Monotonic and core selections in cooperative games with transferable utility

Carlos Rafels;
Universitat de Barcelona (UB),
crafels@ub.edu

Referring to cooperative games of transferable utility, Young (1985) and Housman and Clark (1998) proved that, for general games with at least four players, no core allocation is coalitional monotonic. Coalitional monotonicity assumes that if the worth of a given coalition increases while the worth of all other coalitions remains the same then the payoff of every member of that coalition should (weakly) increase. From an economic point of view, this is a surprising and interesting result as it shows that there exists a trade-off between two important and intuitive properties: core-selection and coalitional monotonicity. Maschler (1992) pointed out the importance of this fact: "if you want a unique point outcome in the core you must face some undesirable non-monotonicity consequences. On the other hand, if you feel that monotonicity is essential, say, because it provides incentives if imposed on a society then you should discard the core, and the nucleolus is not a solution concept that you should recommend."

The core is an important set-solution concept supported by a wide range of applications. An alternative to the core is the bargaining set la Davis and Maschler (1963, 1967). It is remarkable that this bargaining set is always non-empty and includes any core element. In some situations, the bargaining set makes more sense than the core itself, as Maschler (1976) pointed out in an economic example of a game with a non-empty core. The first part of the talk analyzes whether the Youngs impossibility result still holds if we consider the bargaining set instead of the core.

The above analysis has a connection with the Meggido paper (1974) where he shows an undesirable non-monotonicity property for the nucleolus, the kernel and the bargaining set. Meggido shows a nine-player cooperative game where the above solution concepts are not aggregate-monotonic, that is, an increasing of the worth of the grand coalition does not benefit all players. Latter, Hokari (2000) shows that the nucleolus is not aggregate-monotonic even in the domain of convex games with at least four players. Nevertheless, core-selection and aggregate-monotonicity are compatible properties on the domain of all games (Young et al. 1982, Moulin 1988).

We introduce the aggregate-monotonic core of a game as the set of allocations attainable by single-valued solutions that satisfy core-selection and aggregate-monotonicity. Firstly, we provide an explicit description of the aggregate-

monotonic core of a balanced game as a non-empty subset of its core. The main result identifies those games for which the aggregate-monotonic core coincides with the core. Finally, we introduce upper and lower aggregate-monotonicity for set-valued solutions, and we characterize the aggregate-monotonic core using core-selection and upper and lower aggregate-monotonicity.