of the “option value” associated with the choice not to go into retirement today.
- Empirical studies show that workers react to the financial incentives that accompany either early retirement or continued wage-earning.
- Most countries in the OECD have put in place a pension system enabling workers to receive income when they retire from the labor market.
An Application to Retirement

- The typical system is funded by mandatory contributions coming from employers and workers.
- It offers around 40-70 percent of the last net wage upon total retirement.
The pension system creates incentives for workers to take their retirement earlier or later.

It usually specifies a legal age (the “normal” retirement age), past which people can begin, if they wish, to draw full benefits without reduction for early retirement.

But every individual obviously has the right to retire before or after this legal age.

In this case, she receives a smaller or larger pension the farther the age at which she ceases to work lies below or above the legal age.

Hence the decision to retire brings into play a number of elements that emerge very clearly with the help of the life-cycle model, significantly modified.
Option value in the life-cycle model

Let us consider a person employed on date/age $\tau$

Let us suppose that this person decides to retire on date $s \geq \tau$.

The evolution of his wealth starting from date $\tau$ is given by:

$$A_t = (1 + r_t)A_{t-1} + B_t + w_t(1 - L_t) - C_t, \quad \forall t \geq 0$$ (22)

we will suppose that the agent does not work at all after date $s$: $L_t = 1$ for $t \geq s$.

We will use $B_t(s)$ to denote the income expected in the period $t \geq s$, composed of pension payments over the period $t$ and other income available (savings...).

we will use $B_t(0)$ to designate the non-earned income of the agent while he is still working, hence for $t < s$, and

we will use $C_{et}$ and $C_{rt}$ respectively to designate his consumption of physical goods before and after retirement.
An Application to Retirement

Option value in the life-cycle model

For given $s$, the agent solves the following problem:

$$\begin{align*}
\max_{C_{et}, C_{rt}, L_t} & \left[ \sum_{t=\tau}^{s-1} U(C_{et}, L_t, t) + \sum_{t=s}^{T} U(C_{rt}, 1, t) \right] \\
\text{subject to constraints:} & \\
A_t &= \begin{cases} 
(1 + r_t)A_{t-1} + B_t(0) + w_t(1 - L_t) - C_{et} & \text{if } \tau \leq t \leq s - 1 \\
(1 + r_t)A_{t-1} + B_t(s) - C_{rt} & \text{if } s \leq t \leq T
\end{cases}
\end{align*}$$

Thus, in this case, the agent age $\tau$ chooses the date $s$ on which to end his working life by solving the following problem:

$$\begin{align*}
\max_s V_\tau(s) \quad \text{s.t.} \quad T_m \geq s \geq \tau
\end{align*}$$

where $V_\tau(s)$ is the value of welfare of the consumer.
An Application to Retirement

Option value in the life-cycle model

These problems never lend themselves to an explicit resolution, and are generally solved numerically.

In practice, we have to specify the utility function and the manner in which the replacement income is assembled to arrive at a model capable of being simulated or estimated empirically.

Moreover, the decision to retire is made in an environment marked by numerous uncertainties about wealth, health, etc.

We need to deal with a stochastic model where we have to maximize expected utility.

In this case, the solution is to retire today (corner solution) or continue to work one more period.

This leads us to examine the option value attached to the decision not to take retirement right now: \( V_\tau(s^*) - V_\tau(\tau) \).

If it is positive the agent continues to work. If it is not, he goes into retirement.