Chapter 2: LABOUR DEMAND

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BASIC REFERENCE: Cahuc & Zylberberg (2004), Chapter 4
In this chapter we will see:

- How firms choose their factors of production
- Substitution between capital and labor
- Substitution between different types of labour
- The tradeoff between workers and hours
- What the estimates of the elasticities of labour demands with respect to the costs of the inputs are
- What the effects of the adjustment costs of labour are
Chapter 1 has been devoted to the supply side of the labour market.

But the level of employment does not only depend on decisions of workers.

The desire to work a certain amount of work at a given wage must also meet the plans of employers.

The theory of labour demand is part of a wider context, that of the demand for the factors of production;

the basic assumption is that firms utilize the services of labour by combining them with other inputs (capital), in order to maximize their profits.
INTRODUCTION

- An entrepreneur has an interest in hiring a worker whenever the income that worker generates is greater than his or her cost.
- The demand for labour must therefore depend on the cost of labour, but also on the cost of the other factors, and on elements that determine what the firm can earn.
- The efficiency of labour depends upon the technology available and the quantities of the other factors of production.
- It also depends on the qualities of each worker, which depend in turn on different individual characteristics.
In this chapter, it is helpful to make a distinction between short-run decisions and long-run ones.

- In the short run firms adjust their quantity of labour; we take its stock of capital as given.
- In the long run, however, it is possible for firms to substitute capital for certain categories of workers.

We will also distinguish the “static” theory of labour demand from the “dynamic” theory.

- The static theory sets aside the *adjustment costs* of labour, i.e. the costs connected to *changes* in the volume of this factor.
- If such costs do not exist, there is really no dynamics, since nothing prevents labour demand from reaching its desired level immediately.
- By not taking adjustment delays into account, static theory throws the basic properties of labour demand – the “laws” as they are sometimes called –.
These “laws” set the directions in which the quantity of labour demanded varies as a function of the costs of all the factors (elasticities of labour demand).

Knowing the orders of magnitude of these elasticities is essential when it comes to assessing the effects of economic policy.

For example, knowledge of the elasticity of unskilled labour with respect to its cost allows us to set out in approximate figures the changes in the demand for this category of wage-earners in the wake of a reduction in social security contributions, or a rise in the minimum wage.
In the short run, we can make the assumption that only the volume of labour services is variable.

We will see that labour demand depends on the real wage and the market power of the firm.

But in the long term, there exist possibilities of substituting capital for labour that substantially change the determinants of labour demand.

When we do set the time horizon farther out, we can no longer study labour demand by focusing narrowly on just two aggregate factors – capital and labour –

The firm can also, for example, change the composition of its workforce by changing the structure of skills.
1. MARKET POWER

The demand $Y(P)$ for a particular good depends, among other things, on the price $P$ at which a firm sells its product.

To make the explanation easier, it is preferable to work with the inverse relationship $P = P(Y)$, called the inverse demand function.

It is assumed to be decreasing and we shall denote its elasticity by $\eta^P_Y \equiv YP'(Y)/P(Y)$.

we will assume for simplicity’s sake that the elasticity $\eta^P_Y$ is a constant independent of $Y$. 
1. MARKET POWER

- When $\eta^P_Y = 0$, the price of the good does not depend on the quantity produced by the firm.
  - This situation characterizes perfect competition and the firm is then described as a “price taker.”

- On the contrary, if $\eta^P_Y < 0$ the firm finds itself in a situation of imperfect competition and we then say that it is a “price maker.”

- Thus, the absolute value $|\eta^P_Y|$ of this elasticity constitutes an indicator of the market power of the firm.

- But the price $P$ does not depend only on the quantity $Y$ produced by the firm (partial equilibrium, however).
2. FIXED AND FLEXIBLE FACTORS

Factors of production comprise different types of manpower (for example, skilled and unskilled) and different types of plant (machinery and factories).

For simplicity, the latter will be represented by a single factor bearing the generic name capital.

Some factors of production cannot be adjusted in the short run (fixed): capital belongs to that category.

Conversely, factors whose level can be altered in the short run are called flexible: labour, if measured by hours.

All factors of production are flexible in the long run.
3. Cost of labour and marginal productivity

We begin the study of labour demand by assuming that all the services performed by this factor can be represented by a single aggregate $L$ which is flexible in the short run,

The other inputs being taken to be rigid at that horizon. Their levels can therefore be considered as given.

We may, without risk of confusion, represent the production process by a function with a single variable, or $Y = F(L)$.

We shall assume that this function is strictly increasing and strictly concave.
3. Cost of labour and marginal productivity

If we designate the price of a unit of labour by $W$, and set aside the costs tied to the utilization of fixed factors, the firm’s profit is written this way:

(1) \[ \Pi(L) = P(Y)Y - WL \quad \text{with} \quad Y = F(L) \]

The entrepreneur’s only decision is to choose his or her level of employment so as to maximize his or her profit.

The first-order condition is obtained simply, by setting the derivative of the profit to zero with respect to $L$, so that:

(2) \[ \Pi'(L) = F'(L)[P(Y) + P'(Y)Y] - W = F'(L)P(Y)(1 + \eta_Y^P) - W = 0 \]
3. Cost of labour and marginal productivity

Where \((1 + \eta^P_Y) > 0\), the labour demand is defined by:

\[
F'(L) = \nu \frac{W}{P} \quad \text{with} \quad \nu \equiv \frac{1}{1 + \eta^P_Y}
\]

This relation shows that the profit of the firm attains its maximum when the marginal productivity of labour is equal to real wage \(W/P\) multiplied by a markup \(\nu \geq 1\).

The latter is an increasing function of the absolute value \(|\eta^P_Y|\) of price elasticity with respect to production.

The markup constitutes a measure of the firm’s market power.

In a situation of perfect competition, the firm has no market power \((\eta^P_Y = 0)\) and marginal productivity is equal to the real wage.
3. Cost of labour and marginal productivity

The concept of cost function allows us to interpret the optimality condition (3) differently.

In this model, with just one factor of production, this function simply corresponds to the cost of labour linked to the production of a quantity $Y$ of a good, or $C(Y) = WL = WF^{-1}(Y)$.

Since the derivative of $F^{-1}$ is equal to $1/F'$, the marginal cost is defined by $C'(Y) = W/F'(L)$, and relation (3) is written:

$$P = \nu \frac{W}{F'(L)} = \nu C'(L)$$

In other words, the firm sets its price by applying the markup $\nu$ to its marginal cost $C'(Y)$.

In the situation of perfect competition ($\nu = 1$), the price of a good exactly equals the marginal cost.
3. Cost of labour and marginal productivity

The expression of labour demand allows us to study the impact of a variation in the cost of labour on the volume of labour.

Differentiating relation (3) with respect to $W$, we find again that:

$$\frac{\partial L}{\partial W} = \nu / \left( F'' P' + PF'''' \right) < 0$$

Hence short-run labour demand and thus the level of supply of the good are decreasing functions of labour cost.
3. Cost of labour and marginal productivity

On the other hand, the selling price of the good produced by the firm rises with $W$.

It could be shown that labour demand and the level of production diminish, while the price rises, when the markup $\nu$ grows larger.

Thus, the determinants of short-run labour demand are:

- the cost of labour,
- the determinants of demand for the good produced by the firm,
- the firm’s technology,
- the structure of the market for goods – the markup $\nu$ and the elasticity $\eta_Y^P$ –.

In the long run, the firm may contemplate substituting part of its workforce with machines...
We shall now shift to a long-run perspective, in which capital $K$ also becomes a flexible factor.

In order better to appreciate the different elements that bear on demands for the factors of production, it will be helpful to conduct the analysis in two stages.

**FIRST STAGE:** the level of production is taken as given, and we look for the optimal combinations of capital and labour through which that level can be reached.

**SECOND STAGE:** we look for the volume of output that maximize the firm’s profit.

This approach makes it possible to distinguish:

- **SUBSTITUTION EFFECTS:** which occur when the volume of production is fixed (first stage)
- **SCALE EFFECTS:** which are confined to the second stage, in which the optimal level of production is set.
A technology with two inputs

We begin by analyzing the first stage of the producer’s problem.

The first stage makes it possible to define and characterize the cost function of the firm.

We can then deduce the properties of the so-called conditional factors demands.

Assuming once more that labour can be represented by a single aggregate $L$, the production function of the firm will now be written $F(K, L)$.

The conditional demands for these inputs will depend only on the relative price of each.
A technology with two inputs

We shall assume from now on that to attain a given level of production, capital and labour can always combine in different proportions.

- Factors possessing this property are said to be *substitutable*.
- More precisely, we shall posit that the production function is strictly increasing with each of its arguments, so that its partial derivatives will be strictly positive:
  \[ F_K > 0 \text{ and } F_L > 0. \]
- We shall also assume that this function is strictly concave, which means that the marginal productivities of each factor diminish with its quantity.
  - We will thus have \( F_{KK} < 0 \) and \( F_{LL} < 0 \).
A technology with two inputs

In order to make certain results clearer, it will sometime be useful to assume that the production function is homogeneous.

We may note that if \( \theta > 0 \) designates the degree of homogeneity.

This property is characterized by the following equality:

\[
F(\mu K, \mu L) = \mu^\theta F(K, L) \forall \mu > 0, \forall (K, L)
\]

Parameter \( \theta \) represents the level of returns to scale.

We say that returns to scale are decreasing if \( 0 < \theta < 1 \), constant if \( \theta = 1 \), and increasing if \( \theta > 1 \).
Cost function and factor demand

The optimal combination of inputs is obtained by minimizing the cost linked to the production level $Y$.

Let us designate by $R$ and $W$ respectively the price of a unit of capital and labour.

The quantities of inputs corresponding to this choice are given by the solution of the following problem:

\[
\min_{\{K, L\}} (WL + RK) \quad \text{s.t.} \quad F(K, L) \geq Y
\]

The solutions, denoted $\bar{L}$ and $\bar{K}$, are called respectively the conditional demand for labour and the conditional demand for capital.

The minimal value of the total cost, or $(W\bar{L} + R\bar{K})$, is then a function of the unit cost of each factor and the level of production.

This minimal value is called the cost function of the firm: $C(W, R, Y)$. 
Cost function and factor demand

This curve, the *isoquant*, designates the set of values of $K$ and $L$ allowing a given level of production to be attained: $F(K, L) = Y$. 
Cost function and factor demand

The slope of this curve is negative, and the absolute value of its derivative is equal to the *technical rate of substitution* between capital and labour, or $|K'(L)| = F_L/F_K$.

This technical rate of substitution defines the quantity of capital that can be saved when the quantity of labour is augmented by one unit.

In this figure we have also represented an *iso-cost* curve $(C_0)$.

This corresponds to the values of $K$ and $L$ such that $WL + RK = C_0$, where $C_0$ is a positive given constant.

An iso-cost curve is thus a straight line with a slope $-(W/R)$ moving out towards the north-west when $C_0$ increases.

It is evident, then, that if the iso-cost line is not tangent to the isoquant (at $E'$), it is always possible to find a combination of factors $K$ and $L$ satisfying the constraint $F(K,L) \geq Y$ and leading to a cost inferior to that of the combination represented by point $E'$. 
Cost function and factor demand

For that, we need only cause line \((C_0)\) to move in towards the origin (for example, at point \(E''\) the total cost of production is inferior to its value at point \(E'\)).

To sum up, the producer’s optimum lies at point \(E\) where the iso-cost line is tangent to the isoquant.

The property of strict convexity of the isoquant guarantees that point \(E\) represents a unique minimum for the cost of production.

At this point, the technical rate of substitution is equal to the ratio of the costs of inputs. The conditional demands for capital and labour are thus defined by:

\[
\frac{F_L(\bar{K}, \bar{L})}{F_K(\bar{K}, \bar{L})} = \frac{W}{R} \quad \text{and} \quad F(\bar{K}, \bar{L}) = Y
\]
The properties of the cost function

Relation (8) shows that $\bar{K}$ and $\bar{L}$ depend only on $Y$ and $W/R$.

Evidently we could deduce the properties of the conditional demands using the two equations of relation (8).

In fact, it is simpler to proceed indirectly by relying on the properties of the cost function:

1. It is increasing in its arguments and homogeneous of degree 1 in $(W, R)$.
2. It is concave in $(W, R)$.
3. It satisfies Shephard’s lemma, or:

   \begin{equation}
   \bar{L} = C_W(W, R, Y) \quad \text{and} \quad \bar{K} = C_R(W, R, Y)
   \end{equation}

   where $C_W$ and $C_R$ are the partial derivatives of the cost function.
4. It is homogeneous of degree $1/\theta$ with respect to $Y$ when the production function is homogeneous of degree $\theta$. 

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These properties of the cost function allow us to derive the properties of the conditional factor demands very easily.

The most important properties of the conditional demands for labour and capital have to do with the way they vary in the wake of a rise or a fall in the prices of these factors.

The extent of these variations depends on:

1. the elasticity of substitution between capital and labour
2. the share of each factor in the total cost.
Variations in factor prices

The differentiation of the first relation of Shephard’s lemma (9) with respect to $W$ entails:

$$\frac{\partial \bar{L}}{\partial W} = C_{WW} \leq 0$$

The conditional labour demand is thus decreasing with the price of this factor.

Since the first-order conditions (8) show that conditional demand in reality depends only on the relative price of labour, i.e. on $W/R$, we can state that it increases with the price of capital.

Symmetrically, we could show that the conditional demand for capital diminishes with $R$ and increases with $W$.

Shephard’s lemma allows us to characterize more precisely the cross effects of a change in the price of a factor on the demand for the other factor.
Variations in factor prices

Thus relation (9) immediately entails:

\[
\frac{\partial L}{\partial R} = \frac{\partial K}{\partial W} = C_{WR}
\]

Since it was shown above that the conditional demand for a factor is increasing with the price of the other factor, we can deduce that the cross derivative $C_{WR}$ is necessarily positive.

The equality (11) portrays the symmetry condition of cross-price effects.

It means that at the producer’s optimum, the effect of a rise of one dollar in the price of labour on the volume of capital is equal to the effect of a rise of one dollar in the price of capital on the volume of labour.

This (astonishing) equality is no longer verified in terms of elasticities.
Cross elasticities and the elasticity of substitution

Let us recall first that the cross elasticities $\bar{\eta}_R^L$ and $\bar{\eta}_W^K$ of the conditional demand for a factor with respect to the price of the other factor are defined by:

\[
\bar{\eta}_R^L = \frac{R \partial \bar{L}}{\bar{L} \partial R} \quad \text{and} \quad \bar{\eta}_W^K = \frac{W \partial \bar{K}}{\bar{K} \partial W}
\]  

(12)

At the producer’s optimum, relation (11) then entails $\bar{\eta}_R^L = (R\bar{K}/W\bar{L})\bar{\eta}_W^K$.

Consequently, leaving aside the exceptional case where the cost $W\bar{L}$ of manpower would equal the cost $R\bar{K}$ of capital, the cross elasticities will always be different.

They do not, therefore, constitute a significant indicator of the possibilities of substitution between these two factors.
Cross elasticities and the elasticity of substitution

To get round this problem, it is preferable to resort to the notion of *elasticity of substitution* which is the elasticity of the variable $\bar{K}/\bar{L}$ with respect to relative price $W/R$.

The elasticity of substitution between capital and labour, denoted by $\sigma$, is defined by:

$$\sigma = \frac{W/R}{\bar{K}/\bar{L}} \frac{\partial (\bar{K}/\bar{L})}{\partial (W/R)}$$

This formula indicates that the capital-labour ratio increases by $\sigma\%$ when the ratio between the price of labour and the price of capital increases by 1%.

The figure with the isoqant shows that a rise (or a fall) of the relative price $W/R$ increases (or diminishes) the slope of the straight lines of iso-cost and therefore shifts point $E$ towards the left (or the right) along the isoquant.
Cross elasticities and the elasticity of substitution

- In other words, the ratio $\frac{\bar{K}}{\bar{L}}$ varies in the same direction as the relative price $\frac{W}{R}$.
- The elasticity of substitution between capital and labour is thus always *positive*.
- When the value of the elasticity of substitution is high, that means that to obtain a given level of production, the entrepreneur has the possibility of diminishing “greatly” the utilization of one factor and “greatly” increasing that of the other, in the wake of a change in the relative price of the factors.
- Thus, when $W$ rises or $R$ falls, the firm’s interest in diminishing the utilization of labour so as to minimize the total cost is all the greater, the higher the value of $\sigma$ is. That explains why the elasticities of conditional labour demand are increasing, in absolute value, with the elasticity of substitution $\sigma$. 
Variation in the level of output

The effects of an exogenous change in the level of output $Y$ on the total cost are easily characterized if total cost is defined by $C = W\bar{L} + R\bar{K}$ with $F(\bar{K}, \bar{L}) = Y$.

It suffices to differentiate these last two equalities with respect to $Y$ and to take account of the optimality condition (8) to get the following expression of the marginal cost:

$$C_Y(W, R, Y) = \frac{W}{F_L} = \frac{R}{F_K}$$

In the first place, it is apparent that the marginal cost is always positive.

That the total cost rises with the level of output.

Conversely, it is not possible to know the direction of variations in factor demands without supplementary hypotheses.

Clearly, factor demands do not diminish simultaneously when production increases.
Variation in the level of output

Thus a rise in production simply requires that the volume of one of the factors increase, but the volume of the other factor is not obliged to do so; it can even decrease.

However, when the production function satisfies the homogeneity hypothesis (6) a more precise conclusion emerges.

The factor demands are then homogeneous of degree $1/\theta$ with respect to $Y$ — see property (iv) of the cost function set out in section 1.2.1 — and relation (??) clearly shows that the conditional demands for labour and capital then rise simultaneously with the level of output.

Minimization of cost for a given level of output constitutes the first stage of the problem of the firm; we must now examine how the optimal volume of output is determined.
The entrepreneur is generally in a position to choose his or her level of production.

The desired quantities of the factors are then distinguishable from their conditional demands.

The analysis of substitution and scale effects yields highly general properties for labour demand;

among other things, it brings into play the elasticity of substitution between capital and labour, the share of each factor in the total cost, and the market power of the firm.

Let us again designate by \( P(Y) \) the inverse demand function.

Then, profit \( \Pi(W, R, Y) \) linked to a level of production \( Y \) when the unit costs of labour and capital are respectively \( W \) and \( R \), takes the following form:

\[
(15) \quad \Pi(W, R, Y) = P(Y)Y - C(W, R, Y)
\]
SCALE EFFECTS

The first-order condition is obtained by setting the derivative of this expression to zero.

Rearranging terms, we find that the optimal level of production is characterized by:

$$P(Y) = \nu C_Y(W, R, Y)$$

with
$$\nu \equiv 1/(1 + \eta_Y^P)$$

We can see that the firm sets its price by applying the markup $\nu$ to its marginal cost $C_Y$.

Taking into account expression (14) of marginal cost, the optimality condition (16) takes the following form:

$$F_L(K, L) = \nu \frac{W}{P} \quad \text{and} \quad F_K(K, L) = \nu \frac{R}{P}$$

In other words, at the firm’s optimum the marginal productivity of each factor is equal to its real cost multiplied by the markup.
When the competition in the market for the good produced by the firm is perfect \((\nu = 1)\), we rediscover the usual equalities between the marginal productivity of a factor and its real cost.

The values of \(K\) and of \(L\), defined by equations (16) and (17), are called the long-run or unconditional demands for capital and for labour.

We will study now THE LAWS OF DEMAND which refer to the manner in which unconditional demands for the factors of production vary with the unit costs of these same factors.

They combine substitution and scale effects.
UNCONDITIONAL FACTOR DEMANDS

Decreasing relation between the factor demand and its cost

- We will see that the unconditional demand for a factor is decreasing with its cost.
- This property possesses a very general character: in particular, it does not depend on the production function of the firm being homogeneous.
- To demonstrate this result, we have to use the profit function, denoted by $\Pi(W, R)$, equal to the maximal value of profit for given values of the costs of the inputs.
- The cost function $C(W, R, Y)$ being concave in $(W, R)$ for all $Y$, relation (15) signifies that function $\Pi(W, R, Y)$ is convex in $(W, R)$ whatever the value of $Y$ may be.
- Moreover, Shephard’s lemma (9) states that the partial derivative $C_{W}(W, R, Y^{*})$ is equal to unconditional labour demand $L^{*}$.
- An analogous rationale evidently applies to the unconditional capital demand $K^{*}$.
- We thus arrive at the following relations, known as Hotelling’s lemma:

$$\Pi_{W}(W, R) = -L^{*} \quad \text{and} \quad \Pi_{R}(W, R) = -K^{*}$$
LABOUR DEMAND ELASTICITIES

It is possible to be more exact about unconditional labour demand $L^*$ by noting that it always satisfies Shephard’s lemma (9).

Thus we have $L^* = C_W (W, R, Y^*)$.

Differentiating this equality with respect to $W$, we get: $\frac{\partial L^*}{\partial W} = C_W W + C_{WY} \frac{\partial Y^*}{\partial W}$

When we multiply the two members of this relation by $W/L^*$, we bring to light the elasticities $\eta^L_W$ and $\eta^Y_W$ of unconditional labour demand and of the level of output with respect to the wage.

The result is:

$$\eta^L_W = \frac{W}{L^*}C_W W + \left(\frac{Y^*C_{WY}}{L^*}\right) \eta^Y_W$$

Since $L^* = C_W (W, R, Y^*)$, the terms $(W/L^*)C_W W$ and $(Y^*/L^*)C_{WY}$ designate respectively the elasticity $\tilde{\eta}^L_W$ of the conditional labour demand and the elasticity of this demand with respect to the level of output taken at point $Y = Y^*$, $\tilde{\eta}^Y_Y$. 
We thus finally obtain:

\[ \eta^L_W = \bar{\eta}^L_W + \bar{\eta}^L_Y \eta^Y_W \]  

This relation reveals the different effects of a rise in wage on the demand for labour:

- We may start by isolating a *substitution effect* represented by the elasticity \( \bar{\eta}^L_W \) of conditional labour demand.
- We have seen before that this term is always negative, since for a given level of production, a rise in the cost of labour always leads to reduced utilization of this factor (and increased utilization of capital).
- Relation (20) likewise brings out a *scale effect* represented by the product \( \bar{\eta}^L_Y \eta^Y_W \).
- The direction of this scale effect is obtained by noting that the second-order conditions of the firm’s profit maximization dictate that \( \eta^Y_W \) should be of opposed sign to \( C_{WY} \).
- Following Shephard’s lemma (9), \( \bar{\eta}^L_Y \) is of the same sign as \( C_{WY} \), so the scale effect is always *negative* and therefore accentuates the substitution effect.
LABOUR DEMAND ELASTICITIES

Using the same procedure, it is possible to calculate the cross elasticity $\eta^L_R$ of the unconditional labour demand with respect to the cost of capital. This comes to:

$$\eta^L_R = \tilde{\eta}^L_R + \eta^L_Y \eta^Y_R$$

(21)

In the case of two inputs, we have shown that the conditional demand for a factor rises when the price of the other factor rises.

The substitution effect, the term $\tilde{\eta}^L_R$, is thus positive.

Conversely, the scale effect, represented by the term $\eta^L_Y \eta^Y_R$, is a priori ambiguous.

The sign of cross elasticity $\eta^L_R$ is thus undetermined.

- If $\eta^L_R > 0$, labour and capital are qualified as *gross substitutes*: a rise in the price of capital causes demand for this factor to fall and that of labour to rise.

- If $\eta^L_R < 0$, labour and capital are qualified as *gross complements*: a hike in the price of one of these factors signifies that demand for both of them falls of.
The static theory of labour demand furnishes valuable indications about what determines elasticities, and about the possibilities of substitution over the long run between the different inputs.

But it gives no detail about the manner in which the inputs reach their long-run values or about the length of time that these adjustments take.

Moreover, it does not take into account the fact that firms are faced with an ongoing process of reorganization arising from technological constraints, market fluctuations, and manpower mobility.

In order to be able to assess these phenomena, we have to resort to the notion of adjustment cost.

Firms may incur adjustment costs when they decide to change their level of employment.

But the fact that firms must deal with quits by workers entails that they may incur adjustment costs simply in order to maintain a constant level of employment.
No real consensus has yet been reached as regards the analytical representation of these costs, but the quadratic symmetric form, historically the one most frequently utilized, is today gradually being abandoned.

We will examine the dynamics of labour demand in a setting without uncertainty, although there exist important stochastic elements in these problems.

labour adjustment costs arise from variations in the volume of employment, and from the replacement of former employees by new ones.

Numerous studies show that the size of these costs is far from insignificant, and for that reason they play a large role in decisions to hire and fire.

In France, the average cost of a separation represents 56% of the annual cost of labour, whereas a hire represents only 3.3%.

Most assessments conclude that employment protection is less strict in the US, Canada, and the UK than in continental Europe.

In Europe, a large part of the cost of termination is regulatory in nature (period of advance notice, administrative procedure, etc.). The result has been a massive recourse to short-term contracts.
For ease of analysis, adjustment costs have most often been represented using a convex symmetric function (in general quadratic) of net employment changes.

But this way of specifying them does not allow us to explain asymmetric and discontinuous adjustments in employment and the consequence of gross employment changes.

For this reason, it is now gradually being replaced by a representation including fixed costs, linear costs, quadratic costs and gross employment changes.

**Quadratic costs**

This representation was introduced by Holt et al. (1960), who viewed net adjustment costs as equal to \( b(\Delta L_t - a)^2 \), \( a, b > 0 \), with \( \Delta L_t = L_t - L_{t-1} \) or \( \Delta L_t = \dot{L}_t \) according to whether time was represented discretely or continuously.

This specification has the advantage of introducing an asymmetry between the cost of positive and negative variations in employment \((a > 0)\).
Linear costs

The specification of adjustment costs in the form of a piecewise linear function offers the advantage of achieving a more realistic representation of labour demand, in which firms hire in some circumstances, let employees go in others, and sometimes leave their workforce unchanged.

The utilization of piecewise linear costs has greatly expanded in the 1990s, with the works of Bentolila and Bertola (1990), who examine linear adjustment costs of the form:

\[ C(\Delta L) = c_h \Delta L \text{ if } \Delta L \geq 0 \text{ and } C(\Delta L) = -c_f \Delta L \text{ if } \Delta L \leq 0, \quad c_h > 0, \quad c_l > 0 \]

(22)

The coefficients \( c_h \) and \( c_f \) represent the respective unit costs of a hiring and a termination. The adjustment of employment is asymmetric, since \( c_h \neq c_f \).
Quadratic and symmetric adjustment costs

We here consider a firm situated in a deterministic environment, which must support adjustment costs when it alters its workforce.

To make things easier from a technical point of view, a large part of the literature has assumed that these costs were symmetric and could be represented by a quadratic function.

We shall work with a dynamic model in continuous time, in which, at each date, \( t \geq 0 \), the adjustment cost is restricted to labour alone.

When the firm utilizes a quantity \( L_t \) of this factor, it obtains a level of output \( F(L_t) \) that is strictly increasing and concave with respect to \( L_t \).

Taking other inputs into account, like capital for example, greatly complicates the analysis without changing the import of the results which we want to highlight.

We likewise simplify by leaving quits out of the reckoning, on the assumption that net variations in employment are equal to gross variations.
Quadratic and symmetric adjustment costs

Let $\dot{L}_t$ be the derivative with respect to $t$ of the variable $L_t$;

we shall assume that variations in the level of employment are accompanied at every date $t$ by an adjustment cost represented by the quadratic function $(b/2)\dot{L}_t^2$, $b \geq 0$.

To simplify the notations and calculations, from now on we will omit the index $t$ and assume that at every date the cost of labour and the interest rate are exogeneous constants denoted respectively by $W$ and $r$.

At date $t = 0$, the discounted present value of profit, $\Pi_0$, is written:

\[
\Pi_0 = \int_0^{+\infty} \left[ F(L) - WL - \frac{b}{2} \dot{L}^2 \right] e^{-rt} dt
\]  

(23)

In this environment, free of random factors, the firm chooses its present and future levels of employment so as to maximize the discounted present value of profits $\Pi_0$. 

Quadratic and symmetric adjustment costs

This is a classic problem of calculus of variations for which the first-order condition is given by the Euler equation.

After several simple calculations, we find that the employment path is described by a non-linear second-order differential equation that takes the form:

\[ b\ddot{L} - rb\dot{L} + F'(L) - W = 0 \]  

(24)

The stationary value \( L^* \) of employment is obtained by making \( \dot{L} = \ddot{L} = 0 \) in this equation.

Given the difficulty of this model, we will skip the analysis of the dynamics of employment in this case so we will continue by studying the case of linear adjustment costs.
Linear and asymmetric adjustment costs

It is possible to distinguish the costs of hiring and firing by adopting a piecewise linear specification.

The hypothesis of linearity also brings out the fact that, contrary to the model with quadratic costs, employment adjustment can take place immediately.

Let $c_h$ and $c_f$ be two positive constants, and let us assume from now on that the adjustment costs are represented by the function:

\[ C(\dot{L}) = c\dot{L} \quad \text{with} \quad c = c_h \quad \text{if} \quad \dot{L} > 0 \quad \text{and} \quad c = -c_f \quad \text{if} \quad \dot{L} < 0 \]

Parameters $c_h$ and $c_f$ allow us to distinguish hiring costs ($\dot{L} > 0$) from termination costs ($\dot{L} < 0$).

As in the previous model with quadratic adjustment costs, it is assumed, for the sake of simplicity, that there are no quits.
Linear and asymmetric adjustment costs

The problem of the firm consists of choosing, at date $t = 0$, levels of employment that maximize the discounted present value of profit $\Pi_0$. The latter is expressed thus:

\[
\Pi_0 = \int_0^{+\infty} \left[ F(L) - WL - C(\dot{L}) \right] e^{-rt} dt
\]

(26)

Once again, this is a problem of calculus of variations to which the Euler equation (??) applies when the quadratic function $-(b/2)\dot{L}^2$ is replaced by the linear function $C(\dot{L}) = c\dot{L}$.

After several simple calculations, we find that the employment path is defined by the equation $F'(L) = W + rc$, which entails:

\[
F'(L) = W + rc_h \quad \text{if} \quad \dot{L} > 0, \quad \text{and} \quad F'(L) = W - rc_f \quad \text{if} \quad \dot{L} < 0
\]

(27)
Linear and asymmetric adjustment costs

These conditions signify that the firm hires when marginal productivity is sufficiently high to cover the wage $W$ and the hiring cost $rc_h$.

Conversely, the firm fires when productivity is so low that it just equals wage $W$ less the provision $rc_f$ for the termination cost.

In all other cases, i.e. when productivity lies in the interval $[W - rc_f, W + rc_h]$, the firm has no interest in altering the size of its workforce, for the gains due to hiring and firing are less than the costs incurred by adjusting employment.

Labour adjustments take a particularly instructive form when the parameters $W, r, c_h$ and $c_f$ are constants, which we have assumed.

Let us define the employment levels $L_h$ and $L_f$, by the equalities:

\[
F'(L_h) = W + rc_h \quad \text{and} \quad F'(L_f) = W - rc_f
\]
Linear and asymmetric adjustment costs

In this case, the optimal values $L_h$ and $L_f$ do not depend on date $t$.

That means that labour demand *immediately* (i.e. in $t = 0$) “jumps” to its stationary value.

The firm adjusts its workforce to the value $L_h$ (or $L_f$) if the latter is superior (or inferior) to the initial value $L_0$ of employment.

In the opposite case, i.e. if $L_0$ falls in the interval $[L_h, L_f]$, the optimal solution for the firm consists of making no change to the size of its workforce.

In sum, labour demand is defined by:

$$L = \begin{cases} L_h & \text{if } L_0 \leq L_h \\ L_0 & \text{if } L_h \leq L_0 \leq L_f \\ L_f & \text{if } L_f \leq L_0 \end{cases}$$

(29)
Linear and asymmetric adjustment costs

Relations (28) show us that the costs of hiring and firing have *opposing* effects on labour demand.

If the size of the workforce is low at the outset \( L_0 \leq L_h \), then optimal employment is equal to \( L_h \) and a rise in the hiring cost \( c_h \) reduces employment.

Conversely, if there is a large number of workers at the outset \( L_f \leq L_0 \), the optimal level of employment takes the value \( L_f \) and we clearly see that a *rise* in the termination cost \( c_f \) has the effect of *increasing* employment.

We should not, however, conclude on the basis of this analysis that a rise in the termination cost (or a fall in the hiring cost) “augments” the firm’s labour demand.

In reality, since this demand immediately jumps to \( L_h \) or \( L_f \) (unless it simply remains at \( L_0 \)), the level of employment is always equal to one of the three quantities \( L_h, L_f \) or \( L_0 \).
Linear and asymmetric adjustment costs

Let us suppose that the number of workers is $L_f$, a rise in the termination cost $c_f$ will augment $L_f$ up to a certain value $L_f^+$, and will thus have the effect of placing the outset level of the workforce (now equal to $L_f$) somewhere in the interval $[L_h, L_f^+]$.

In this case, relation (29) describing labour demand shows that the firm then has an interest in remaining at $L_f$.

In other words, a rise in the cost of terminating hinders the firm from going ahead with reductions in personnel, but gives it no incentive to hire.

An analogous line of reasoning would show that a rise in the costs of hiring has the effect of discouraging further recruitment, but does not lead to a reduction in employment.

Conversely, a reduction in hiring costs always has a positive effect on employment to the extent that it increases the value $L_h$ of optimal employment.