Peer Group Effects and Optimal Education System*

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Abstract

The belief that the characteristics of their peers influence the behavior and outcomes of students in school has been important in shaping education policy. How peers affect individuals depends on the prevailing educational system. I analyze two polar cases -tracking and mixing- and I propose several criteria for comparing them. I find that at compulsory level, average human capital across the population is maximized under tracking, although tracking does not dominate mixing according to first order stochastic dominance. The education system that maximizes college attendance depends on the income level in the population and on the opportunity cost of college attendance.

Keywords: Peer Effects, Tracking, Mixing, Income Premium

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1 Introduction

The belief that peers' characteristics in school influence the behavior and outcomes of students has been important in shaping education policy. By "peer effects" we refer to the effect on an individual's academic performance of the ability distribution of their peers. Interest in social interactions, neighborhood effects, and social dynamics has recently undergone a revival. A small body of literature has emerged that studies the generation of persistent inequality among a population due to neighborhood effects of various kinds. One of these neighborhood effects are the peer group influences.[1] Peer group effects have played an important role in a number of policy debates, including ability tracking and school desegregation. The peer group composition of schools is, therefore, undeniably important in the minds of parents as well as policy makers. If peer effects exist, the government should take them into account in order to better achieve its policy objectives. An example of this is the choice between streaming (or tracking) and mixing students of different abilities in public schools.

There is a great deal of controversy regarding the practice of tracking (grouping students in classrooms according to their ability level). The main argument is that by narrowing the range of students' abilities within the classroom, teachers can target instruction to a level more closely aligned with students' needs than in more heterogeneous environments. The critics of ability grouping argue that when students are segregated, disadvantaged students lose any positive peer influences that might be gained from being with more able students. In keeping with this view, there has been considerable movement in the US towards eliminating the practice of grouping students according to ability.[2] In Europe there is currently an intense debate in response to the publication of PISA 2000 and 2003 Reports.

The influence of peers' ability on own educational achievement is well documented but still controversial. Most papers focus on the average innate ability within the classroom as the main characteristic of the student's classmates which can affect achievement. On the one hand, for example, Evans, Oates and Schwab (1992) find

[1] Roemer and Wets (1994) and Streufert (2000) show how economic segregation can lead to inaccurate assessments of the economic payoff to education. The basic idea in this type of analysis is that by depriving children in poor neighborhoods of successful role models (which is an inevitable consequence of economic segregation), they make inferences on the benefits of education that are biased downward.

[2] For example, data from the Schools and Staffing Survey suggest than 20% of schools with programs for gifted children in 1990 had eliminated the programs by 1993 (Figlio and Page (2000)).

a significant peer group effect that vanishes when they control for endogeneity. On the other hand, Henderson, Mieszkowski, and Sauvageau (1978), Summers and Wolfe (1977) and more recently Zimmer and Toma (2000) report significant positive influences of higher achieving peers on achievement.

The existence of peer effects and their links with different policies of grouping students have been studied theoretically as well as empirically. Most authors focus on the effects of grouping students by ability, and conclude that, relative to outcomes in mixed groups, students placed in the low track lose out while those placed in the high track gain. The next question is whether the losses of the former are offset by the gains of the latter. There is no clear evidence on that respect (see, for example, Argys et al.(1996), Betts and Shkolnik (2000) and Figlio and Page (2000)). Theoretical contributions are fewer in number. Among others we find the works by de Bartolome (1990) and Epple, Newlon and Romano (2002) and Arnott and Rowse (1987).

The aim of this paper is to study public intervention in education when the government has to decide the optimal education system taking into account peer effects on students' achievement levels. I analyze two different education systems. The first one, tracking, consists of grouping students based on their innate ability. The second one, mixing, consists of pooling students of different ability levels within the same classroom. Both education systems must be understood as polar cases. I use a model in which individuals live for two periods. In the first period individuals attend compulsory education and accumulate human capital. The acquisition of human capital reflects the influence of family and peers. I consider two different education systems at compulsory level: mixing and tracking. At some point of the first period, students must also decide whether to attend college or not. If they do, they spend the second part of this first period at college. If they do not, they immediately enter the labour market as unskilled workers. By attending college they become skilled workers. During the second period all individuals work. My goal is to compare mixing and tracking, using several criteria that have not been used previously in this literature. Since I propose a normative theory, I need to define the objectives of the government. Most debates have focused on which system leads to higher achievement.

This paper contributes to the relevant literature in two directions. First, at the compulsory level, the paper advances the existing literature by comparing both systems in terms of the induced distributions of human capital at the end of compulsory school. I find that tracking is the system that maximizes average human capital although there is no education system that dominates the other in the sense of first or second order stochastic dominance.

The education system at the compulsory level has some effects on individuals' outcomes later in life, such as college choice and occupational attainment. The second contribution of my paper lies in analyzing these effects. As far as I know there is no previous literature on peer effects on college attendance decisions. When we focus on the college level, we could look at which system maximizes college attendance. I find that this depends on the opportunity cost of college attendance and on how wealthy society is. In particular, if the opportunity cost is low enough, mixing is the system that maximizes college attendance, while the reverse occurs if the opportunity cost is high enough.

The second criterion that I want to study is equality of opportunities, which implies guaranteeing that individuals decide whether to attend college independently of their parents' income. The surprising finding is that tracking is the most equitable system in most cases, except when at the same time there is a high level of inequality in the population and the minimum level of human capital required to attend college is sufficiently low.

The rest of the paper is organized as follows. In Section 2 I describe the model and the main features of human capital distribution under the two education systems at compulsory level. Section 3 analyzes individuals' decision whether to attend college. Two criteria are proposed to compare the two systems at this education level: maximum college attendance and equality of opportunities. Section 4 concludes.

2 Model

2.1 Individuals

I consider an economy in which individuals live for two periods. The population size is constant at 1. Individuals in each generation differ in two aspects: their innate ability, θ_0, and their family background, denoted by z.[3] To make the model tractable I will assume that θ_0 is uniformly distributed over the interval $[0, 1]$ and that family background z takes only two values, 1 and $x > 1$ with probabilities $1 - \lambda$ and λ, respectively. I assume that the two characteristics are independently distributed.

In the first period of their lives, individuals accumulate human capital. At the beginning of this period they attend compulsory education, which is free of charge,

[3] Therefore z could be either the parents' level of income or the parents' human capital.

and they are not allowed to work. At some point of this first period, they decide whether to attend college or not. I denote by γ, where $\gamma \in [0, 1]$, the fraction of the first period that it is left after attending compulsory education. Those who attend college spend fraction γ of the first period at college, while the rest work fraction γ as unskilled workers. By attending college they become skilled workers.[4]

During the second period of their lives all individuals have one unit of time and they all work. Those who went to college work as skilled workers and those who did not go as unskilled. The wage they receive is proportional to their own level of human capital.

2.2 Production of Human Capital

At compulsory level individuals are separated into different groups. To simplify matters, I will assume that there are only two groups or classrooms. The production of human capital at compulsory level depends on three factors. The first is the individual's innate ability, θ_0. The second is the "formal schooling" or "peer group" effect. It depends on the characteristics of the group in which the individual is placed. These characteristics are summarized by the mean ability of the group j or "peer" effect, denoted by $\overline{\theta_0}^j$. The third is "informal schooling" and refers to family background effects, captured by z. After attending compulsory education an individual with innate ability θ_0 ends up with a level of human capital θ_1:

$$\theta_1(\theta_0, \overline{\theta_0}^j, z) = \theta_0(1 + r(\overline{\theta_0}^j, z)), \tag{1}$$

where r is the individual rate of return. In particular I propose:

$$r(\overline{\theta_0}^j, z) = (\overline{\theta_0}^j)^{\alpha}(z)^{1-\alpha}. \tag{2}$$

The acquisition of human capital reflects the influence of family and peers, with respective weights of $1 - \alpha$ and α, where $\alpha \in [0, 1]$ [5].

The main properties of r are as follows. First, regarding family background, individuals' level of human capital is an increasing function of the parental level of

[4] Note that the parameter γ can be interpreted as the cost of investment in human capital, or the fraction of earnings that would have been received in the absence of the investment.

[5] This technology of production of human capital (Equation (2)) is commonly used in this literature. See for example Benabou (1996) or Epple and Romano (1998 and 2002).

human capital but at a decreasing rate, $r_2 > 0$ and $r_{22} < 0$. In addition, note that regarding the peer group effect we have $r_1 > 0$, $r_{11} < 0$ and $r_{12} > 0$.[6]

Empirical evidence establishes that the peer group effect is non-linear: the achievement level of individual students rises with an improvement in the average quality of their classroom, but this positive effect has decreasing returns.[7]

From Equations (1) and (2) we observe that the peer effect becomes more effective in the production of human capital as the level of innate ability or parents' income increases, that is $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial \theta_0} > 0$ and $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial z} > 0$.[8]

In the second part of the first period, each individual decides whether to attend college or not. After attending college they will enjoy a further increase in their level of human capital acquired during compulsory level. I denote that increase by δ and, thus, those individuals who decide to attend college will end up with the following level of human capital:

$$\theta_2 = \theta_1(1 + \delta(\theta_1)). \tag{3}$$

The findings of recent empirical literature show that factors that arise in the early stages of life are crucial determinants of children's later success.[9] Therefore we assume that the acquisition of human capital at college is not directly affected by the family or by peers. We assume that this increase, which reflects the efficacy of higher education, is an increasing function of the human capital acquired at compulsory level, but at a decreasing rate ($\delta_1 > 0, \delta_{11} < 0$).

It is important to note that the characteristics of the group in which the individual is placed affect her final level of human capital θ_2 through two different channels.

[6] The importance of parents' education in the acquisition of human capital of the individual has been explored theoretically as well as empirically. Recently and among others see Feinstein and Symons (1999) find that parental interest is the principal way in which the attainments of each generation are passed to the next. They also suggest the complementarity between parental interest and peer effect.

[7] See Summers and Wolfe (1977), Henderson, Mieszkowski, and Sauvageau (1978), de Bartolome (1990).

[8] The empirical evidence regarding these properties is still mixed. Henderson et al. (1978) find no interaction between own ability and the benefits of an improved peer group, i.e. $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial \theta_0} = 0$. Argys et al. (1996) suggest $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial \theta_0} > 0$. Summers and Wolfe (1977) find some support for higher peer group benefits to lower ability students, that is, $\frac{\partial^2 \theta_1}{\partial \overline{\theta_0}^j \partial \theta_0} < 0$. See Appendix A, where I propose another human capital production function to allow for the possibility that $\overline{\theta_0}^j$ and θ_0 are substitutes.

[9] In particular, Neal and Johnson (1996) find that differences in educational achievements by the time of high-school completion account for almost all the observed black-white wage gap.

First, there is a direct effect since peers affect the human capital acquired in compulsory education. Second, there is also an indirect effect since this level of human capital determines the efficacy of higher education and, thus, as we will see below, the decision of the individual whether to undertake college education or not.

Therefore, it is crucial to analyze the different compositions of groups at school, which are determined by the prevailing education system. This composition is important in determining the distribution of human capital across the population and, as we will see below, in individuals' decisions of whether to attend college or not. In the next section I study the two different education systems.

2.3 Education Systems at Compulsory Level

As indicated in the Introduction, grouping students based on ability measurements (tracking) is very common in the USA and in Europe.[10] In this section I describe the two polar education systems of mixing and tracking and analyze the distribution of human capital at the end of compulsory school under each system.

2.3.1 Mixing

Under mixing the ability distribution is the same in both classrooms. We denote the average ability in each classroom by $\overline{\theta_0}^m$. It coincides with the average ability in the population, m. That is, $\overline{\theta_0}^m = 1/2$.

However, as individuals differ in their parents' level of human capital, there will be two income groups within each classroom: the rich and the poor. I now look at the distribution of θ_1, the human capital at the end of compulsory education. Note that, with probability λ, θ_1 follows a uniform distribution on $(0, b')$, and with probability $(1-\lambda)$, θ_1 follows a uniform distribution on $(0, a')$, where b' and a' denote the human capital θ_1 acquired by the "best" individual (most able) in the rich and the poor income group, respectively:

$$a' = 1 + (1/2)^\alpha \tag{4}$$

$$b' = 1 + (1/2)^\alpha x^{1-\alpha}. \tag{5}$$

Under mixing, therefore, the C.D.F. (cumulative distribution function) of human capital at the end of compulsory education, denoted by $F_M(\theta_1)$, is:

[10] For the US case, public school teachers reported that only 14.4% and 10.8% of tenth-grade students were in heterogeneous (untracked) math classes in 1988 and 1990 respectively, see Rees et al. (1996).

$$F_M(\theta_1) = \begin{cases} 0 & if \quad \theta_1 \le 0 \\ \left(\frac{\lambda}{b'} + \frac{(1-\lambda)}{a'}\right)\theta_1 & if \quad 0 \le \theta_1 \le a' \\ (1-\lambda) + \frac{\lambda}{b'}\theta_1 & if \quad a' \le \theta_1 \le b' \\ 1 & if \quad \theta_1 > b'. \end{cases} \tag{6}$$

I denote by $E_M(\theta_1)$ the expected value of θ_1 under mixing, where:

$$E_M(\theta_1) = (1-\lambda)\frac{a'}{2} + \lambda\frac{b'}{2} = \frac{1}{2}\left(a' + \lambda(b'-a')\right),$$

or, using Equations (4) and (5):

$$E_M(\theta_1) = \frac{1}{2}\left[(1 + (1/2)^\alpha) + \lambda(1/2)^\alpha(x^{1-\alpha} - 1)\right]. \tag{7}$$

Thus $E_M(\theta_1)$ is an average of the mean values of θ_1 in the two income groups, with respective weights $(1-\lambda)$ and λ. From the above equation we observe also that $E_M(\theta_1)$ is an increasing function of both x and λ.

2.3.2 Tracking

Under tracking students are grouped based on innate ability. For the sake of simplicity, I permit at most two tracks. Thus, the median level of innate ability, m, is used as a threshold ability to group students. Students are assigned to the high (low) track as long as their ability θ_0 is above (below) the median.

The distribution of human capital within each track is uniform but with different parameters. I denote by $\overline{\theta_0}^h$ and $\overline{\theta_0}^l$ the average ability in the high and low track respectively. Thus, given the distributional assumption on θ_0, I have that $\overline{\theta_0}^h = 3/4$, whereas $\overline{\theta_0}^l = 1/4$.

Again, there will be two income groups within each track. In the low track θ_1 follows a uniform distribution on $(0, c)$ with probability λ, and it follows a uniform distribution on $(0, a)$ with probability $(1-\lambda)$, where a and c denote the human capital acquired by the "best" individual (most able) in the poor and rich groups, respectively, that is:

$$a = \frac{1}{2}\left(1 + (\frac{1}{4})^\alpha\right) \tag{8}$$

$$c = \frac{1}{2}\left(1 + (\frac{1}{4})^\alpha x^{1-\alpha}\right). \tag{9}$$

Likewise, in the high track θ_1 follows a uniform distribution on (b, e) with probability λ, and it follows a uniform distribution on (d, f) with probability $(1 - \lambda)$. We denote by b and d the human capital θ_1 acquired by the "worst" individual (least able) in the poor and rich groups, respectively. We denote by e and f the human capital θ_1 acquired by the "best" individual (most able) in the poor and rich groups, respectively, i.e.:

$$b = \frac{1}{2}\left(1 + (\frac{3}{4})^{\alpha}\right) \quad (10)$$

$$e = \left(1 + (\frac{3}{4})^{\alpha}\right) \quad (11)$$

$$d = \frac{1}{2}\left(1 + (\frac{3}{4})^{\alpha} x^{1-\alpha}\right) \quad (12)$$

$$f = \left(1 + (\frac{3}{4})^{\alpha} x^{1-\alpha}\right). \quad (13)$$

From Equations (8) to (13) above we have that $a < c, b < d$ and $e < f$. That is, independently of the ability group in which the individual is placed, given two individuals with the same level of innate ability, the one whose parents have the higher income will always attain a higher level of human capital. Second, we have that $a > 0, c > 0, e > b$ and $f > d$. This means that, independently of the ability group in which the individual is placed, given two individuals whose parents have the same income level, the one with the highest ability will always attain a higher level of human capital.

Now I need to introduce some assumptions to ensure that the support of θ_1 is a connected set under tracking, i.e. the density function under tracking denoted by $f_T(\theta_1)$ is strictly positive for all θ_1 in the interval $[0, f]$.

Assumption 1 (A.1): $c > b$.

This assumption ensures that the support of θ_1 in the low track overlaps the support of θ_1 in the high track. In other words, the "best" (the richest and most able) individual in the low track obtains more human capital than the "worst" individual in the high track (the poorest and least able). This assumption implies a restriction on both x and α. For a fixed α this implies that x has to be above a threshold level : $x > \underline{x}(\alpha) = 3^{\frac{\alpha}{1-\alpha}}$. That is, x must be high enough to offset the disadvantage of being in the low track.

Assumption 2 (A.2): $a' > d$.

This assumption implies that under mixing the "best" individual among the poor obtains a higher level of human capital than the "worst" individual among the rich in the high track. As in Assumption 1, it implies a restriction on both x and α. For fixed α it requires that x must be below a threshold level: $x < \overline{x}(\alpha) = ((4/3)^{\alpha}(1 + 2^{1-\alpha}))^{\frac{1}{1-\alpha}}$.[11]

In addition Assumption 2 implies that the two intervals within the high track overlap, as in the case of the low track. That is, the "best" individual in the low income group has more human capital than the "worst" individual in the high income group.

From the two assumptions above I have that for any $\alpha \in (0, 1)$ the income level of the rich must belong to the following interval :

$$\underline{x}(\alpha) < x < \overline{x}(\alpha). \tag{14}$$

Inversely, one might consider a restriction on α for any x. In Figure 1 I illustrate the different intervals for θ_1 and the relationship between them, for both education systems. Now, under Assumptions A.1 and A.2, the C.D.F. of θ_1 under tracking, denoted by $F_T(\theta_1)$ is as follows:

$$F_T(\theta_1) = \begin{cases} 0 & if \quad \theta_1 \leq 0 \\ \left(\dfrac{\lambda}{2c} + \dfrac{(1-\lambda)}{2a}\right)\theta_1 & if \quad 0 \leq \theta_1 \leq a \\ \dfrac{(1-\lambda)}{2} + \dfrac{\lambda}{2c}\theta_1 & if \quad a \leq \theta_1 \leq b \\ \left(\dfrac{\lambda}{2c} + \dfrac{(1-\lambda)}{e}\right)\theta_1 & if \quad b \leq \theta_1 \leq c \\ \dfrac{\lambda}{2} + \dfrac{(1-\lambda)}{e}\theta_1 & if \quad c \leq \theta_1 \leq d \\ \left(\dfrac{\lambda}{f} + \dfrac{(1-\lambda)}{e}\right)\theta_1 & if \quad d \leq \theta_1 \leq e \\ (1-\lambda) + \dfrac{\lambda}{f}\theta_1 & if \quad e \leq \theta_1 \leq f \\ 1 & if \quad \theta_1 > f. \end{cases} \tag{15}$$

When $\alpha = 0$ we have that $F_T(\theta_1) = F_M(\theta_1)$, since the peer effect plays no role on human capital accumulation.

[11] One might consider that mixing represents the public education system whereas tracking represents a private system where only individuals with high levels of innate ability (and wealth) are accepted. Thus, A.2 implies that the best public school student can achieve a higher level of human capital than the worst private school student.

The expected value of θ_1 under tracking is:

$$E_T(\theta_1) = (1-\lambda)\frac{a}{4} + (1-\lambda)\frac{3b}{4} + \lambda\frac{c}{4} + \lambda\frac{3d}{4}, \tag{16}$$

and using Equations (10) to (13):

$$E_T(\theta_1) = \frac{1}{8}\left[(4 + (1/4)^{\alpha}\left(1+3^{\alpha+1}\right)) + \lambda(1/4)^{\alpha}\left(1+3^{\alpha+1}\right)\left(x^{1-\alpha}-1\right)\right]. \tag{17}$$

As in the case of mixing, the expected value of θ_1 is a weighted average of the mean value of θ_1 in the four income groups analyzed above. It is also increasing in both x and λ.

2.3.3 A preliminary comparison of mixing and tracking

In this section I compare both systems in terms of the induced distributions of human capital at the end of compulsory school. The first criteria that could come to our minds is first order stochastic dominance. If the distribution of human capital under a particular system dominates the other, we could say that the former system is better because all individuals prefer it.

However, it is easy to see that neither system dominates the other according to this criterion. To see this, note that from $F_T(\theta_1)$ in Equation (15) and $F_M(\theta_1)$ in Equation (6) we have that for any $\theta_1 \in (0, a]$, $(F_T(\theta_1) - F_M(\theta_1)) > 0$ for every λ, α and x. It can also be checked, using Equations (15) and (6), that for any $\theta_1 \in [d, f]$, $(F_T(\theta_1) - F_M(\theta_1)) < 0$. Similarly it is easy to see that the criterion of second order stochastic dominance is inconclusive as well (see the Appendix B for a proof).

Therefore we can conclude that given the choice between tracking and mixing there will be no unanimity in society as to which system to choose: some individuals will prefer tracking and others will prefer mixing, which means that in going from one system to another there will always be winners and losers.

One final possibility is just to compare both systems in terms of average human capital. In the next proposition I show that average human capital is always maximized under tracking. While the result is an almost immediate consequence of the model, it is worth stating formally since it considerably facilitates the analysis of the rest of the paper.

Proposition 1 *Let $\alpha > 0$. Then, $E_T(\theta_1) - E_M(\theta_1) > 0$ for all x and λ.*
Proof. *See Appendix B.* ■

If the goal of government is to maximize average human capital across the population at the end of the compulsory level, it should choose the education system that groups students according to ability. It is important to note that this result may be driven by previous assumptions about the human capital production function, in particular the complementarity between peer effect, $\overline{\theta_0}^j$, and innate ability θ_0 [12]. I propose in Appendix A an alternative production function to allow for the possibility that $\overline{\theta_0}^j$ and θ_0 are substitutes. The main (numerical) results imply that the difference $E_T(\theta_1) - E_M(\theta_1)$ decreases as $\overline{\theta_0}^j$ and θ_0 become better substitutes. In fact, when $\overline{\theta_0}^j$ and θ_0 are close substitutes, it may happen that $E_T(\theta_1) - E_M(\theta_1) < 0$.

3 Comparing Education Systems at College

Now I turn to how individuals decide whether to attend college. I am interested in how the system chosen at compulsory level, tracking or mixing, might affect this decision. I assume that individuals want to maximize their consumption, which is equal to their lifetime income, which is assumed to be a linear function of the level of human capital. First, if an individual does not go to college, she will work as an unskilled worker for fraction γ of the first period and for the whole second period. Her lifetime income will be $\theta_1(1+\gamma)$. Second, the lifetime income of those individuals who decide to go to college is the skilled wage, i.e. the increased level of human capital after attending college, $\theta_2 = \theta_1(1 + \delta(\theta_1))$. Finally, individuals take the education system, either tracking or mixing as given. Then, for all individuals who decide to attend college, the following condition must hold:

$$\theta_1(1 + \delta(\theta_1)) \geq \theta_1(1 + \gamma),$$

or,

$$\delta(\theta_1) \geq \gamma. \tag{18}$$

This condition determines a minimum level of human capital accumulated through compulsory education, θ_1^*, such that only individuals above that threshold will attend college.[13] That is, $\theta_1^* \in (0, f)$ is the value that satisfies the previous Equation with equality. Determining the interval in which the threshold level θ_1^* is placed is crucial,

[12] See Arnott and Rowse (1987) for an analysis of the optimal allocation of students and resources when peer effects are present. They conclude that the optimal allocation, when the objective is to maximize mean performance, depends on the properties of the education production function.

[13] To ensure that θ_1^* is interior we will assume that $\delta(\theta_1 = 0) < \gamma < \delta(\theta_1 = f)$.

since this determines the composition of the student body under tracking. It depends on both the efficacy of higher education described by $\delta(\theta_1)$, and the opportunity cost of attending college measured by γ. In particular, for a given function $\delta(\theta_1)$, an increase in γ will increase θ_1^* meaning that a lower proportion of individuals will attend college. In addition, for a fixed γ, an upward shift of $\delta(\theta_1)$ implies that a higher proportion of individuals will attend college.

Now I turn to a comparison of the two systems once individuals have decided on college attendance. First, I analyze which system maximizes college attendance. Next I propose a criterion of equality of opportunities that consists of minimizing the income premium, where this income premium is defined as the difference between the rich and the poor in the probability of attending college.

3.1 Proportion of college students

One might think of higher education as positive for the individual and her well-being, but one might also think of the positive externalities generated by more highly educated people for the society as a whole. In this regard, Moretti (2004) finds empirical evidence suggesting that an increase in the supply of college graduates not only increases wages among the less-educated, but also among the high-school educated. In addition it is well known the significance of human capital accumulation for economic growth, and as a result there is much policy focus on promoting human capital formation (see, for example, PISA 2003 Report).

One important point in this regard is the extent to which the demand for higher education is affected by the education system prevailing at compulsory level through the existence of peer effects. So, we want to analyze which system -tracking or mixing-provides higher education to the highest number of individuals.[14]

I denote by π_s the proportion of individuals attending college under education system s, for $s = M, T$, that is $\pi_s = 1 - F_s(\theta_1^*)$. I also define the probability of attending college conditional on individual background. I denote by $\pi_{1,s}$ the probability of attending college for an individual with poor parents ($x = 1$), that is $\pi_{1,s} = 1 - F_s(\theta_1^* \mid x = 1)$, and by $\pi_{x,s}$ the probability of attending college for an individual with rich parents, that is, $\pi_{x,s} = 1 - F_s(\theta_1^* \mid x > 1)$. Both probabilities depend on the minimum level of human capital required to attend college, θ_1^*.

[14] See Schofield (1995) for a discussion of the possible determinants of the impact of tracking on college attendance from a sociological point of view.

To compare total college attendance under the two education systems I need first to analyze the value of θ_1 for which the two cumulative distribution functions cross, since the relationship between this value and θ_1^* is crucial in determining which system maximizes college attendance. I find that the crucial parameter to determine the value of θ_1 where they cross is λ, the proportion of rich individuals in the population. To see this, Propositions 2 and 3 below show that which system maximizes college attendance depends on the particular income level in the population.

In Section 2 we saw that F_M cuts F_T from below for any value of λ, α and x. The next Proposition shows that the threshold value of θ_1 must be in one of only two different intervals, depending on the value of λ.

Proposition 2 *There is a unique $\widetilde{\theta_1} \neq 0$ such that $F_T(\widetilde{\theta_1}) = F_M(\widetilde{\theta_1})$. There is a threshold value $\widehat{\lambda}$ such that, if $\lambda < \widehat{\lambda}$, then $\widetilde{\theta_1} \in (a, b)$. If $\lambda > \widehat{\lambda}$, then $\widetilde{\theta_1} \in (c, d)$.*

Proof. See Appendix B. ■

Thus, we have that if $\theta_1 \in (0, \widetilde{\theta_1})$ then $F_T(\theta_1) - F_M(\theta_1) > 0$, and if $\theta_1 \in (\widetilde{\theta_1}, f)$ then $F_T(\theta_1) - F_M(\theta_1) < 0$. In other words, the density function of θ_1 under tracking accumulates more probability in the tails than under mixing. Moreover I find that $\widetilde{\theta_1}$ is an increasing function of the proportion of rich individuals in the economy λ, independently of the interval where $\widetilde{\theta_1}$ is located.[15] When there are few rich people (λ is low), F_M surpasses F_T for a low value of θ_1. The intuition is that family background cannot offset the peer effect, which is stronger under tracking than under mixing. As society gets richer, average human capital increases as we saw in Section 2.3, and the crossing point $\widetilde{\theta_1}$ moves to the right. In other words, the C.D.F. under mixing will be below the C.D.F. under tracking for a larger interval of values of θ_1.

Now, remember from Equation (18) that the minimum level of human capital required to attend college is an increasing function of the opportunity cost of college attendance, γ. I define two particular values of this opportunity cost that correspond to two different compositions of the college student body under tracking. I denote by $\underline{\gamma}$ the opportunity cost such that $\delta(a) = \underline{\gamma}$. That is, when the opportunity cost is $\underline{\gamma}$, we get $\theta_1^* = a$. This value $\underline{\gamma}$ is such that when $\gamma > \underline{\gamma}$, with tracking the only poor individuals attending college are those in the high track (see Figure 1). I denote by $\overline{\gamma}$ the opportunity cost such that $\delta(d) = \overline{\gamma}$. That is, when the opportunity cost is $\overline{\gamma}$,

[15] Note from Equation (15) that if $\widetilde{\theta_1} \in (a, b)$ then, $\widetilde{\theta_1} = \frac{(1-\lambda)a'2cb'}{2(a'\lambda(2c-b')+(1-\lambda)2cb')}$. If $\widetilde{\theta_1} \in (c, d)$ then, $\widetilde{\theta_1} = \frac{\lambda a'eb'}{2(\lambda a'e+(1-\lambda)b'(e-a'))}$.

we get $\theta_1^* = d$. In other words, if $\gamma < \overline{\gamma}$ all rich individuals in the high track go to college (see Figure 1).

The next Proposition shows that the education system that maximizes college attendance depends on both the opportunity cost of college attendance γ, and the proportion of rich individuals in the economy, λ.

Proposition 3 *The education system that maximizes college attendance is:*

(i) Mixing, if $\gamma \leq \underline{\gamma}$ *for any* λ.

(ii) Tracking, if $\gamma \geq \overline{\gamma}$ *for any* λ.

(iii) Mixing (Tracking) if $\lambda \geq (\leq)\ \lambda(\gamma)$*, where* $\lambda(\gamma)$ *is a threshold value weakly increasing in* γ*, for any* $\gamma \in (\underline{\gamma}, \overline{\gamma})$.

Proof. See Appendix B. ■

In Figure 2 I illustrate this result. First note that in the extreme cases $\gamma \leq \underline{\gamma}$ and $\gamma \geq \overline{\gamma}$, the proportion of rich individuals in the population plays no role in the choice of the education system that maximizes the proportion of college students. When $\gamma \leq \underline{\gamma}$, in other words, when the minimum level of human capital required to attend college is very low, college attendance is maximized under mixing. Intuitively, the variance of θ_1 under tracking is higher than under mixing.[16] If the opportunity cost is low (and more than half of all individuals attend college) then mixing increases college attendance. When $\gamma \geq \overline{\gamma}$, i.e. when the opportunity cost of college attendance is high (and, as a result, less than half of all individuals attend college), tracking maximizes college attendance.[17]

When $\gamma \in (\underline{\gamma}, \overline{\gamma})$, the education system that maximizes college attendance depends on the income level in the population. In particular, as the proportion of rich individuals rises, the optimal education system changes from tracking to mixing. The intuition is as follows. Take as given a minimum level of human capital required to attend college, θ_1^*. In poor societies we ensure that the probability of attending college

[16]It is important to stress the fact that this result does not depend on the properties of the human capital production function.

[17]The case of Spain during the eighties could be suitable to illustrate this result. Given the low rates of college attendance in those years, the priority of the government was to maximize the proportion of college students. The low opportunity cost of college attendance at that time, together with a mixing education system at compulsory level yielded an extraordinary increase in the number of college students (from 744,115 in 1983/84 to 1,508,842 in 1995/96. See Estadística Universitaria (2003)).

will be as high as possible by doing tracking. In this case the minimum level of human capital required is such that, under tracking, all poor students attending college come from the high track. For all those individuals, θ_1 is higher under tracking than under mixing since the peer variable is stronger. By doing mixing, the proportion of individuals who enjoy a higher $\theta_1$than under tracking, i.e. those with $\theta_0 < 1/2$, is smaller than the proportion of individuals who enjoy a lower θ_1 than under mixing, i.e. those with $\theta_0 > 1/2$.

In rich societies we maximize college attendance by doing mixing. Now the minimum level of human capital required is such that, under tracking, all rich students attending college come from the low track. But we already know that the human capital of those individuals is higher under mixing since the peer variable for them is stronger. By doing mixing, the proportion of individuals who enjoy a higher θ_1, i.e. those with $\theta_0 < 1/2$, is higher than the proportion of individuals who enjoy a lower θ_1, i.e. those with $\theta_0 > 1/2$.

Finally, note that as the minimum level of human capital required to attend college increases a lower proportion of students will benefit from being under mixing, i.e. those with $\theta_0 < 1/2$, in particular the rich ones. Therefore, the proportion of rich students in the population needs to be high enough to offset the lower θ_1 achieved by those individuals with $\theta_0 > 1/2$, in particular the poor ones.

3.1.1 Rawlsian Criteria

Throughout the paper it is implicitly assumed that individuals derive utility from income, which they get in the labor market, either as skilled or unskilled workers. Individuals maximize utility when deciding whether to attend college or not.

Suppose now that the government wants to maximize the utility of the worst-off individuals in society. To do this we have to define first who are the worst-off. If, for example, we take as the worst-off those with innate ability below the median level and with poor parents, the result is quite straightforward. Mixing is always better. This comes directly from the properties of the human capital production function (Equations (1) and (2)), since maximizing the utility of these individuals implies maximizing their human capital at compulsory level θ_1, which in turn increases their probability of college attendance.[18]

[18]Note that this applies to all individuals with $\theta_0 < m$, except for any individual with $\theta_0 = 0$. It applies to all individuals for any other uniform distribution whose lower bound is strictly positive.

Therefore, I propose to widen the concept of worse-off individuals by considering all individuals with poor parents as worst-off . Thus, I assume that the government chooses the education system that maximizes the utility of this group by maximizing their probability of attending college. I define two particular values of the opportunity cost of college attendance. I denote by γ_0 the opportunity cost such that $\delta(a'/2) = \gamma_0$. That is, if the opportunity cost is γ_0 we get that $\theta_1^* = a'/2$.

In the next Proposition I show that when the opportunity cost of college attendance γ is low, mixing maximizes college attendance, and the reverse is true for high levels of γ.

Proposition 4 *If* $\gamma < (>)\ \gamma_0$ *then mixing (tracking) maximizes college attendance of the worst-off.*

Proof. See Appendix B. ■

So, when $\theta_1^* < (1/2)a'$, the proportion of individuals above θ_1^* is higher under mixing than under tracking. In particular, the human capital acquired under mixing is equal to $\theta_0 a'$ whereas under tracking is equal to $\theta_0 2a$. The reverse occurs when $\theta_1^* > (1/2)a'$. In that particular case the human capital acquired under mixing is equal to $\theta_0 a'$ whereas under tracking is equal to $\theta_0 e$.

In Table 1 below I summarize the main results in Propositions 3 and 4:

Table 1. *Maximizing College Attendance*

$\lambda \setminus \gamma$	$\gamma < \underline{\gamma}$	$\underline{\gamma} < \gamma < \overline{\gamma}$		$\gamma > \overline{\gamma}$
		$\gamma < \gamma_0$	$\gamma > \gamma_0$	
Rich Societies| Total/Poor	M/M	M/M	M/T	T/T
Poor Societies| Total/Poor	M/M	T/M	T/T	T/T

Note: *Total=Total College Attendance;Poor=College Attendance of the poor;M=Mixing;T=Tracking*

Observe that for poor societies tracking maximizes college attendance for both the poor and the total population when the opportunity cost of attending college is high enough. For rich societies, provided that the opportunity cost of attending college is low enough, it is mixing that maximizes college attendance for both the poor and the total population.

3.2 Equality of Opportunities

In this section I study which system better guarantees that an individual's decision whether or not to attend college is taken independently of her parents' income. For the two systems I define the income premium $p_s(\theta_1^*)$ or income gap, as the difference in the probability of attending college between the rich and the poor under education system s, for $s = M, T$:

$$p_s(\theta_1^*) = \pi_{x,s} - \pi_{1,s}. \tag{19}$$

If the government wants to guarantee equality of opportunities, it should choose the system s that makes $p_s(\theta_1^*) = 0$. Since this is not possible because $p_s(\theta_1^*)$ is always strictly positive, an education system is called "equitable" if it minimizes the income premium $p_s(\theta_1^*)$. As we will see below, which system is the most equitable depends on the minimum level of human capital required to attend college. Observe also that the income premium is defined only for strictly positive values of both $\pi_{x,s}$ and $\pi_{1,s}$, that is, when the minimum level of human capital required to attend college in each system is such that there are individuals with poor and rich parents attending college. This is equivalent to saying that the income premium is defined for every $\theta_1^* \leq a'$.

In the following proposition I state which system is more equitable. I find that this depends both on the minimum level of human capital required to attend college and on the level of income inequality in the population.

Proposition 5 *Define $\widetilde{x}(\alpha) = 2^{3\left(\frac{\alpha}{1-\alpha}\right)}$. There is a minimum level of human capital required to attend college $\eta_1 \in (a, b)$ such that:*

(i) When $\theta_1^* \leq \eta_1$ the following two statements hold:

(i.1) If $x \leq \widetilde{x}(\alpha)$ tracking is the most equitable system.

(i.2) If $x \geq \widetilde{x}(\alpha)$ mixing is the most equitable system.

(ii) When $\theta_1^* > \eta_1$ tracking is the most equitable system independently of the level of income inequality.

Proof. See Appendix B. ■

I illustrate this result in Figure 3. First of all note that when $\theta_1^* \leq \eta_1$ every individual placed in the high track will attend college, and thus individuals from both rich and poor families share the same probability of attending college, which is 1/2. This implies that the income premium under tracking is capturing only the income gap for those individuals placed in the low track.

The proposition above shows that when income inequality is low, tracking is the most equitable system, independently of the level of human capital required to attend college. If this is not the case and income inequality is high, tracking becomes the most equitable system as θ_1^* rises.

In addition, the above proposition shows that when the minimum level of human capital required to attend college is high enough, tracking is the most equitable system. The intuition is that in that case there are no poor individuals from the low track attending college. Since the positive effect of the peer variable is higher for the most skilled individuals under tracking than under mixing and, in particular, for those students placed in the high track, their family background is not a crucial factor to determine their final level of human capital. However, under mixing, the backgrounds of individuals with the same level of innate ability, have a higher relative weight, and as a result the income premium is crucially affected by this variable.

4 Final Comments

In this paper I analyze public intervention in education when the government has to decide how to group students. I analyze two different education systems: tracking and mixing. Thus, the objective of this paper is to evaluate the two education systems using several criteria.

Some previous works have studied the optimal education system at compulsory level by focusing on mean achievement. My paper contributes to this line of research in two directions. The first contribution of the paper is to compare both systems in terms of the induced distributions of human capital at the end of compulsory school. The second one lies on highlighting the importance of peer effects on college choices.

I believe my results are relevant for several issues in the literature on the economics of education. Studies that link persistent levels of inequality in the population to neighborhood effects provide an interesting example. Empirical investigation of college choices and the impact of different financial schemes on college attendance decisions could also benefit from our analysis.

In this paper I assume that the achievement of individual students rises with an improvement in the average level of their classmates, but at a decreasing rate. Another type of non-linearities of peer group effects is through the "distance" impact. There is empirical evidence that suggests that peer effects are stronger when the distance between the individual's innate ability and the average innate ability in the classroom is small, and that as this distance increases, peer effects become almost negligible. Although I do not model this effect it can be checked that it will only reinforce my main results without adding additional insights. For example, regarding Proposition 1 observe that, under tracking, students in the high track gain more because the peer effect is stronger and because the distance to the average ability in the group is lower. Individuals in the low track first lose because of a lower average ability in the group, but now there is a new positive effect since the distance to the average ability in the group is lower.[19]

The paper allows for some extensions. On the one hand it might be interesting to check the robustness of the main results to particular features of the model. It could be important to relax some assumptions, in particular some properties of the human capital production function. For example, I could introduce different measures of the peer effect, e.g. for example some measure of the level of heterogeneity in the group. Moreover, it might be interesting to consider other distributions of innate ability. Other possible ways of modelling the tracking system could be considered. For example, introducing the possibility of students being placed in tracked classes for only a subset of subjects as in Epple, Newlon and Romano (2002). In addition to adding realism, incorporating this possibility would help to determine the design of an optimal education system. On the other hand it might also be interesting to compare the two education systems in a dynamic set up.

[19] See for example Manski and Wise (1983).

5 Appendix A

In Proposition 1 we saw that under some particular assumptions about the human capital production function, average human capital at compulsory level is maximized under tracking. Since this result might be sensitive to technological assumptions, and in particular to the fact that $\overline{\theta_0}^j$ and θ_0 are complements, here I propose a different production function to see how the result of Proposition 1 may change:

$$\theta_1 = z^{1-\beta_1}((1-\varepsilon)\theta_0^{\beta_2} + \varepsilon(\overline{\theta_0}^j)^{\beta_2})^{\frac{\beta_1}{\beta_2}}, \tag{20}$$

where β_1, β_2 and $\varepsilon \in (0,1)$. It can be checked that the properties of Equations (1) and (2) hold except for the relationship between $\overline{\theta_0}^j$ and θ_0. In particular, for β_2 close to 0, both $\overline{\theta_0}^j$ and θ_0 have some level of complementarity as in Equation (1) and as β_2 tends to 1 the two factors become perfect substitutes.

Due to the complexity of the above human capital production function, I cannot obtain clear analytical results regarding the maximization of average human capital. However we can extract some conclusions using numerical simulations. The most important one is that the difference between average human capital under the two systems, $E_T(\theta_1) - E_M(\theta_1)$, decreases with β_2. In the following table I present the value of β_2, for some fixed values of β_1 and ε, such that $E_T(\theta_1) - E_M(\theta_1) = 0$:

Table 2. *Maximizing Average Human Capital*				
$\varepsilon \setminus \beta_1$	.2	.4	.6	.8
1/2	.549	.649	.885	.941
2/3	.542	.640	.740	.860
3/4	.449	.530	.650	.810

(21)

Thus, we can conclude that if β_2 is close to 0, i.e. when $\overline{\theta_0}^j$ and θ_0 have some level of complementarity, then average human capital is always maximized under tracking. As β_2 tends to 1, meaning that the two factors become closer substitutes, average human capital can be maximized under mixing under some parameter values, in particular when the peer effect is strong (ε is high) and family background does not play an important role in human capital accumulation (β_1 is high). However, it can also be checked that with this human capital technology (Equation (20)) neither education system dominates the other in the sense of first order stochastic dominance.

6 Appendix B

Proposition 6 $F_r(\theta_1) \not\succeq_{SOSD} F_s(\theta_1)$ *for* $r, s = M, T$ *and* $r \neq s$.

Proof. (i) $F_T(\theta_1) \not\succeq_{SOSD} F_M(\theta_1)$. Using $F_T(\theta_1)$ from (15) and $F_M(\theta_1)$ from (6) we can check that, $\int_0^b (F_T(\theta_1) - F_M(\theta_1))d\theta_1 > 0$, for every λ, α and x. (ii) $F_M(\theta_1) \not\succeq_{SOSD} F_T(\theta_1)$. Recall that the expected value of a random variable defined on $[0, \overline{z}]$ can be written as: $E[z] = \overline{z} - \int_0^{\overline{z}} F(z)dz$. But then, if $F_M(\theta_1) \succeq_{SOSD} F_T(\theta_1)$ then the following inequality should hold: $f - E_M(\theta_1) \leq f - E_T(\theta_1)$. The final result is immediate from Proposition 1. ■

Proof of Proposition 1: From Equations (7) and (17), and rearranging terms, we have that the difference between the average human capital under tracking and mixing, $E_T(\theta_1) - E_M(\theta_1)$ is positive if and only if the following condition holds:

$$\frac{1}{8}\left(\frac{1}{2}\right)^{\alpha}\left(\lambda x^{1-\alpha} + (1-\lambda)\right)\left(\left(\frac{1}{2}\right)^{\alpha} + 3\left(\frac{3}{2}\right)^{\alpha} - 4\right) > 0.$$

This expression is positive if and only if $\left(\left(\frac{1}{2}\right)^{\alpha} + 3\left(\frac{3}{2}\right)^{\alpha} - 4\right) > 0$ for all α. But this expression is positive and strictly increasing when $\alpha > 0$ and is zero when $\alpha = 0$. This proves the claim. ■

Proof of Proposition 2: I denote by $\widehat{\lambda}$ the proportion of rich individuals in the economy such that $\frac{\widehat{\lambda}}{1-\widehat{\lambda}} = \Lambda$, where Λ is defined as $\Lambda = \dfrac{2cb'(e-a')}{(b'-2c)a'e}$. In Section 2 it is shown that $F_T(\theta_1) - F_M(\theta_1) > 0$ for all $\theta_1 \in (0, a)$, whereas $F_T(\theta_1) - F_M(\theta_1) < 0$ for all $\theta_1 \in (d, f)$. It is easy to verify also that $F_T(b) - F_M(b) < (>)0$ if and only if $F_T(c) - F_M(c) < (>)0$. If we evaluate the two C.D.F. under mixing and tracking for $\theta_1 = c$, we can check that, $F_M(c) = \dfrac{c}{b'}\lambda + \dfrac{c}{a'}(1-\lambda)$ and $F_T(c) = \dfrac{c}{e}(1-\lambda) + \dfrac{\lambda}{2}$. Thus, from Equation (15) $F_T(c) > F_M(c)$ if and only if $\dfrac{\lambda}{1-\lambda} > \Lambda$. The final result follows immediately from the definition of $\widehat{\lambda}$. ■

Proof of Proposition 3: From Proposition 2 and the fact that F_M always cut F_T from below, a necessary and sufficient condition to ensure that $\pi_M(\theta_1^*) > (<)\ \pi_T(\theta_1^*)$ is that $\widetilde{\theta}_1 > (<)\ \theta_1^*$. If $\gamma < \underline{\gamma}$ then, from Proposition 2 we have that $\widetilde{\theta}_1 > \theta_1^*$ for all λ. Now assume $\gamma \in (\underline{\gamma}, \overline{\gamma})$. If γ is such that $\theta_1^* \in (b, c)$ then, from Proposition 2 we have that if $\lambda < \widehat{\lambda}$ then, $\widetilde{\theta}_1 < \theta_1^*$ and if $\lambda > \widehat{\lambda}$ then, $\widetilde{\theta}_1 > \theta_1^*$. Now let γ be such that

$\theta_1^* \in (a,b)$ or $\theta_1^* \in (c,d)$. Then, for each θ_1^*, there is one λ, denoted by $\widetilde{\lambda}$, such that $\widetilde{\theta}_1(\widetilde{\lambda}) = \theta_1^*$. Thus, since $\widetilde{\theta}_1$ is increasing with λ, we have that, if $\lambda < \widetilde{\lambda}$ then $\widetilde{\theta}_1 < \theta_1^*$ and if $\lambda > \widetilde{\lambda}$ then, $\widetilde{\theta}_1 > \theta_1^*$. From Proposition 2 we have that $\widetilde{\lambda} < \widehat{\lambda}$. Finally, if $\gamma > \overline{\gamma}$ then, from Proposition 2 we have that $\widetilde{\theta}_1 < \theta_1^*$ for all λ. ■

Proof of Proposition 4: If $\theta_1^* \in (0,a')$ then $\pi_{1,M} = 1 - \frac{\theta_1^*}{a'}$ whereas if $\theta_1^* > a'$ then $\pi_{1,M} = 0$. If $\theta_1^* \in (0,a)$ then $\pi_{1,T} = \frac{2a-\theta_1^*}{2a}$ and for any $\theta_1^* \in (a,b)$ we have that $\pi_{1,T} = \frac{1}{2}$. Following the same reasoning, if $\theta_1^* \in (b,e)$ then $\pi_{1,T} = (\frac{1}{2}) - \frac{1}{2}\frac{(\theta_1^*-b)}{(e-b)}$. The proof follows just by comparing $\pi_{1,M}$ and $\pi_{1,T}$ for the different intervals. ■

Proof of Proposition 5: From Assumptions 1 and 2 $\widetilde{x}(\alpha) \in [\underline{x}(\alpha), \overline{x}(\alpha)]$. From Equations (6) and (19) we have that when $\theta_1^* \in (0,a')$, $p_M(\theta_1^*) = \theta_1^* \left(\frac{b'-a'}{a'b'}\right)$. Take first any $\theta_1^* \in (0,a)$ and then from Equations (15) and (19), $p_T(\theta_1^*) = \theta_1^* \left(\frac{c-a}{2ac}\right)$. By A.2 we have that $a < a'$. Thus, merely by comparing the two income premiums and from Equations (4) and (5) for $p_M(\theta_1^*)$, and Equations (8) and (9) for $p_T(\theta_1^*)$ we can check that $p_T(\theta_1^*) \geq (\leq)\ p_M(\theta_1^*)$ if and only if $x \geq (\leq)\ \widetilde{x}(\alpha)$. Now take any $\theta_1^* \in (a,b)$. We check that $p_T(\theta_1^*) = \frac{1}{2}\left(1 - \frac{\theta_1^*}{c}\right)$. Let $\eta_1 = \frac{a'b'c}{2c(b'-a')+a'b'}$ be a level of human capital strictly lower than b. Then, we find that a sufficient condition to ensure that $p_T(\theta_1^*) \leq p_M(\theta_1^*)$ is $x \leq \widetilde{x}(\alpha)$. If $x > \widetilde{x}(\alpha)$ then, merely by comparing the two income premiums it can be checked that $p_T(\theta_1^*) \geq (\leq)\ p_M(\theta_1^*)$ if and only if $\theta_1^* \leq (\geq)\ \eta_1$. Take any θ_1^* that belongs to (b,c). From Equations (15) and (19) we obtain $p_T(\theta_1^*) = \theta_1^* \left(\frac{c-b}{2bc}\right)$. As $b < a'$ we find that $p_M(\theta_1^*) = \theta_1^* \left(\frac{b'-a'}{a'b'}\right)$. From Equations (4) and (5) for $p_M(\theta_1^*)$ and Equations (9) and (10) for $p_T(\theta_1^*)$, we can check that $p_M(\theta_1^*) > p_T(\theta_1^*)$ is equivalent to the following expression:

$$\left(x^{1-\alpha} - 1\right)(1 + (\tfrac{3}{4})^\alpha)(1 + (\tfrac{1}{4})^\alpha x^{1-\alpha}) > (\tfrac{1}{2})^\alpha \left(x^{1-\alpha} - 3^\alpha\right)(1 + (\tfrac{1}{2})^\alpha)(1 + (\tfrac{1}{2})^\alpha x^{1-\alpha}).$$

This inequality holds for every x and α. Now take any θ_1^* that belongs to (c,d). From Equations (15) and (19) we obtain $p_T(\theta_1^*) = \frac{1}{2}\left(\frac{\theta_1^*}{b} - 1\right)$. From A.2 we have that $d < a'$, and thus, $p_M(\theta_1^*) > p_T(\theta_1^*)$ implies that the following inequality must hold: $\theta_1^* < \frac{a'b'b}{a'b'-2b(b'-a')}$. A sufficient condition to ensure it is $\frac{a'b'b}{a'b'-2b(b'-a')} \leq d$. This last inequality is equivalent to: $\frac{b'-a'}{a'b'} \geq \frac{d-b}{2bd}$. From Equations (4) and (5) for $p_M(\theta_1^*)$ and Equations (10) and (12) for $p_T(\theta_1^*)$ we check that the last inequality holds if and only if $x \geq (2^\alpha(4/3)^\alpha)^{\frac{1}{1-\alpha}}$. But this is always true since $\underline{x}(\alpha) > (2^\alpha(4/3)^\alpha)^{\frac{1}{1-\alpha}}$. Now, if θ_1^* belongs to (d,a') from Equations (15) and (19) we have that $p_T(\theta_1^*) = \theta_1^* \left(\frac{d-b}{2bd}\right)$. But we have just seen that $\frac{b'-a'}{a'b'} \geq \frac{d-b}{2bd}$, and thus $p_M(\theta_1^*) > p_T(\theta_1^*)$ in this interval. ■

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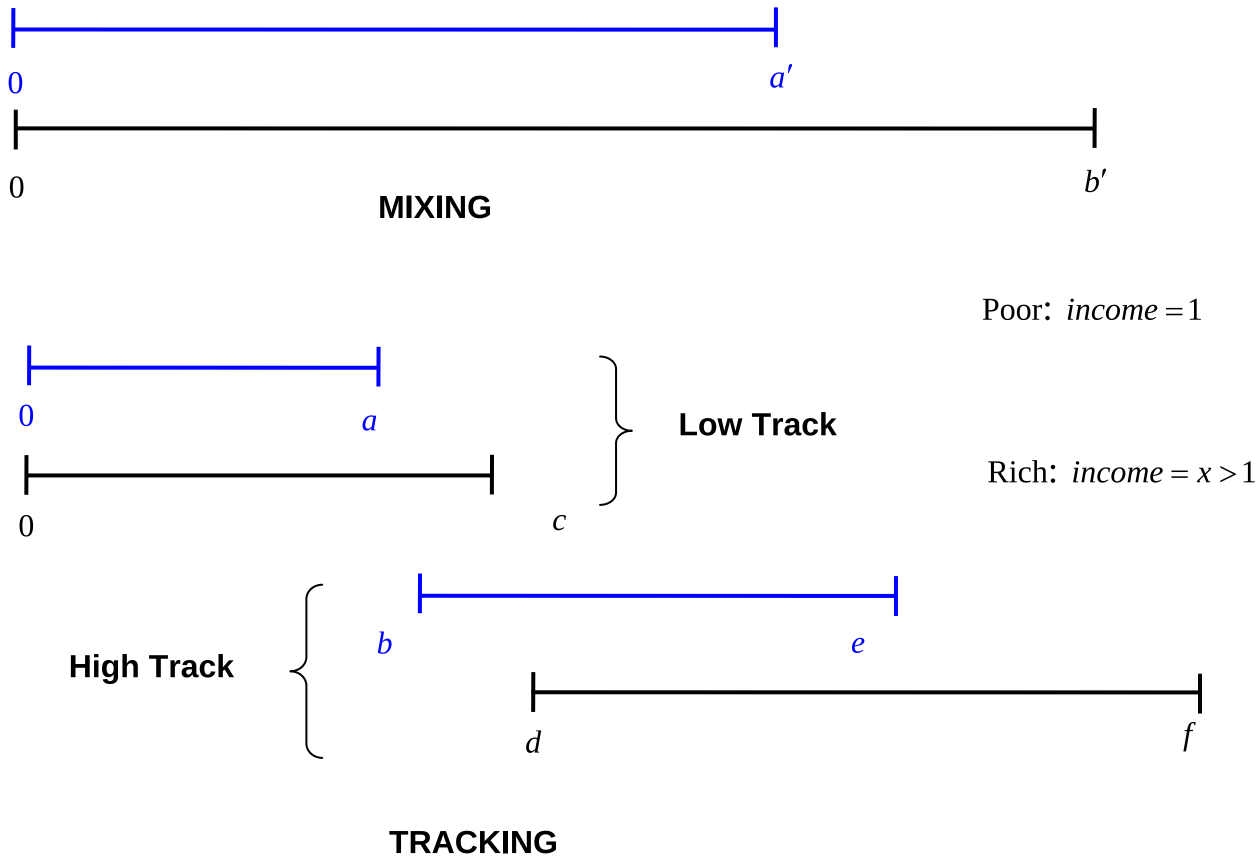

FIGURE 1.- EDUCATION SYSTEMS

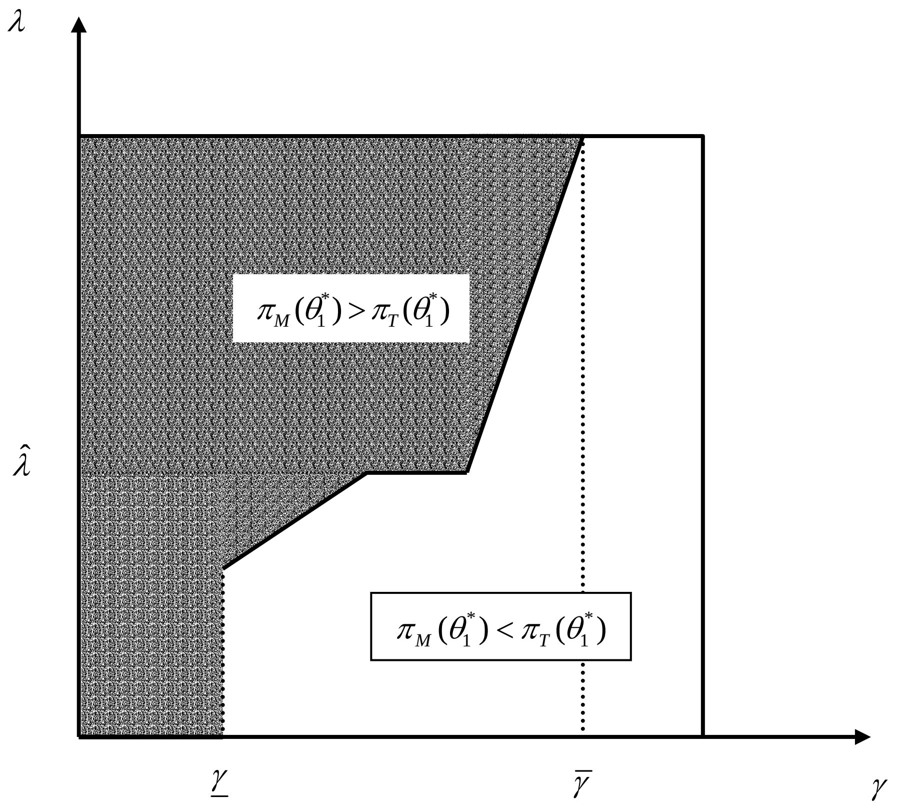

FIGURE 2.- PROPORTION OF COLLEGE STUDENTS

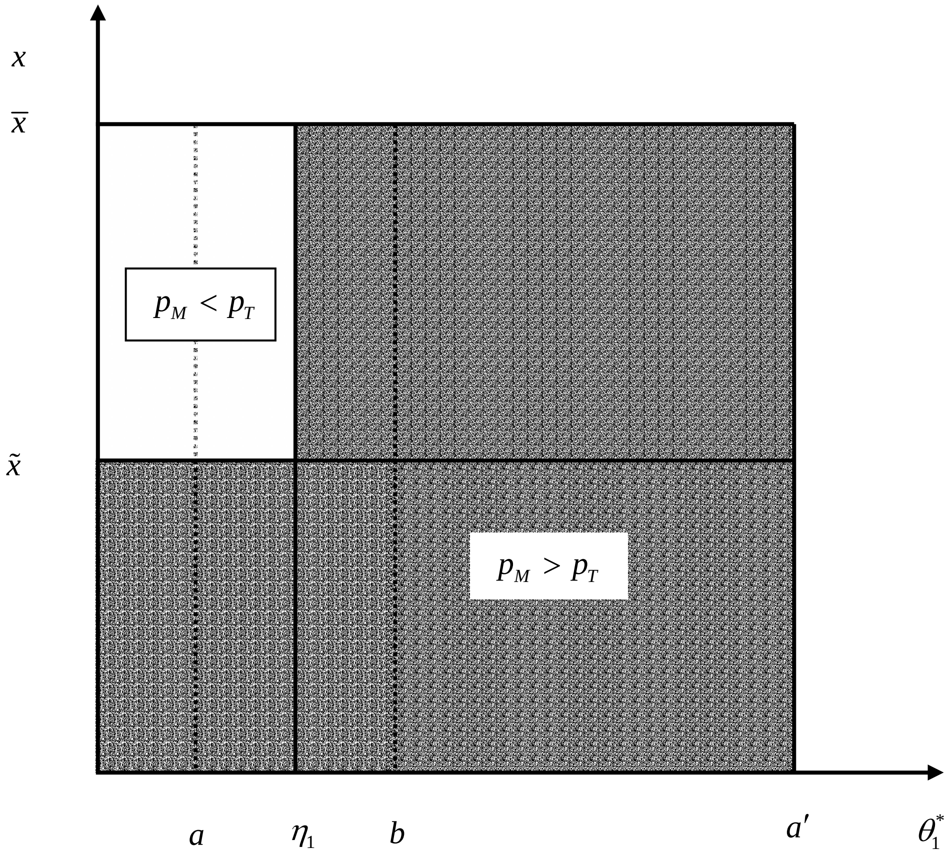

FIGURE 3: EQUALITY OF OPPORTUNITIES