

Random–Walk–Based Segregation Measures*

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March 22, 2011

Abstract

In this paper, we propose an intuitive way of measuring residential segregation in social networks. Individuals are located in different nodes on a network that are interconnected through links. In this setting, we consider the process according to which, every period, an individual either moves to an adjacent node or she stops walking over the network and define the *segregation index* as the probability that a randomly chosen individual meets an individual of the same social group when this random-walk terminates. It is shown that the segregation index is a natural generalization of the well-known isolation index to networks, that it has a closed-form relation to the PageRank index applied by Google in order to determine the importance of webpages, and that it is continuous when two separated components of the networks are integrated. The spectral segregation index proposed by Echenique and Fryer (2007) does not satisfy the latter property. Finally, we apply the segregation index to the Spanish 2009 census tract data.

Keywords: Isolation, Clustering, Network, PageRank, Random-Walk, Segregation.

JEL–Numbers: C0, D85, Z13.

*We are very grateful to Antonio Cabrales, Matthew Jackson and Yves Zenou for very fruitful discussions. We also thank the seminar/conference participants at ASSET (Alicante), IMEBE (Bilbao), FEDEA, Lisbon, Murcia, Saarbrücken, and Valencia for many very helpful comments. It would have been impossible to conduct the empirical analysis without the financial support of FEDEA.

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1 Introduction

Residential segregation—the degree to which social groups live separate— is known to have a variety of effects on socioeconomic variables. Benabou (1993) studies theoretically a model with ex ante identical individuals who have to choose their level of education (they become either high-skilled, low-skilled, or drop out of the labor market) and where to live. Since education is assumed to be a club good, there is a benefit from being close to those who share the same characteristics, which implies that high-skilled workers are willing to pay more for their houses in a neighborhood with more high-skilled. It is then shown that the resulting stratification may not only turn neighborhoods with lesser educated into unproductive ghettos, the overall efficiency may even be inversely related to the level of segregation. Borjas (1995), on the other hand, investigates empirically the relation between residential segregation and human capital and finds that those with a higher ethnic capital—the average skills of the ethnic group in the parents generation—are more mobile. Cutler and Glaeser (1997) examine the effects of segregation for schooling, employment, and single parenthood in the U.S. Controlling for the choice of where to live, it is shown that blacks in more segregated areas have significantly worse outcomes than blacks in less segregated areas. Using data from a natural experiment in Sweden, Edin et al. (2003) establish that segregation can also have positive effects on labor market outcomes: less skilled immigrants earn more when living in an “ethnic enclave” and members of a group with a relative higher income benefit more from being in an enclave than members of a group with a relative lower income. Finally, the interim evaluation of the Moving To Opportunity program in the U.S. by Kling et al. (2007) uncovers that the beneficiaries of the intervention do not fare better than the control group in terms of their economic self-sufficiency and physical health but in terms of their mental health.

The most basic framework that addresses the question of how to actually measure residential segregation starts with some urban space (*i.e.*, a city) that is divided into smaller units called census tracts or neighborhoods. Also one takes the distribution of the different social groups (the population can be partitioned using individual characteristics such as ethnicity, religion, or nationality) over census tracts as given. Within this setting, the classical work of Massey and Denton (1988) introduces the following five dimensions of residential segregation: *evenness* or the extent to which a group is distributed homogeneously over neighborhoods; *exposure* or the degree of potential contact to other groups; *concentration* or the amount of physical space a group occupies; *centralization* or the degree to which a group resides close to the city center; and *clustering* or the extent to which individuals from the same group tend to live in neighborhoods next to each other. The authors then review a total of 20 measures from which they isolate with the help of a factor analysis one for each dimension.

In a recent major contribution, Echenique and Fryer (2007) apply graph theory to derive a measure of (residential) segregation. The social interaction framework studied is based on the assumption that individuals divide their time among their neighbors/friends and the more time they spend with individuals from their group, the more segregated the group as a whole is. The segregation of a group is in this sense directly related to the level of within-group interactions. More formally, individuals are located on the vertices of a graph with the edges representing social relations. Using this as the only information, the *Spectral Segregation Index (SSI)* for a given group is calculated as follows. One determines first the subgraph that only contains the interactions between members of the considered group. Since such a subgraph consists generally of more than one connected component—a connected component is a maximal set of individuals such that there is a path between any pair of individuals belonging to the set—it is suggested to determine the segregation for each connected component

separately and to aggregate these values afterwards through a weighted average. In particular, the *SSI* of a connected component is set equal to its spectral radius, the largest absolute value of the eigenvalues of the matrix representing the social ties in that component. One advantage of this approach is that one can also obtain a level of segregation for every individual by rescaling the entries of the eigenvector corresponding to the spectral radius in such a way that their average is exactly equal to the respective spectral radius.

Our setting departs from the framework of Echenique and Fryer (2007) only in that we allow multiple individuals to be located in the same node. This allows for more straightforward empirical applications in situations when one group is in majority in *almost all* census tracts, which happens to be the case for our data. More importantly, the measure we develop is different in nature than the *SSI* and, in particular, it is related to the following process: pick any two individuals from a given group s at random and suppose that the first of the two individuals starts to move over the network (city) from her position (area of residence) in such a way that each period, she either advances to some adjacent node with probability $\alpha \geq 0$ (*i.e.*, if node i has k adjacent nodes each of these nodes might be reached with probability α/k) or the process stops with probability $1 - \alpha$. Thus, $1 - \alpha$ can be regarded as a parameter that accounts for the degree of viscosity of social relationships. The *normalized segregation index* for group s , denoted by $\bar{\sigma}_s$, is then defined as the probability that the first randomly chosen individual meets the second one when her random-walk terminates.^{1,2}

¹An alternative description of $\bar{\sigma}_s$ is as follows: suppose one individual wants to pass a message to another individual from the same social group. Every period, the current holder of the message gives it to a random individual she knows but the process stops with some probability. The normalized segregation index can then be envisioned as the probability that the process stops when the message is at its destination.

²The use of a random-walk based model is appealing in this setting because social interactions are often random. People meet friends, friends of friends, or higher-order acquaintances in a random fashion. Also, socioeconomic relationships are usually recursive, self-enforcing, and with feedback effects. This implies that social interactions are not necessarily explained by shortest-path arguments, and a model based on random-walks is a convenient tool in order to take into account both the randomness and the recursive nature of social interactions. For example, Jackson and Rogers (2007) show that a random link formation process matches empirical networks very well in various situations.

Intuitively, the segregation index $\bar{\sigma}_s$ is controlling for the group size given that *both* individuals are chosen at random. The measure is also robust to the number of census tracts and the population size and, in this sense, it is a normalized or relative coefficient of segregation. But there is a more important feature hidden in these invariance properties: the measure is robust to the definition of the boundaries of the census tracts. That is, if we assume that a census tract is the basic unit of interaction between citizens, then changing the boundary of a census tract will not much affect $\bar{\sigma}_s$. The geographical network can then be interpreted as a network of people, where each node is an individual, and not merely as a network of arbitrarily defined nodes. This makes the choice of boundaries more natural—less arbitrary—given that, in fact, the frontier of social interactions start at every social actor. Nevertheless, it can still be convenient to apply a group-size dependent measure, so we also define the *segregation index* σ_s .³ The interpretation of σ_s is then akin to that of the isolation index, an important measure of exposure (see, Lieberman 1981); *i.e.*, the probability that a randomly chosen individual of group s will meet *any* individual from the same group when the random-walk terminates.

Our main theoretical result is a dual theorem (Propositions 1 and 2) showing that the segregation index can be interpreted in two different ways. First, it is a natural spatial⁴ generalization of the isolation index to networks. In fact, σ_s reduces to the isolation index of group s if the socio-geographical network is empty or, alternatively, if the transition probability α is zero, but generally it takes into account social ties to neighborhoods that are not too far away. Thus, σ_s explicitly accounts for the intensity of interactions within group s , yet the

³There are a variety of measures in the literature that are either relative or absolute with respect to the group size. For instance, the isolation index, the exposure index, and the mutual information index are sensitive to group sizes, even though the first two allow for straightforward normalizations. By contrast, the dissimilarity index, the Gini, and the Atkinson index do not depend on the group size.

⁴We refer here to an abstract notion of space. Space could be regarded as the geographical or social setting where individuals interact.

measure is nevertheless directly related to the level of interaction with other groups as well because more intra-group interactions prevents the group from socializing with others.

Second, the segregation index is also related to the dimension of centralization because groups that are closer to the socio-geographical center of the network are more likely to interact with individuals from the same group everything else equal. The relevance of centralization as a dimension of segregation has been reported in some countries like the U.S., where socioeconomic factors can cause some groups to move to the city center. In particular, the spatial mismatch literature emphasizes that blacks who tend to live closer to the city center have worse labor market outcomes (see, Ihlanfeldt and Sjoquist 1998 and Gobillon et al. 2007). We show that the segregation index of a group s is a weighted sum over the vector of group proportions (the fraction of individuals that belong to group s in a given node), where the weight assigned to each node is assessed through the PageRank index used by Google to determine the importance of webpages in the World Wide Web. Thus, more central nodes receive more weight when measuring the segregation of a group as the likelihood of interaction is more prominent in these nodes. Observe that the notion of centrality is not absolute but relative to the target group s ; *i.e.*, it refers to how reachable a node is by a typical individual of group s .

A third theoretical contribution of the paper is related to a drawback of the *SSI* introduced in Echenique and Fryer (2007). We present an instructive example showing that the *SSI* is not continuous in the strength of the social ties because in situations when two separated components are merged by a new link, the *SSI* converges to the spectral radius of the bigger component instead of a weighted average over both connected components. We believe — and argue to more detail in the concluding discussion— that continuity is a very important property because both social and geographical networks change over time and are subject to

a continuous addition and deletion of links. In particular, abrupt changes in the *SSI* may occur if two individuals from completely disconnected components decide to interact for the first time. By considering the network structure our approach maintains the postulate of Echenique and Fryer (2007) that segregation is formed through social interactions, but we amend their drawback by developing a continuous measure.

In the last part of the paper, we apply the segregation index to the Spanish 2009 census tract data. Groups are determined at the level of nationalities because social interactions naturally emerge between individuals from the same nationality as crucial aspects like culture, language, and patriotism are often shared. In the last decade, Spain has experienced a considerable flow of immigrants, mainly attracted by the favorable conditions of the labor market, resulting in a total of 10% of non-national residents in the country. During these years, incoming residents have settled at different places not only based on economic factors like the location of the workplace and housing prices, but also on purely social reasons like proximity to close friends or family or feelings of empathy towards the location's social environment. Ours is therefore the first attempt to explore this geographic or social close-knitness of foreign residents in Spain since the start of the migration flow. Following Echenique and Fryer (2007), we use geographical data of residents' households in order to compute our measure. This will not only allow us to explain patterns on the geographical distribution of groups (nationalities) in the country, it will also shed some light on the social clustering of these groups since two individuals with closer ties have usually a smaller physical distance attached between them.⁵

Our empirical analysis shows that the southern part of the Mediterranean coast is the most segregated area in the country, while northern autonomous communities of Galicia and

⁵Observe that the segregation index takes the dimension of clustering as defined in Massey and Denton (1988) into account by definition because the random walk starting at node i is more likely to terminate at node j than at node k if the distance between i and j is smaller than the distance between i and k .

the Basque Country together with the southern region of Andalusia are the least segregated ones. With respect to the different nationalities, we find that the English and the Germans are among the most segregated groups, immigrants from Latin America, on the other hand, turn out to be rather integrated. The empirical study also reveals that network effects—the change in the measure due to putting the network structure on top of the isolation index—is strong for some groups like the Pakistanis and immigrants from African countries but weak for immigrants from Western Europe. Observe that the described network effects are directly related to the degree of clustering of the groups. To see this, imagine two equally segregated groups that only differ in the fact that the first group is evenly spread over the city while the second group is clustered in a city area. Now, since an increase in the continuation probability α allows for new interactions with neighboring census tracts, the network effect will be higher for the less clustered group. Finally, it also turns out that the segregation index and the isolation index are positively correlated to the *SSI* but not strongly so.

We proceed as follows. In Section 2, we introduce the segregation index. In Section 3, the measure is related to various dimensions of segregation. In Section 4, it is shown that the *SSI* is discontinuous. In Section 5, the segregation index is applied to the Spanish data from 2009. In Section 6, we conclude. Some numerical results are relegated to the Appendix.

2 The Segregation Index

Consider a finite set $N = \{1, 2, \dots, n\}$ of n *individuals*. For simplicity we will call N a *society*. Individuals live in a city that is composed of a finite set $M = \{1, 2, \dots, m\}$ of m *nodes* representing neighborhoods or census tracts. To be consistent with that interpretation, we assume that every individual is located in exactly one node but that each node possibly inhabits multiple individuals. Also, there is a set $K = \{1, 2, \dots, k\}$ of k *groups* that form a

partition of the society. One can think of a group as a subset of members of the population that share a particular attribute such as religion or ethnicity. For any group $s \in K$, the number of individuals located in node i is given by $n_{s,i}$. The number of individuals belonging to group $s \in K$ is then equal to $n_s = \sum_{i \in M} n_{s,i}$. Similarly, $n_i = \sum_{s \in K} n_{s,i}$ is the number of individuals located in node $i \in M$. Finally, the column vectors \mathbf{b}_s and \mathbf{c}_s with the generic entries $b_{s,i} = n_{s,i}/n_s$ and $c_{s,i} = n_{s,i}/n_i$ are referred to as the vectors of *group densities* and *group proportions*, respectively.

The different nodes in a city are interconnected through links. Formally, \mathbf{A} is an $m \times m$ matrix such that $a_{i,j} = 1$ if there is a connection between i and j and $a_{i,j} = 0$ otherwise. The general intuition is that two nodes are connected if they are geographically adjacent or if individuals in these nodes interact with each other. Also observe that there is no need to assume that the graph is symmetric.⁶ Let $a_i = |\{j \in M : a_{i,j} = 1\}|$ be the number of nodes i is connected to. Finally, it is also assumed that $a_{i,i} = 1$ so that each node can be assessed from itself.⁷ This implies that $a_i > 0$.

An $m \times m$ matrix \mathbf{G} is said to be a *row stochastic matrix associated with \mathbf{A}* whenever the following conditions hold: $g_{i,j} = 0$ whenever $a_{i,j} = 0$, $g_{i,j} > 0$ whenever $a_{i,j} = 1$, and $g_i \equiv \sum_{j \in M} g_{i,j} = 1$. One way of constructing \mathbf{G} is to set $g_{i,j} = a_{i,j}/a_i$ for all (i, j) -entries; that is, the entries in row i are normalized by the number of connections of node i . We will apply this construction technique in our later application, however theoretically one can allow for arbitrary values in the associated matrix as long as the restrictions above

⁶Asymmetric graphs occur for instance when \mathbf{A} represents a social communication structure. Also, friendship networks do not necessarily satisfy the property of symmetry. Finally, the mobility between two different social strata could be more sticky depending on the direction of movement. Consequently, our framework can also be applied to more abstract settings like social relationships and is not restricted to the geographical dimension of segregation alone.

⁷Thus, individuals from the same neighborhood can interact directly with each other. This assumption is purely technical and simplifies the analysis without losing generality. Even more importantly, it reduces the computational time for calculating the *SSI* of Echenique and Fryer (2007) as done in Section 5.

are satisfied. With this notation at hand we can now formally define a *city* as a tuple $C = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G} \rangle$.

The advantage of working with a stochastic \mathbf{G} is that it provides a natural interpretation in terms of node-to-node transition probabilities. Formally, a *walk* is a sequence $\omega = (\omega_0, \omega_1, \dots)$ of nodes with the restriction that two consecutive nodes must be connected (*i.e.*, $g_{\omega_j, \omega_{j+1}} > 0$ for all $j = 0, 1, \dots$). It is assumed that a walk continues every period $t = 0, 1, \dots$ to an adjacent node with probability $\alpha \in [0, 1)$ and that it ends with probability $1 - \alpha$. In case of continuation, the walk currently at node i passes to node j with probability $g_{i,j}$. Thus, we define an (α, \mathbf{G}) -*random-walk* as a random variable whose realization is a particular walk, where the probability of a walk $\omega = (\omega_0, \omega_1, \dots, \omega_h)$ of length $|\omega| = h$ is given by

$$\text{Prob}_\omega(\alpha, \mathbf{G}) = \alpha^h (1 - \alpha) g_{\omega_0, \omega_1} g_{\omega_1, \omega_2} \cdots g_{\omega_{h-1}, \omega_h}.$$

Observe that an (α, \mathbf{G}) -random-walk is completely defined through the matrix \mathbf{G} and the continuation probability α . Since $\alpha < 1$, a realization ω of an (α, \mathbf{G}) -random-walk is of finite length with probability 1. Let $\Omega(\alpha, \mathbf{G})$ be the set of all possible realizations of an (α, \mathbf{G}) -random-walk. The expected length $l(\alpha, \mathbf{G})$ of such a walk is⁸

$$l(\alpha, \mathbf{G}) = \sum_{k=0}^{\infty} k \sum_{\omega \in \Omega(\alpha, \mathbf{G}): |\omega|=k} \text{Prob}_\omega(\alpha, \mathbf{G}) = \frac{\alpha}{1 - \alpha}.$$

This equation shows that the continuation probability α is associated with the the expected walk length. For instance, $\alpha = 0.85$ corresponds to an expected walk length of about 6 steps and $\alpha = 0.99$ to about 99 steps. So, $1 - \alpha$ can be regarded as a coefficient of social viscosity reflecting the expected flow of interactions an individual is likely to have.⁹

⁸Note that $l(\alpha, \mathbf{G})$ can be rewritten recursively through the formula $l(\alpha, \mathbf{G}) = \alpha(1 + l(\alpha, \mathbf{G}))$. This is so because every node has an out-link ($a_i > 0$), which implies that the expected length of a walk is independent of its origin. Consider now any starting node ω_0 . The random-walk stops with probability $(1 - \alpha)$ having length 0. It continues with probability α to some node ω_1 . The random-walk starting at ω_1 has, due to the independence of the origin, an expected length of $l(\alpha, \mathbf{G})$. Consequently, $l(\alpha, \mathbf{G}) = (1 - \alpha) \cdot 0 + \alpha(1 + l(\alpha, \mathbf{G}))$.

⁹Throughout, we are going to assume that α is identical for all groups. This is a simplifying assumption

Given the transition matrix \mathbf{G} and the continuation probability $\alpha \in [0, 1)$, let $\mathbf{P}(\alpha, \mathbf{G})$ (or simply \mathbf{P}) be the $m \times m$ matrix where $p_{i,j}(\alpha, \mathbf{G})$ (or simply $p_{i,j}$) is the probability that a walk starting at node i ends at node j . One of our first objectives will be to provide a closed formula of \mathbf{P} in terms of the parameters \mathbf{G} and α , however it can already be seen at this point that if $\alpha = 0$ (every walk stops immediately) or if $\mathbf{G} = \mathbf{I}$ (all nodes are disconnected), no interaction takes place across neighborhoods and \mathbf{P} reduces to the identity matrix. Similarly, if α tends to 1, the expected walk length grows arbitrarily large and the notion of social proximity vanishes (*i.e.*, $p_{i,j}$ becomes independent of the origin within a connected city).

We are now ready to suggest a measure of residential segregation that is related to the level of within-group interaction, taken into account through the matrix \mathbf{P} . In particular, the (non-normalized) segregation index σ_s for group $s \in K$ in city C is defined as the probability that a randomly chosen individual of group s meets an individual of the same group in the node where the random-walk terminates.

Definition 1. Given city $C = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G} \rangle$ and the continuation probability $\alpha \in [0, 1)$, the *segregation index of group $s \in K$* is

$$\sigma_s(C, \alpha) = \sum_{i \in M} b_{s,i} \sum_{j \in M} p_{i,j} c_{s,j} = \mathbf{b}_s^\top \mathbf{P} \mathbf{c}_s.$$

Observe that σ_s takes values in the interval $(0, 1]$ because the probability that the randomly chosen individual stops walking immediately is $1 - \alpha > 0$. Nevertheless, σ_s gets arbitrarily close to zero if, for example, s consists of a single individual who lives in a very populated neighborhood. We would also like to remark that σ_s is increasing in the relative size n_s/n of group s . To see this, take any city C^1 and consider the city C^2 that can be obtained because the mobility of a group is likely to be related to various socioeconomic factors like income, education, and age. Still, the model can be straightforwardly extended to take these kind of dependencies into account.

from C^1 by doubling the fraction of individuals of group s in each node maintaining the total population in each node fixed. Thus, $\mathbf{b}_s^2 = \mathbf{b}_s^1$, $\mathbf{c}_s^2 = 2\mathbf{c}_s^1$ and $\mathbf{P}^2 = \mathbf{P}^1$, which implies that $\sigma_s(C^2, \alpha) = 2\sigma_s(C^1, \alpha)$. Consequently, the segregation of group s has doubled simply because the group has grown but not because its distribution over nodes or the network has changed. The reason for this dependence is that it is only required that an individual meets *any* (and not a randomly chosen) individual from the same group. This effect can be accounted for (as we do in the application in Section 5) with the following normalization.

Definition 2. Given city $C = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G} \rangle$ and the continuation probability $\alpha \in [0, 1)$, the *normalized segregation index of group $s \in K$* is

$$\bar{\sigma}_s(C, \alpha) = \left(\frac{n_s}{n}\right)^{-1} \sum_{i \in M} b_{s,i} \sum_{j \in M} p_{i,j} c_{s,j} = \left(\frac{n_s}{n}\right)^{-1} \mathbf{b}_s^\top \mathbf{P} \mathbf{c}_s.$$

If group s is distributed homogeneously over all nodes—that is, $c_{s,i} = n_s/n$ for all $i \in M$ —, then $\sigma_s(C, \alpha) = n_s/n$ and $\bar{\sigma}_s(C, \alpha) = 1$. So if $\bar{\sigma}_s(C, \alpha) > 1$, group s is (on average) overrepresented in census tracts compared to its overall population share. The size-independence becomes even more visible if the measure is rewritten as

$$\bar{\sigma}_s(C, \alpha) = n \sum_{i \in M} b_{s,i} \sum_{j \in M} n_j^{-1} p_{ij}(\alpha, \mathbf{G}) b_{s,j}.$$

According to this equation, $\bar{\sigma}_s$ is equal to the probability (up to the multiplicative scalar n) that a randomly chosen individual from group s meets another randomly chosen individual from the same group. The fact that *both* individuals are chosen at random prevents this probability from depending on the representativeness of this group in the society. Finally, observe that the *segregation index of city C* can be straightforwardly defined as the weighted average over the segregation indices for each group; that is, $\sigma(C, \alpha) = \sum_{s \in K} n_s/n \cdot \sigma_s(C, \alpha)$. Intuitively, $\sigma_s(C, \alpha)$ is the probability that a randomly chosen individual meets an individual

from the same group when her random-walk terminates. The normalized segregation index of city C is defined accordingly.

3 Exposure, centralization, and clustering

In this section, we establish that the segregation index incorporates directly three of the five dimensions introduced by Massey and Denton (1988), namely those of exposure, centralization, and clustering. The segregation index of a group can on one hand be interpreted as the degree of within-group interactions in the neighborhoods across the city and, in particular, we are going to show that the measure is a natural generalization of the well-known isolation index to social networks (Proposition 1). On the other hand, the segregation index can also be expressed in terms of centrality. By this we mean that everything else equal, groups that are more central in the socio-geographical network are more likely to interact with individuals from the same group. Hence, groups that are located more central are more segregated everything else equal. To formally relate the segregation index to the notion of centrality, we present an equivalent definition on the basis of Google’s PageRank index that measures the importance of websites in the World Wide Web (Proposition 2). Observe that both propositions are essential as they derive specific formulae for the computation of the segregation index in empirical applications as in Section 5. Finally, we show with the help of an example that the segregation index incorporates the dimension of clustering—groups that live in neighborhoods next to each other are more segregated—by construction.¹⁰

¹⁰Among the axioms discussed in Frankel and Volij (2010), the segregation index satisfies symmetry (relabelling groups or census tracts does not affect the measure), scale invariance (the measure does not change if all groups increase by the same percentage preserving the distribution in each node), the group division property (the measure does not change if a group is split into two preserving its distribution across nodes)—this axiom is only satisfied by the normalized version—and independence (adding isolated nodes to two cities does not change the relative segregation whenever the distribution of groups is the same across cities). Also, σ_s is *robust* in the sense that (a) if two disconnected cities are merged, then the segregation of a group in the new city is a weighted average of their original levels and (b) if a census tract is divided into homogenous neighborhoods and the new nodes maintain the same connections as the original nodes, then the measure does not change.

3.1 The dimension of exposure

Isolation, the opposite of exposure, refers to the tendency to interact with individuals from the same group. This notion is captured through the *isolation index* $I_s(C)$, which is defined as the likelihood that a typical individual of group s faces a member of the same group in her neighborhood:

$$I_s(C) = \sum_{i \in M} b_{s,i} c_{s,i} = \mathbf{b}_s^\top \mathbf{c}_s.$$

Even though the relation between the isolation index and the segregation index is clearly visible, the following proposition helps us to relate them more directly.

Proposition 1. *Given city $C = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G} \rangle$ and the continuation probability $\alpha \in [0, 1)$, the segregation index of group $s \in K$ is*

$$\sigma_s(C, \alpha) = (1 - \alpha) \mathbf{b}_s^\top (\mathbf{I} - \alpha \mathbf{G})^{-1} \mathbf{c}_s.$$

Proof. By definition $\sigma_s(C, \alpha) = \mathbf{b}_s^\top \mathbf{P} \mathbf{c}_s$. Hence, we only have to show that $\mathbf{P} = (1 - \alpha)(\mathbf{I} - \alpha \mathbf{G})^{-1}$. Consider $p_{i,j}$, which is the probability that a walk starting at node i ends at node j . In the first period, the walk starts with probability α . Given that the walk starts, the individual reaches node k at the end of period 1 with probability $g_{i,k}$. Now, the probability that a walk starting at node $k \in M$ ends up at node j is, by definition, equal to $p_{k,j}$. Therefore, we have that for $i \neq j$, $p_{i,j} = \alpha \sum_{k \in M} g_{i,k} p_{k,j}$. Following a similar argument, one can show that $p_{i,i} = (1 - \alpha) + \alpha \sum_{k \in M} g_{i,k} p_{k,i}$. Rewriting these equations in matrix form yields $\mathbf{P} = (1 - \alpha)\mathbf{I} + \alpha \mathbf{G} \mathbf{P}$, which is equivalent to $\mathbf{P} = (1 - \alpha)(\mathbf{I} - \alpha \mathbf{G})^{-1}$, where the inverse is well-defined and non-negative because $\mathbf{G} \geq \mathbf{0}$ and $\alpha \in [0, 1)$. \square

It follows from Proposition 1 that the segregation index $\sigma_s(C, \alpha)$ can be calculated as the average proportion of group s in the neighborhood that is reached by a typical s -member in

a random-walk. Note that when all census tracts are disconnected —that is, if $\mathbf{G} = \mathbf{I}$ — or when there is no node-to-node transition —that is, if $\alpha = 0$ — the segregation index $\sigma_s(C, \alpha)$ reduces to the isolation index $I_s(C)$.¹¹ Hence, the segregation index $\sigma_s(C, \alpha)$ generalizes the isolation index $I_s(C)$ by taking the connectivity among census tracts explicitly into account. The measure can also be interpreted as a weighted average over the entries of the vector $\mathbf{v}_s(C, \alpha) \equiv \mathbf{P}\mathbf{c}_s = (1 - \alpha)(\mathbf{I} - \alpha\mathbf{G})^{-1}\mathbf{c}_s$, where the weight of each node $i \in M$ is equal to $b_{s,i}$. Since \mathbf{v}_s contains the contributions of all census tracts to the segregation of group s , we will call it the vector of *local isolations* of group s , and $v_{s,i}$ represents the average proportion of group s around node i . Following this interpretation, the segregation index $\sigma_s(C, \alpha) = \mathbf{b}_s\mathbf{v}_s$ can be seen as the weighted average over all local isolations. We conclude with an example showing a way to compute the segregation index.

Example. Consider the city C depicted in Figure 1. There are two ethnic groups, blacks and

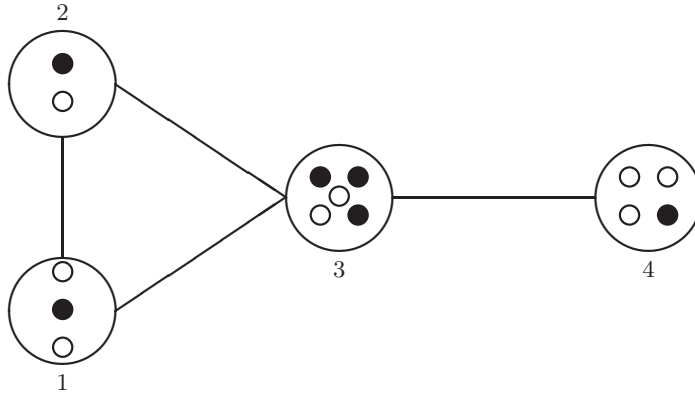


Figure 1: Calculation of the segregation index.

whites. It is assumed that 2 whites live in the nodes 1 and 3, 3 whites live in node 4, and 1 white lives in node 2. On the other hand, 1 black lives in the nodes 1, 2 and 4 while 3 blacks live in node 3. If we suppose that an individual moves to all adjacent nodes with the same

¹¹If $\alpha \rightarrow 1$ and \mathbf{G} is strongly connected, then $\sigma_s \rightarrow \mathbf{z}^\top \mathbf{c}_s$, where \mathbf{z} is the principal right eigenvector of \mathbf{G} .

probability, the transition matrix corresponding to the network structure in Figure 1 is

$$\mathbf{G} = \begin{pmatrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}.$$

The distribution of individuals across nodes implies that $\mathbf{b}_b^\top = (1/6, 1/6, 1/2, 1/6)$, $\mathbf{b}_w^\top = (1/4, 1/8, 1/4, 3/8)$, $\mathbf{c}_b^\top = (1/3, 1/2, 3/5, 1/4)$, and $\mathbf{c}_w^\top = (2/3, 1/2, 2/5, 3/4)$. One can then verify that $\sigma_b(C, 0.5) = 1/2 \cdot \mathbf{b}_b^\top (\mathbf{I} - 1/2 \cdot \mathbf{G})^{-1} \mathbf{c}_b = 0.46$ and that $\sigma_w(C, 0.5) = 0.54$. \square

3.2 The dimension of centralization

Generally speaking the centrality of a node in a network captures its well-connectedness. Depending on the specific context, it can for example be assessed using the notions of degree (the number of connections a node has), betweenness (determine the shortest paths between any two nodes and calculate then for each node on how many of these shortest paths it belongs to), or closeness (the mean geodesic distance between a node and all nodes reachable from it). In our case, the segregation index belongs to the class of eigenvector-based centrality measures and, in particular, it is related to the PageRank index of Brin and Page (1998) that underlies Google’s search engine.¹²

The main idea of the PageRank vector is that a webpage is more relevant given a query and therefore listed higher up by the search engine when it is linked by other relevant pages. This idea is formalized using a finite Markov chain according to which an individual “surfs” randomly across the web. The entries of the PageRank vector are then just the stationary probabilities of this process. More formally, an individual starts at any node (webpage) and surfs the web randomly according to the (column-stochastic) matrix \mathbf{S} that represents the actual links of the web. Additionally, at any point in time, the surfer is “teleported” to another

¹²See Wasserman and Faust (1994) for an excellent overview of centrality measures.

page j (even if the current page i is not directly linked with j) with probability $1 - \beta$, while she continues her random-walk by crossing links normally according to \mathbf{S} with probability β . In case of teleportation, the surfer is taken to page j with probability q_j . This process defines a Markov chain whose stationary probabilities are gathered in the PageRank vector \mathbf{r} . The vector $\mathbf{q} = (q_i)_{i \in M}$ is the *personalization vector* of the PageRank index summarizing the teleporting mechanism of the Markov chain. It turns out that the PageRank vector \mathbf{r} is equal to the principal eigenvector of the matrix $\mathbf{T} = (1 - \beta)\mathbf{q}_s\mathbf{1}^\top + \beta\mathbf{S}$, which is the formula used by Google in order to assess the importance of webpages.

This notion of centrality can be adapted to our framework in a straightforward way: an individual randomly walks in the city according to the transition matrix \mathbf{G} (that is, $\mathbf{S} = \mathbf{G}$) and is teleported to node j with probability $(1 - \alpha)b_{s,j}$ (that is, $\beta = \alpha$ and $\mathbf{q} = \mathbf{b}_s$).

Definition 3. Given city $C = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G} \rangle$ and the continuation probability $\alpha \in [0, 1)$, the *PageRank vector* \mathbf{w}_s of group $s \in K$ is the principal eigenvector of the matrix $\mathbf{Q} \equiv (1 - \alpha)\mathbf{b}_s\mathbf{1}^\top + \alpha\mathbf{G}$.

Note that \mathbf{w}_s captures the centrality of the nodes *for group* s . This means that the PageRank $w_{s,i}$ of node $i \in M$ depends on the considered group s (because the personalization vector \mathbf{q} is set equal to the group density vector \mathbf{b}_s) and therefore, the PageRank vector does not refer to an absolute notion of centrality of nodes in the city or social network. In particular, $w_{s,i}$ is the probability that a typical member from group s arrives at node $i \in M$; that is, $w_{s,i}$ accounts for the reachability of node i for members of group s .

The following proposition is the dual of Proposition 1 stating that the segregation index of group s is a weighted average over the vector of group proportions \mathbf{c}_s , where the weight of each node is given by its PageRank vector for the considered group.

Proposition 2. Given city $C = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G} \rangle$ and the continuation probability $\alpha \in [0, 1)$, the segregation index of group s is

$$\sigma_s(C, \alpha) = \mathbf{w}_s^\top \mathbf{c}_s.$$

Proof. By definition $\sigma_s(C, \alpha) = \mathbf{b}_s^\top \mathbf{P} \mathbf{c}_s$, which can be rewritten as $\sigma_s(C, \alpha) = \mathbf{c}_s^\top \mathbf{P}^\top \mathbf{b}_s$. Following the very same line of argumentation as in Proposition 1, it can then be shown that $\mathbf{P}^\top = (1 - \alpha)(\mathbf{I} - \alpha \mathbf{G}^\top)^{-1}$. Hence, it only remains to be shown that $\mathbf{w}_s = (1 - \alpha)(\mathbf{I} - \alpha \mathbf{G}^\top)^{-1} \mathbf{b}_s$. By definition, \mathbf{w}_s is equal to the principal eigenvector of the matrix $\mathbf{Q} = (1 - \alpha) \mathbf{b}_s \mathbf{1}^\top + \alpha \mathbf{G}^\top$. Since \mathbf{Q} is column stochastic, its principal eigenvalue is 1 and the principal eigenvector \mathbf{w}_s satisfies the equation $\mathbf{Q} \mathbf{w}_s = \mathbf{w}_s$. Taking an \mathbf{w}_s whose coordinates sum up to 1, we get that $(1 - \alpha) \mathbf{b}_s + \alpha \mathbf{G}^\top \mathbf{w}_s = \mathbf{w}_s$. Solving this equation we get that $\mathbf{w}_s = (1 - \alpha)(\mathbf{I} - \alpha \mathbf{G}^\top)^{-1} \mathbf{b}_s$. \square

Proposition 2 establishes that more central individuals contribute more to the overall segregation of their group because they are easier to reach by the other group members. Note that when there is no network ($\mathbf{G} = \mathbf{I}$ or $\alpha = 0$), we have that $\mathbf{w}_s = \mathbf{b}_s$ and $\sigma_s(C, \alpha) = I_s(C)$. If network effects are present, social groups that are located more central in the network have a higher possibility of intra-group interactions and, as a consequence, a higher segregation index.¹³ This is made visible with the help of the following example.

Example. Consider the city C depicted in Figure 2. There are eleven individuals belonging to three ethnic groups (blacks, whites, and diamonds). At every node in the network, there is exactly one individual. The particularity of the considered social network structure is that the blacks and the whites are allocated in a very similar way. Every black is connected to two other blacks plus herself, one white, and one diamond. Every white is also connected to two

¹³There is a vast literature that recognizes the importance of centrality in determining socioeconomic outcomes. Examples includes education (Calvo-Armengol et al. 2009), criminal behavior (Haynie 2001), worker's performance (Mehra et al. 2001), power in organizations (Brass 1984) and the formation and performance of R&D networks (Boje and Whetten 1981, Powell et al. 1996 and Uzzi 1997).

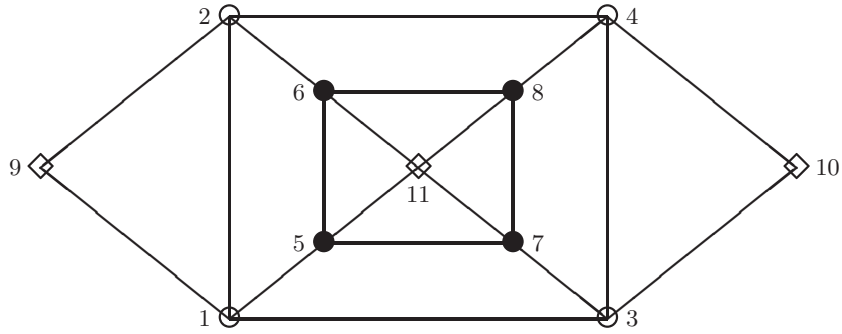


Figure 2: Centrality.

other whites plus herself, one black, and one diamond. The important difference between the two groups is that all blacks are connected to the same diamond (individual 11), while two whites are connected to diamond 9 and two whites to diamond 10. Hence, diamond 11 is the most central individual of her group. This implies that the blacks are located relative more central than the whites and are thus more segregated.

Since \mathbf{c}_b and \mathbf{c}_w are completely symmetric, σ_s incorporates the differences between the two groups entirely through the PageRank indexes \mathbf{w}_b and \mathbf{w}_w . In particular, we obtain for $\alpha = 0.85$ that $\mathbf{w}_{b,i} = 0.1392$ whenever i is a black and that $\mathbf{w}_{w,i} = 0.1373$ whenever i is a white. As a consequence, $\sigma_b(C, 0.85) = 4 \cdot 0.1392 > 4 \cdot 0.1373 = \sigma_w(C, 0.85)$.¹⁴ \square

3.3 The dimension of clustering

Following Massey and Denton (1988), clustering is the extent to which individuals from the same social groups live in neighborhoods next to each other. This definition has to be distinguished from the corresponding concept in the literature on social networks, where clustering usually refers to the probability that two neighbors/friends of a given individual have a direct

¹⁴It will become clear in the next section that the *SSI* is not directly related to the dimension of centralization because it only considers the network of within group interactions. In the example, the two subgraphs are the same so that the *SSI* takes identical values for both groups. In particular, the *SSI* misses that the blacks are connected to a more central diamond than the whites, which makes it more likely that two blacks interact.

link between themselves. It is rather intuitive that the segregation index incorporates the idea of clustering as expressed by Massey and Denton (1988) in the proper definition: an individual who starts a random-walk in her area of residence is more likely to end up in a node that is closer to her neighborhood than in a node that is further away as soon as $\alpha > 0$. To say it differently, two individuals are more likely to meet the closer they live to each other. If one looks for example at Figure 3, it can be seen that $p_{4,5}(B, \alpha) > p_{2,5}(A, \alpha)$ and that $p_{5,4}(B, \alpha) > p_{5,2}(A, \alpha)$ for all $\alpha \in (0, 1)$. Thus, the segregation index for the blacks takes a higher value in City B than in City A.

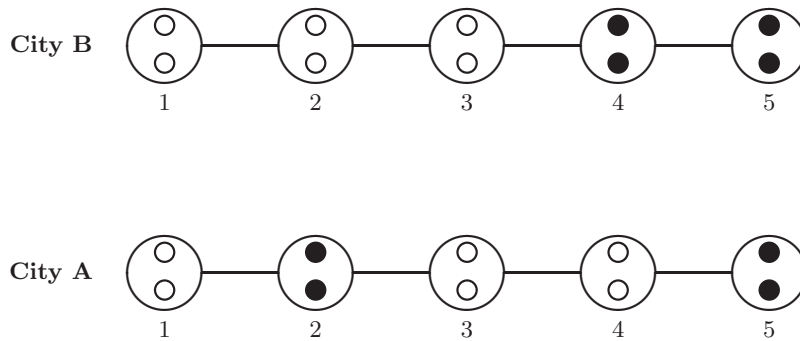


Figure 3: Clustering.

According to Massey and Denton (1988), the higher segregation of the blacks in City B is entirely due to clustering. They write on page 293:

..., suppose we have two urban areas with the same number of minority members, who comprise the same proportion of the total population. In each place, no minority member shares a common residential area with a majority member, all minority areas are located the same average distance from the central business district, and all areas are of the same geographic size. In this case, both urban areas would display identical measures of evenness, exposure, concentration, and centralization. However, if all minority areas in one of the urban areas were contiguous to one another, but in the other area they were separated from one another, then we would probably consider the former urban area to be more segregated, since all

minority members live within one single homogeneous ghetto, compared to the latter area, where they reside in minority neighborhoods that are scattered throughout the urban area.

Figure 3 exactly represents the situation described in the quote (node 3 should be seen as the city center from an absolute point of view) and, as suggested, the segregation index for the blacks is higher in City B.

4 Discontinuity of the *SSI*

In this part of the paper, it is shown that the *SSI* proposed in Echenique and Fryer (2007) is not continuous in the strength of the social ties. Even though cities do not experience abrupt changes in the short run, people may move within the city or change their pattern of social interactions and we therefore think that a measure of segregation should behave in a stable way to these changes. Since the only difference between the model by Echenique and Fryer (2007) and ours is that they assume a one-to-one mapping between nodes and individuals while we allow for multiple individual to reside in the same neighborhood, it is assumed for the time being that $n_i = 1$ for all $i \in M$.¹⁵

Given the matrix \mathbf{G} , the *SSI* for group s is calculated by determining first the subgraph \mathbf{H} of \mathbf{G} that only contains interactions between members of s . All individuals belonging to a different group are eliminated from the network and, consequently, from the matrix \mathbf{G} . Generally such a subgraph \mathbf{H} of \mathbf{G} consists of more than one strongly connected components because it reflects the interactions within a group, even if the city is itself connected. Hence, the next step is to calculate the *SSI* for each component separately.¹⁶ In particular, the *SSI*

¹⁵A natural extension of the *SSI* to a setting with multiple individuals in the same node is presented in the next section.

¹⁶For the sake of simplicity, given any such matrix \mathbf{H} , we will also use \mathbf{H} to refer to the network corresponding to \mathbf{H} . Formally, the subgraph \mathbf{H}^γ is said to be a *strongly connected component* of the network represented by \mathbf{H} if \mathbf{H}^γ is a network of maximal size satisfying that there is a path between there any pair of its nodes.

for group s in the (strongly connected) component \mathbf{H}^γ of \mathbf{H} —denoted by $SSI(\mathbf{H}^\gamma)$ —is set equal to the spectral radius of the matrix \mathbf{H}^γ , the largest absolute value of the eigenvalues of the matrix \mathbf{H}^γ . The segregation of an individual i of this group in this component is denoted by $SSI_i(\mathbf{H}^\gamma)$ and it is the i -th entry of the principal eigenvector of the matrix \mathbf{H}^γ , normalized so that the vector average is $SSI(\mathbf{H}^\gamma)$.

In order to finally aggregate the segregation over components, an additivity property is imposed in the definition of the SSI : the segregation of group s is a weighted average over the segregation levels of group s in all components of \mathbf{H} . In particular,

$$SSI(\mathbf{H}) = \sum_{\gamma} b_s^\gamma \cdot SSI(\mathbf{H}^\gamma),$$

where $b_s^\gamma = 1/n_s \cdot \sum_{i \in \mathbf{H}^\gamma} n_{s,i}$ is the fraction of individuals from group s that live in \mathbf{H}^γ .¹⁷

Alternatively, the SSI for group s can also be expressed as the average over all individual segregation levels:

$$SSI(\mathbf{H}) = \frac{1}{n_s} \sum_{\gamma} \sum_{i \in \mathbf{H}^\gamma} SSI_i(\mathbf{H}^\gamma).$$

To see how the SSI is applied consider the panel on the left-hand side in Figure 4, which corresponds to the motivating example in Echenique and Fryer (2007). The society living in City 1 is composed of two ethnic groups, blacks and whites. Each dot in the panel represents one individual. It is also assumed that individuals only interact with their horizontal and vertical neighbors. So, for example, individual (A,1) spends 50% of her time with each (A,2) and (B,1), while individual (C,2) interacts 25% of her time with each (C,1), (C,3), (B,2) and

¹⁷Buried in the appendix, in Proposition 13, Echenique and Fryer (2007) provide an alternative interpretation of their measure that deserves some attention: the SSI is proportional to the probability that two individuals from the same social group meet *without* leaving the subgroup network. Hence, the SSI is also related to a probability but, at the same time, differs substantially from the segregation index. Indeed, for the segregation index it does not matter whether i meets somebody from a different group on his way to j , yet the walks undertaken are restricted to within group interactions according to the SSI . This important distinction can lead to substantial differences in applications; i.e., our empirical analysis in the next section shows that the correlation between the segregation index and the SSI can be rather low.

(D,2). As a consequence, the subgraph \mathbf{H} for the blacks in City 1 consists of two connected components \mathbf{H}^1 and \mathbf{H}^2 .

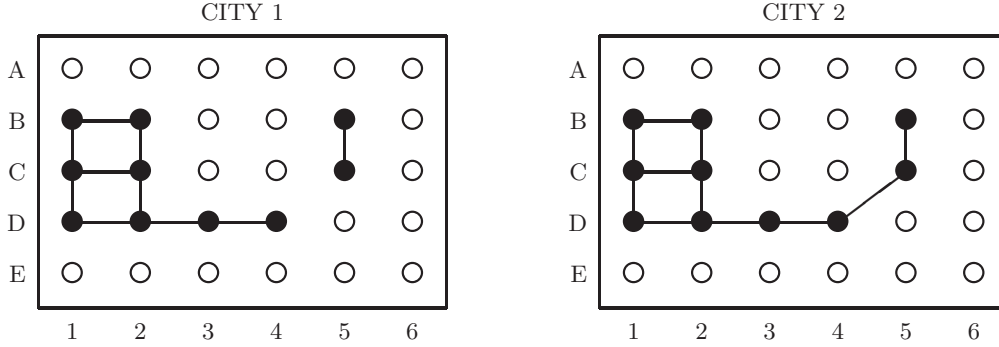


Figure 4: Examples for determining the *SSI*.

The *SSI* for the blacks as a group is determined by taking a weighted average over the spectral radii of the two black connected components. The upper part of Table 1 shows that $SSI(\mathbf{H}^1) = 0.72$ and that $SSI(\mathbf{H}^2) = 0.25$. Using that 80% of the blacks reside in component 1 and 20% in component 2, the segregation for the blacks as a group in City 1 is $SSI(\mathbf{H}) = 0.8 \cdot 0.72 + 0.2 \cdot 0.25 = 0.63$.

To see that the *SSI* is not continuous in the entries of \mathbf{H} suppose that the black individuals (D,4) and (C,5) start communicating with each other 1% of their respective times. It is irrelevant which neighbor(s) receive now relative less attention from (D,4) and (C,5), for the sake of the example just suppose that this extra time is taken away from the white neighbors so that all interactions with the other blacks remain unaffected. This small change in the network leads to City 2, which has a single component and is depicted in the right-hand panel of Figure 4. As it can be seen in the lower part of Table 1, the effect of this change on the segregation of the blacks is substantial, $SSI(\mathbf{H})$ increases from 0.63 to 0.72. In particular, the spectral radius of the integrated component converges to the larger of the two spectral

radii. Finally, one also sees that the segregation levels of individuals (B,5) and (C,5) are now very close to 0 whereas they were 0.25 in City 1.

Components		Blacks								<i>SSI</i>
City 1										
Component 1	(B,1)	(B,2)	(C,1)	(C,2)	(D,1)	(D,2)	(D,3)	(D,4)		
	0.87	0.62	1.26	0.92	0.93	0.75	0.29	0.10		0.72
Component 2	(B,5)	(C,5)								
	0.25	0.25								0.25
<i>Weighted Average</i>										0.63
City 2										
Component	(B,1)	(B,2)	(C,1)	(C,2)	(D,1)	(D,2)	(D,3)	(D,4)	(B,5)	(C,5)
	1.09	0.78	1.58	1.14	1.16	0.93	0.37	0.13	0.00	0.00
										0.72

Table 1: Calculation of the *SSI*.

More formally, $SSI(\mathbf{H})$ is discontinuous in the entries of \mathbf{H} , as opposed to what is stated in Proposition 5 in Echenique and Fryer (2007). The discontinuity becomes evident when two connected components of \mathbf{H} are merged or when one component of \mathbf{H} is split up.¹⁸ The key to understand this is the implicit property of component averaging/additivity that is attached to the *SSI*: the *SSI* of a connected component \mathbf{H}^γ of \mathbf{H} is defined as the spectral radius of the submatrix \mathbf{H}^γ , but the spectral radius of \mathbf{H} —which is continuous in \mathbf{H} —can generally not be averaged over its components. In fact,

$$\rho(\mathbf{H}) = \max_{\gamma} \rho(\mathbf{H}^\gamma),$$

where $\rho(\cdot)$ denotes the spectral radius. So, the property of component additivity inherent to the *SSI* sacrifices continuity. To restore the continuity of the *SSI* one would need to drop this component additivity. However, in this case the segregation of group s would become equal to the largest (and not to the average) segregation of all components of \mathbf{H} , and the segregation

¹⁸However, we also note that problems still arise when \mathbf{H} is connected but near disconnectedness. In that case, the measure can still display very unstable behavior.

levels in all other components of \mathbf{H} would be zero. So, if additivity is dropped, one would still need to explain why this asymmetric outcome is a reasonable measure of segregation.¹⁹

To conclude we point out that σ_s and $\bar{\sigma}_s$ are continuous in all their arguments. In particular, continuity with respect to α and \mathbf{G} follows from the fact that \mathbf{G} is stochastic and that $|\alpha| < 1$. Moreover, our measures are quite stable in applications since we are choosing values for α well below 1. Finally, the next proposition provides an upper bound to the change in segregation when connections change in the social setting. For that, given two cities $C = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G} \rangle$ and $C' = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G}' \rangle$ with a common distribution of individuals over nodes, let U be the subset of nodes whose connection changes across the two cities; that is, $U = \{i \in M : g_{i,j} \neq g'_{i,j} \text{ for some } j \in M\}$.

Proposition 3. *The change in segregation index between $C = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G} \rangle$ and $C' = \langle N, M, K, (n_{s,i})_{s \in K, i \in M}, \mathbf{G}' \rangle$ is bounded above by*

$$|\sigma_s(C', \alpha) - \sigma_s(C, \alpha)| \leq \max_{i \in M} \{c_{s,i}\} \frac{2\alpha}{1 - \alpha} \sum_{j \in U} w_{s,j}.$$

Proof. This follows from Theorem 6.5.1 in Langville and Meyer (2006) and Proposition 2. \square

5 Application

5.1 Segregation in Spain

In this section, the segregation index is applied to the Spanish census tract data from January 2009. Among the developed countries, Spain is particularly interesting to look at because the country attracted a lot of immigrants from many different parts of the world over the last decade. During the boom years, it was easy for the young South Americans to find a job in the construction or the (private) service sector because they were relatively cheaper to

¹⁹Note that Proposition 5 on page 475 indicates continuity of $SSI(\mathbf{H})$ in the entire network \mathbf{H} , which is not true. Indeed, $SSI(\mathbf{H})$ is continuous as long as \mathbf{H} is connected.

hire and had the advantage of speaking the same native language. But also immigrants from the Eastern European countries that recently entered the European Union or Africa were attracted by the vast job opportunities in the labor market. At the same time, Europeans from countries with a relatively higher GDP like England and Germany invested into the new residential areas at the Mediterranean coast. These combined effects led to the current situation that more than 10 % of the 45 million residents in Spain are foreigners.

Nationality	ID	Number of Residents	Share
Spain	E	40,956,149	89.13 %
Europe			
Germany	G	190,716	0.41 %
Bulgaria	BU	164,716	0.36 %
France	F	120,262	0.26 %
Italy	I	175,232	0.38 %
Poland	PO	85,007	0.19 %
Portugal	P	140,801	0.31 %
Great Britain	UK	375,593	0.82 %
Romania	RU	798,869	1.74 %
Russia	R	47,428	0.10 %
Ukraine	UC	82,263	0.18 %
Africa			
Algeria	AR	56,194	0.12 %
Morocco	MA	708,939	1.54 %
Nigeria	NI	42,322	0.09 %
Senegal	SE	56,589	0.12 %
South America			
Argentina	A	142,239	0.31 %
Bolivia	BO	230,693	0.50 %
Brazil	B	126,172	0.27 %
Colombia	CO	296,619	0.65 %
Cuba	CU	54,598	0.12 %
Chile	CH	51,032	0.11 %
Ecuador	EC	421,385	0.92 %
Paraguay	PA	81,549	0.18 %
Peru	PE	139,167	0.30 %
Dominican Republic	RD	88,102	0.19 %
Uruguay	UR	50,422	0.11 %
Venezuela	VE	61,448	0.13 %
Asia			
China	C	147,373	0.32 %
Pakistan	PK	54,100	0.12 %

Table 2: Residents in Spain as of January 2009 according to the country of origin (nationality). The data is made available by the National Statistical Institute (INE) of Spain.

Table 2 presents the population shares of the 29 nationalities with the highest number of residents in Spain. It can be seen that the Rumanians form the largest foreign group with 1.74 % of the total population followed by the Moroccans who account for 1.54 % of the residents. The high number Moroccans in Spain is of no surprise given their long tradition in the country. Next are Ecuador (0.92 %), Great Britain (0.82 %), Colombia (0.65 %) and

Germany (0.41 %). The Nigerians form the smallest foreign group (0.09 %).

The Spanish territory is officially divided into 52 provinces, 47 of those are on the Iberian Peninsula. The remaining 5 ones are the Balearic Islands in the Mediterranean Sea, the Canary Islands in the Atlantic Ocean (2 provinces), and the autonomous cities Ceuta and Melilla in North Africa. We abstain from incorporating Ceuta and Melilla in our analysis because they are too small —each of the two cities has only about 75.000 residents and very few non-Africans. As of January 2009, the National Statistical Institute of Spain (INE) divides the 50 main provinces into a total of 35,757 census tracts. The mean number of residents per census tract is 1,284.95, the corresponding standard deviation is 657.13. As in Echenique and Fryer (2007), in order to define the network, we use the geographical location of the centroids of each census tract. In particular, we define two census tracts to be connected if the distance between their centroids is less than 400 meters. A node has then on average 4.50 connections. The corresponding standard deviation is 5.48. Also, there are 18,114 isolated nodes.

To see that this neighborhood radius is appropriately defined, note that the capital of Madrid is divided into a total of 2,397 census tracts and that its biggest connected component consists of 1,951 census tracts. Also, Echenique and Fryer (2007) apply a neighborhood radius of 1,000 meters in the United States, which is far bigger and less densely populated. In case the neighborhood radius is set to 1,000 meters, the average number of connections increases to 20.78. The corresponding standard deviation is 27.10. Still there are 13,789 isolated nodes.²⁰

To compare the segregation index across provinces, we calculate $\bar{\sigma}(C, \alpha)$ for each province

²⁰Two comments are in order. First, we use purely geographical positioning data in order to construct the network. However, the data nevertheless proxies true social interactions because geographical proximity is naturally correlated to close relationships, even though the new era of information has made weak social links less dependent on physical distance (see, Goldenberg and Levy 2009 and Mok et al. 2010). Thus, in our application we are analyzing not only geographical but also social segregation of residents of different nationalities. Second, the choice of 400 meters is arbitrarily based on the fact that actual neighboring census tracts should also be connected in the constructed network. In particular, we also performed calculations for radii of 1,000 and 1,500 meters, and our qualitative results remain identical.

separately; that is C is defined by the borders of the provinces.²¹ We set α equal to 0.85 because it is a prominent choice in other application related to the PageRank index as well. The numerical results for $\alpha \in \{0.00; 0.25; 0.50; 0.70; 0.99\}$ can be found in the Appendix. The results are graphically represented in Figure 5.

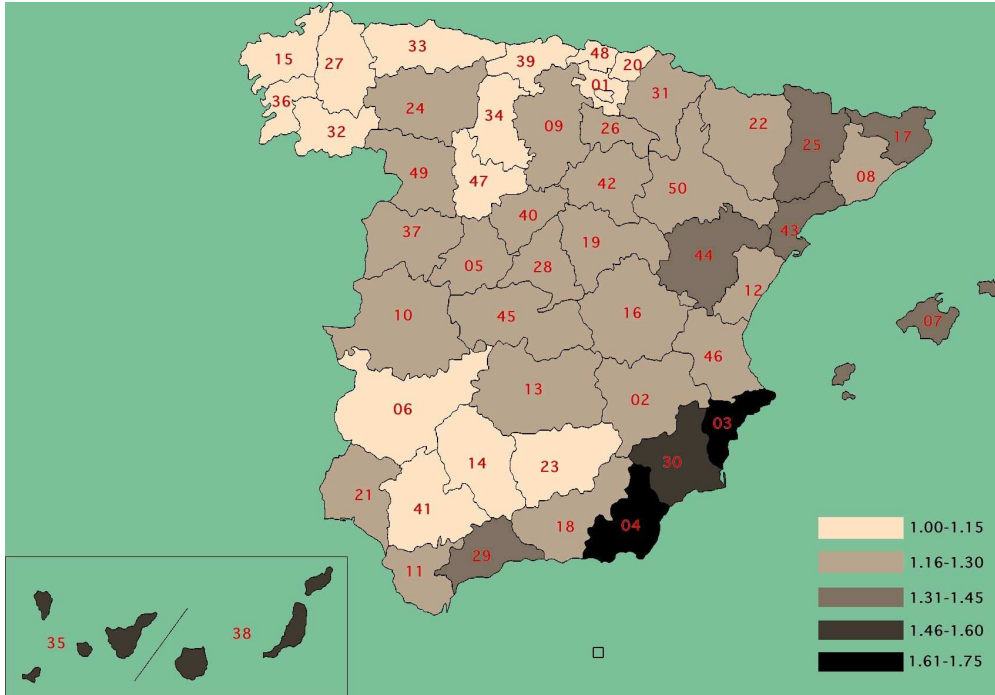


Figure 5: Normalized segregation in Spain by provinces as of January 2009 for $\alpha = 0.85$ and a neighborhood radius of 400 meters. The names of the different provinces can be identified with the help of Tables in the Appendix.

We can see that the segregation index is lowest in the autonomous communities of Galicia (provinces 15, 27, 32, and 36), Asturias (province 33), Cantabria (province 39), and the Basque Country (provinces 01, 20, and 48) that are all in northern part of Spain touching the Cantabrian Sea and the Atlantic Ocean. Also, some parts of Andalusia such as Cordoba, Jaen, and Seville (provinces 14, 23, and 41) have a segregation index between 1.00 and 1.15. The largest part of the country, which includes the capitals Madrid (province 28), Barcelona

²¹We use the normalized measure $\bar{\sigma}$ since it has desirable scale properties like invariance to population size, to relative group size, to city size and to subdivision of census tracts. Recall that it is proportional to the probability that two typical individuals with the same nationality will meet.

(province 08), and Valencia (province 46), has a moderate segregation index between 1.16 and 1.30. The segregation index takes slightly higher values within the autonomous community of Catalonia (provinces 17, 25 and 43) in northeastern part of the country and the Balearic Islands (province 07). The segregation index is high —between 1.46 and 1.60— in the Canary Islands (provinces 35 and 38) and Murcia (province 30). It reaches its maximum in the coastal sides of Alicante (province 03) and Almeria (province 04).

Figure 5 uncovers how segregated different parts of the country are, but so far we have not analyzed which groups are causing the results we see. To say it in different words, we still have to investigate which groups are, on average, more segregated. We detail on this by calculating the normalized segregation index $\bar{\sigma}_s(C, \alpha)$ for each of the 29 groups in the whole country; that is, C consists now of all 35,757 census tracts. Obviously, we take α to be equal to 0.85 again. The respective results are presented in Figure 6.

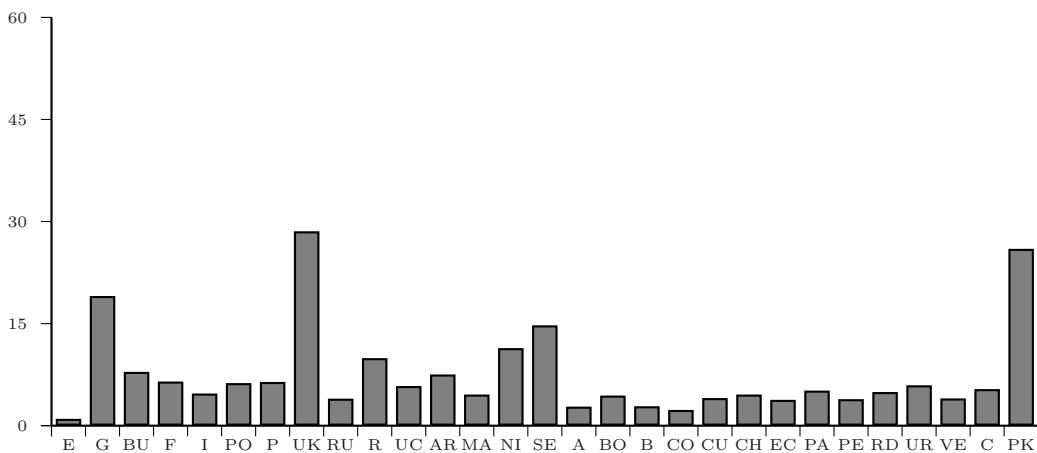


Figure 6: Normalized segregation in Spain by groups as of January 2009 for $\alpha = 0.85$ and a neighborhood radius of 400 meters. The ordering of the different groups corresponds to that in Table 2.

It can be seen that the Spanish are by far the least segregated group ($\bar{\sigma}_E(\text{Spain}, 0.85) = 1.01$). Remembering that the normalized segregation index for group s takes a value of 1 if $c_{s,i} = n_s/n$ for all nodes $i \in M$, this finding supports the interpretation that the Spanish are

very homogeneously distributed over all census tracts. Also, since the normalized segregation index is considerably bigger than 1 for all other nationalities, the foreign groups show the general tendency to stick together.

The most segregated immigrants are the British ($\bar{\sigma}_{GB}(\text{Spain}, 0.85) = 28.57$) followed by the Pakistanis ($\bar{\sigma}_{PK}(\text{Spain}, 0.85) = 25.99$) and the Germans ($\bar{\sigma}_G(\text{Spain}, 0.85) = 19.06$). This means interactions within these groups occur about 20 times more often than if their members were uniformly spread in the city. The Rumanians and the immigrants from the African countries of Algeria, Nigeria, and Senegal have a normalized segregation index of about 10 and are therefore less segregated than the formerly mentioned groups. The immigrants of the South American countries, on the other hand, are the most integrated ones as their segregation index lies between 2.32 for the Colombians and 5.94 for the Uruguayans. Finally, one can use Figure 2 to explain why Alicante is the most segregated province. Overall, 0.82 % of the residents are British and 0.41 % are German. But, in Alicante 6.91 % are British and 1.97 % are German. Consequently, the segregation in Alicante is high because the more segregated groups are overrepresented in this province.

5.2 Network effects

One important question at this point is whether the incorporation of the social network as an additional dimension adds to our understanding of segregation. To investigate this, one has to compare the normalized segregation index for $\alpha = 0.85$ presented above with the normalized segregation index for $\alpha = 0$ when there is no network effect and the measure reduces to the normalized version of the isolation index. So, the analysis captures differences that are due to the fact that social interactions take place not only within census tract but also across neighboring census tracts. This comparison will also reveal which nationalities cluster

in neighborhoods next to each other. To see this, consider the case when there is no actual network interactions ($\alpha = 0$) and imagine that group s is overrepresented in all its census tracts so that its segregation is very high. There are now two opposite scenarios to be investigated: the census tracts with inhabitants of group s are evenly distributed across the city or they are clustered in one particular area. Since the members of s are equally overrepresented in their census tracts in both scenarios, the normalized isolation index takes the same value, however the network effects due to an increase in α will be quite different. If the members of s live in evenly spread census tracts, new connections to neighboring census tracts will increase the interactions with members outside s and the segregation will decrease. On the other hand, if s forms a clustered group in the city, then the segregation will also decrease but now the effect will be smaller because the new interactions are more likely to be intra-group related as the neighboring nodes are mainly populated by s .

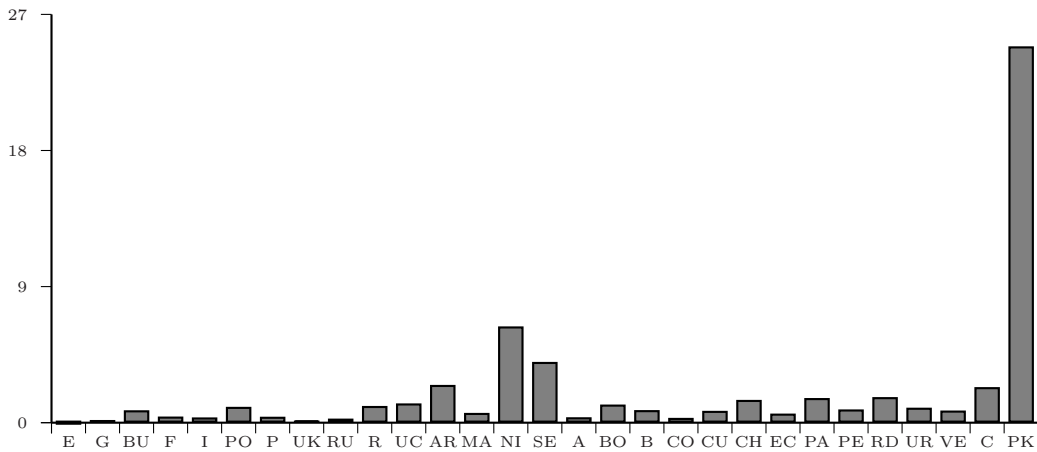


Figure 7: Difference in the normalized segregation index between $\alpha = 0$ and $\alpha = 0.85$ in Spain by groups as of January 2009 for a neighborhood radius of 400 meters. The ordering of the different groups corresponds to that in Table 2.

Figure 7 presents the difference in the segregation between $\alpha = 0$ and $\alpha = 0.85$ for Spain as a whole. One sees that the network effect is substantial for some groups but negligible

for others. The clearest effect can be identified for the Pakistanis: introducing the network reduces the normalized segregation index of this group from more than 50 to about 25. Other nationalities that present a significant network effect are, for example, Nigeria and Senegal. However, it is not only the absolute difference that matters, it is also remarkable that the Pakistanis are by far the most segregated group if $\alpha = 0.00$, while the British are the most segregated group if $\alpha = 0.85$. Actually, the network effect for the British and the Germans is rather small as the segregation index of these two groups is hardly affected by α . As we mentioned before, this difference is due to clustering. Considering Spain our geographical area of study, the British and the Germans are much more clustered in Mediterranean areas, whereas the Pakistanis are spread in many cities, even though they live very clustered within these cities. Thus, a less clustered group has larger network effects due to the additional interactions with individuals from other groups.

Recalling that $\mathbf{v}_s(\text{Spain}, \alpha) = \sum_i b_{s,i}(\text{Spain}, \alpha) v_{s,i}(\text{Spain}, \alpha)$, we now compute the correlation between the vectors of local isolations $\mathbf{v}_s(\text{Spain}, 0.85)$ and $\mathbf{v}_s(\text{Spain}, 0) = \mathbf{c}_s$ to quantify the network effects more accurately. The correlation between the two vectors is highest for the British (0.9964) and the Germans (0.9945). Thus, and as we have argued before, adding the network structure only plays a minor role in determining the segregation of these nationalities as they are highly clustered anyway. The correlation is lowest (0.8172) in case of the Pakistanis. So, network effects are important for understanding the segregation of this group. The average correlation between the two vectors calculated over all groups is 0.9122. The correlations above are calculated at the country level, but they change to some extent if we study the provinces on their own. For example, the average correlation between the two vectors across groups tends to be on the lower end in provinces with bigger cities like Catalo-

nia (0.8304), Madrid (0.8337), Valencia (0.8359), and Vizcaya (0.8056) but high in provinces with few inhabitants and relatively more isolated census tracts like Almeria (0.9549), Avila (0.9711), Guadalajara (0.9666), and Segovia (0.9700).

5.3 Correlation with the *SSI*

Next, we calculate the correlation between the *SSI* and the segregation index. The two measures are intimately related because they both calculate the eigenvector of a (sub-)stochastic matrix, and even though Echenique and Fryer (2007) consider a setting with a one-to-one mapping between nodes and individuals, it is possible to redefine their framework in such a way that a direct comparison to our segregation index becomes possible.

As explained in the former section, the basic idea behind the *SSI* is that individuals in a given node interact with their “neighbors”. If one node corresponds to one individual, the set of neighbors of an individual naturally includes all nodes she is connected to. We allow for multiple individuals living in the same node, but the two frameworks become comparable if one defines the set of neighbors as all those individuals that can be reached within one step. Following this idea, let $\hat{n}_i = \sum_{j \in M} a_{i,j} n_j$ be the total population around node i . Remember that $a_{i,j} = 1$ whenever i and j are connected and that $a_{i,j} = 0$ otherwise. The within-group interaction matrix \mathbf{H} for group s is then defined as

$$h_{i,j} = \begin{cases} n_{s,j}/\hat{n}_i & \text{if } a_{i,j} = 1 \\ 0 & \text{if } a_{i,j} = 0 \end{cases}.$$

Hence, an individual from group s living in node i interacts with all s -members that can be reached within one step with equal probability and the *SSI* can again be defined as

$$SSI(\mathbf{H}) = \sum_{\gamma} b_s^{\gamma} \cdot SSI(\mathbf{H}^{\gamma}).$$

Since the spectral radius for each connected component can be set equal to the weighted average of the corresponding eigenvector, we obtain that

$$SSI(\mathbf{H}) = \sum_{\gamma} b_s^{\gamma} \sum_{i \in \gamma} b_{s,i}^{\gamma} \cdot SSI_i(\mathbf{H}^{\gamma}),$$

where $b_{s,i}^{\gamma} = n_{s,i} / \sum_{j \in \mathbf{H}^{\gamma}} n_{s,j}$ is the fraction of individuals of group s from component γ who are located at node i . Rewriting this equation as

$$SSI(\mathbf{H}) = \sum_{\gamma} \sum_{i \in \mathbf{H}^{\gamma}} b_s^{\gamma} \cdot b_{s,i}^{\gamma} \cdot SSI_i(\mathbf{H}^{\gamma}) = \sum_{i \in M} b_{s,i} \cdot SSI_i(\mathbf{H}^{\gamma(i)})$$

shows that the SSI for a group s can be envisioned as a weighted average over of all nodes. Since the SSI is not invariant to group sizes, we have to compare it to the size dependent version of the segregation index, which according to Proposition 1 is equal to

$$\sigma_s(C, \alpha) = \sum_{i \in M} b_{s,i} v_{s,i}.$$

Consequently, we proceed by calculating the correlation between the two vectors $\mathbf{v}_s = (v_{s,i})_{i \in M}$ and $(SSI_i(\mathbf{H}_s^{\gamma(i)}))_{i \in M}$ of size 35,757 (corresponding to Spain as a whole) for each of the 29 nationalities taking $\alpha = 0.85$ and a direct neighborhood radius of 400 meters.

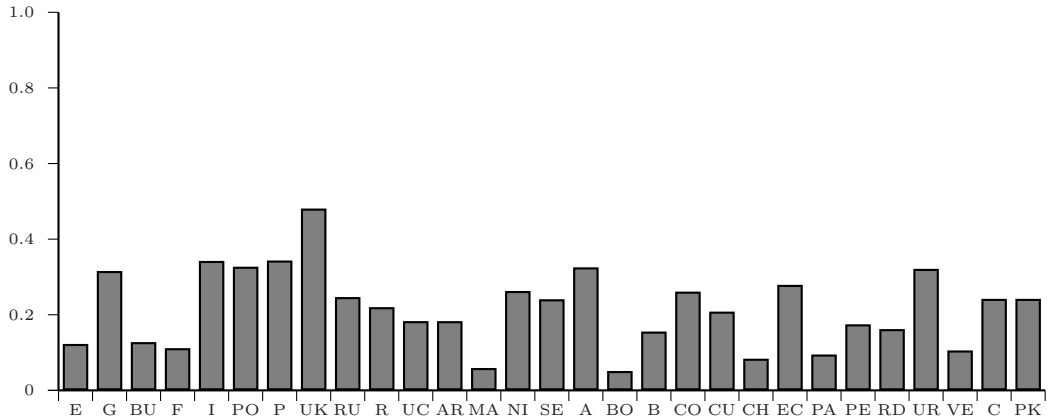


Figure 8: Correlation between $\sigma_s(C, \alpha)$ and the SSI in Spain by groups as of January 2009 for $\alpha = 0.85$ and a neighborhood radius of 400 meters. The ordering of the different groups corresponds to that in Table 2.

Figure 8 shows a positive correlation between the SSI and the segregation index for all 29 nationalities. For some groups like the Italians, the Polish, the Portuguese and the British, the correlation is considerable and lies between 0.38 and 0.48. For the immigrants from Morocco and Bolivia, on the other hand, the correlation is rather small.²² Echenique and Fryer (2007) report a very high correlation between the SSI and the isolation index for the U.S., which seems to contradict our finding that the correlation between the segregation index and the SSI is rather low; a high correlation between $\sigma_s(C, \alpha)$ and $I_s(C)$ and a high correlation between $I_s(C)$ and the SSI should, intuitively, imply a high correlation between the SSI and $\sigma_s(C, \alpha)$ as well. Yet, this apparent contradiction is easily explained: Figure 9 reveals for the Spanish data a low correlation between the SSI and the isolation index.

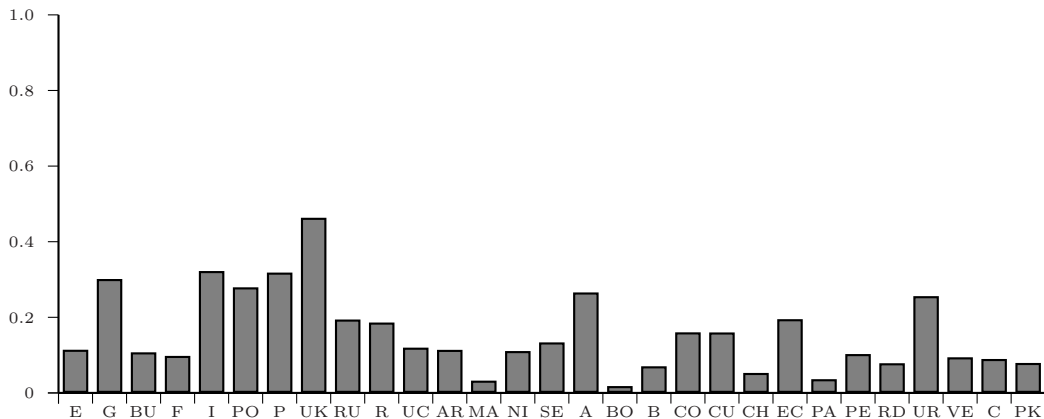


Figure 9: Correlation between $I_s(C)$ and the SSI in Spain by groups as of January 2009 for $\alpha = 0.85$ and a neighborhood radius of 400 meters. The ordering of the different groups corresponds to that in Table 2.

6 Discussion

In this paper, we have developed a new measure of residential segregation. In our theoretical model, the nodes of a network represent neighborhoods or census tracts and links indicate

²²We also determine the correlation of $\bar{\sigma}_s$ with the *dissimilarity index* and the *Gini coefficient*, two important measures of evenness. This is done by using the data across provinces for each nationality s . We find that the average correlation (over nationalities) ranges between 0.72 (for $\alpha = 0$) and 0.52 (for $\alpha = 0.99$) for the dissimilarity index and between 0.60 (for $\alpha = 0$) and 0.50 (for $\alpha = 0.99$) for the Gini coefficient.

which census tracts are adjacent in the urban space. It has also been assumed that every individual belonging to the society is located in only one node but that multiple individuals (from possibly different social groups) can be located in the very same node. Using this information as the only primitive of our analysis, we have studied the following Markov chain: every period, an individual advances to an adjacent node with a given probability or the process stops. Using this framework, the segregation index has then been defined as the probability that a randomly chosen individual from a given group meets an individual from the same social group in the node where her random-walk terminates.

We have shown in our theoretical analysis that segregation index has several favorable aspects. *First*, the measure reduces to the isolation index in case the network is empty or in case the exogenous probability that the random-walk stops is one. Consequently, the segregation index can be interpreted as a natural generalization of the isolation index to spatial networks. *Second*, the segregation index incorporates the idea that social groups that are located closer to the (relative) city center are more segregated everything else equal. In particular, the segregation index turns out to be proportional to the PageRank index applied by Google to determine the importance of webpages in the World Wide Web. *Finally*, the segregation index is a continuous function in the social ties. Indeed, it has also been established that the *SSI* suggested by Echenique and Fryer (2007), who have been the first to suggest a measure of segregation based on social interactions, fails to satisfy this important criterion.

There are several situations/circumstances that can trigger the discontinuity problem of the *SSI* in various types of applications. The reader should observe first that the disconnectedness of a *group* inside a social network causes this group's *SSI* to be discontinuous.

Thus, even if a city or social network is connected, the assumption that *all* groups within this network are connected is likely to be violated in many applications. For instance, it could be possible to find a group whose members are separated within the city, resulting in a subnetwork with several separated components, even though the whole city consists of a single component. Second, if several individuals live in a node as it happens in the data corresponding to the U.S. Census, the majority rule employed in Echenique and Fryer (2007) is a sensible choice to determine the representative individual of a census tract. However one cannot exclude the possibility that the majority group in a node changes (even though the data revealed a considerable margin in most districts) and that this causes two or more smaller components to merge into one big component or vice versa.²³ Third, even if no particular representativeness rule has to be considered because there is only a single individual in each node, it could happen that one individual changes her location in the spatial network, thus changing the network structure of her group by merging previously separated components or separating previously integrated components. Hence, the movement of a single individual can provoke a significant variation in the segregation of her group. Finally, the definition of adjacent nodes in a spatial network is also likely to become problematic if the measure is not continuous. Echenique and Fryer (2007) reasonably assume that two census tracts are connected whenever their centroids are not more than one kilometer away from each other. Yet a small change in this distance could potentially merge or separate components of some groups, triggering an abrupt change in the segregation of these groups. Even though it turns out that the application to the U.S. Census in Echenique and Fryer (2007) is robust with respect to this kind of variation in the network, one cannot exclude the possibility that difficulties

²³Note that we are here not analyzing the correctness of choosing the majority rule but the effects that the discontinuity of the *SSI* has on this choice.

appear in future applications with different datasets.

In our empirical analysis, we have studied the Spanish 2009 census tract data. The main result in this regard is that the provinces on the Mediterranean coast like Alicante and Almeria are the most segregated areas, mainly because the most segregated nationalities, the British and the Germans, are overrepresented in these regions. Immigrants from Latin America, on the other hand, turn out to be very much integrated, an effect that may be related to the fact that these groups speak Spanish natively. We have also seen that clustering effects are important to understand the segregation of some of the some smaller nationalities like the Pakistanis and immigrants from African countries like Nigeria and Senegal but not for the bigger European communities like the British and the Germans. Finally, network effects are substantial in provinces with bigger cities like Madrid, Barcelona, Valencia, and Bilbao.

Finally, we would also like to note that the concept of segregation bares some close relationships with the one of *homophily*, the tendency that an individual is more likely to associate with somebody who is of the same type. In a recent study, Currarini et al. (2009) use the AddHealth database to establish the following three friendship patterns in U.S. high schools: (a) larger ethnic groups form a greater fraction of their friendships with people of their type, (b) larger ethnic groups form significantly more friendships, and (c) only the middle-sized ethnic groups have a strong tendency to *inbreed*—that is, they form friendships with those from the same group at rates that exceed the relative fraction in the population.

Formally, Currarini et al. (2009) define the *homophily index* of an ethnic group as the ratio of the average number of friendships with those who form part of the same group over the average number of overall friendships. Consequently, there is inbreeding as soon as the homophily index is greater than the fraction of individuals that belong to the considered

group. The (normalized) segregation index can be interpreted as an alternative way to measure homophily. In particular, if $\sigma_s(C, \alpha) > \frac{n_s}{n}$ or if, equivalently, $\bar{\sigma}_s(C, \alpha) > 1$, then the probability that an individual interacts with somebody from the same group is greater than the ex ante probability and there is inbreeding homophily. The values in Table 6 from our empirical application therefore reveal that the British inbreed more than any other group and that the Spanish are the only ones who do not inbreed at all. If one matches Table 6 with Table 2, it can however be seen that inbreeding homophily is not related to group sizes but rather to the continent of origin of the immigrants.

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