# Strategic characterization of the index of an equilibrium

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The index of a Nash equilibrium is an integer that is related to notions of "stability" of the equilibrium, for example under dynamics that have equilibria as rest points. The index is a relatively complicated topological notion, essentially a geometric orientation of the equilibrium. We prove the following theorem, first conjectured by Hofbauer (2003), which characterizes the index in much simpler strategic terms:

Theorem. A generic bimatrix game has index +1 if and only if it can be made the unique equilibrium of an extended game with additional strategies of one player. In an mxn game, it suffices to add 3m strategies of the column player.

The main tool to prove this theorem is a novel geometric-combinatorial method that we call the "dual construction", which we think is of interest of its own. It allows to visualize all equilibria of an mxn game in a diagram of dimension m-1. For example, all equilibria of a 3xn game are visualized with a diagram (essentially, of suitably connected n+3 points) in the plane. This should provide new insights into the geometry of Nash equilibria.