

# The division problem with voluntary participation

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Sprumont (1991) *Econometrica* 59: 509-519, studies the problem of dividing a given amount of a good among a set of agents having single peak preferences over the amount they receive. He characterizes the uniform rule as the unique rule satisfying efficiency, anonymity and strategy-proofness.

Sprumont assume that each agent can receive any amount between 0 and the total amount to divide. We think that this assumption becomes unrealistic in many cases because agents can have an outside option, for instance do not participate. Thus, agent preferences are as follows. They first prefer something in  $[l_i, u_i]$ . We assume that preferences over  $[l_i, u_i]$  are single-peak. Secondly, agents prefer do not participate, which is identified with receiving 0. Finally, they prefer to receive something outside  $[l_i, u_i] \cup 0$ . We assume that agents can not be forced to participate if they do not want. Thus a rule should assign to each agent something in  $[l_i, u_i] \cup 0$ . Moreover, the total amount assigned must be the total amount of the good or nothing.

We prove that in our setting there is no rule satisfying efficiency and strategy-proofness. There is no rule satisfying efficiency and anonymity. Even there are rules satisfying strategy-proofness and anonymity (for instance each agent receives 0), these rules does not seem to be very interesting. This means that the result of Sprumont (1991) can not be extended to our setting.

Sonmez (1994) characterizes the uniform rule with four properties: efficiency, consistency, individually rational from equal division, and one-side resource monotonicity. Consistency is defined in the same way in our setting. The last two properties should be adapted to our setting. Then, we characterize the class of rules satisfying these four properties.

We consider the property of admissible contraction. A coalition  $S \subset N$  is admissible according with the profile  $\succsim$  if there is possible to divide the good among the agents in  $S$  without violating the participants constraints given by  $[l_i, u_i]$ . Let  $A(\succsim)$  denote the set of admissible coalitions according with  $\succsim$ . Assume that the rule  $f(\succsim)$  divide the good among agents in  $S \in A(\succsim)$ . Let  $\succsim'$  be a preference profile satisfying that  $S \in A(\succsim') \subset A(\succsim)$ . Thus,  $f(\succsim)$  must divide the good among agents in  $S$ .

Finally we add to the previous characterization the property of admissible contraction. We characterize the set of rules satisfying these five properties.