Consistent versus Non-Consistent Term Structure Models: Some Evidence from the Spanish Market

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Abstract

In this paper we price caps and swaptions in the Spanish market with the Vasicek (1977), Cox, Ingersoll, and Ross (1985), and Hull and White (1990) (HW) models. We show that derivative prices obtained with the Vasicek and CIR models estimated from time-series data are very similar, but they differ substantially from the values given by the HW model fitted to the term structure of interest-rate swap yields (especially for ATM and OTM options). However, when the former models are estimated cross-sectionally, they produce option prices similar to those of the HW model. In samples of caps and swaptions, we find that the Vasicek model estimated cross-sectionally outperforms the HW model. Nonetheless, the Vasicek and CIR models estimated from time series produce very large pricing errors.

There are many ways of modeling the term structure of interest rates, many interest rate models, and many classifications of them. Some models describe the evolution of a given interest rate (usually the short-term rate) and will be consistent by construction with the current value of that interest rate. However, these models, in general, will not be consistent with the rest of the yield curve, and will not price "correctly" (relative to the market) claims as simple as discount bonds; which suggests that the models will do a poor job pricing more complex derivatives. Some of these models use one factor to explain the evolution of interest rates (see, for example, Vasicek (1977) and Cox, Ingersoll, and Ross (1985)), while others employ two factors (Brennan and Schwartz (1979), Schaefer and Schwartz (1984), Longstaff and Schwartz (1992), and Moreno (1996), among others).

From the perspective of derivatives pricing, it seems more convenient to develop models consistent with the market yield curve. This is the approach followed by Ho and Lee (1986) (using bond prices) and Heath, Jarrow, and Morton (1992) (using forward interest rates). An equivalent approach (see Dybvig (1988), and Jamshidian (1988)) is to build models based on the evolution of the short rate (or a function of it), and allow for time-dependent parameters. These parameters can be calibrated so that the model fits the current yield curve and the market prices of a set of interest-rate derivatives (typically caps). Examples of these models can be found in Hull and White (1990) (HW), Black, Derman, and Toy (1990), and Black and Karasinski (1991). Of course, a model fitted to the current term structure of interest rates could be nothing more than a parameterization of the curve and will not necessarily price correctly other interest-rate derivatives such as bond options or spread options.

In this paper, we study whether models consistent with the current term structure of interest-rate swap yields in the Spanish market and models not consistent with it produce similar results when pricing different interest-rate claims. In particular, we analyze two of the most popular one-factor interest rate models, the Vasicek (1977) and the Cox, Ingersoll, and Ross (1985) (CIR hereafter) models, and we compare them with the yield-curve term structure model of HW.

The rest of the paper is organized as follows. Next section reviews the interest-rate models used in this study and the valuation of interest-rate derivatives. In Section 2, we estimate the models using both time-series and cross-sectional data. Then we choose arbitrarily a date (06/30/1997) and we price discount and coupon bonds, discount and coupon bond options, interest-rate caps, and interest-rate swaptions. In Section 3, we study the pricing of caps in the Spanish market during the period from 01/02/1996 to 02/09/1998. In Section 4, we study the pricing of swaptions during the period from 03/22/1996 to 06/26/1997. Finally, we summarize our findings in Section 5.

1 The Interest-Rate Models

The Vasicek (1977) and CIR models assume that the term structure of interest rates at time t is given by the instantaneous interest rate, r, which follows a mean-reverting process of the form

$$dr = \kappa \left(\theta - r \right) dt + \sigma r^{\delta} dz, \tag{1}$$

where δ is equal to 0 and $\frac{1}{2}$ in the Vasicek and CIR models respectively, σ, κ, and θ are positive constants, and z is a Wiener process. In these models, the interest r is pulled towards its long-term mean θ at the rate κ.

These models are easy to implement since there exist closed-form solutions for the price of discount bonds and bond options. However, to price bond options with the CIR model, the noncentral χ^2 distribution needs to be used. We approximate this distribution using the results of Johnson and Kotz (1970).

One drawback of the Vasicek model is that r can become negative, although with a small probability in practice (as we shall see later).

To value coupon bonds we simply decompose the bonds into a portfolio of discount bonds and we price each of them accordingly.

Hull and White (1990) study a version of the Vasicek model with time-dependent parameters, which is known as the extended Vasicek model. The most general expression of the model is

$$dr = (\theta(t) - a(t)r)\, dt + \sigma(t) dz,$$

where $\theta(t)$ is chosen so that the model exactly matches the initial term structure of interest rates, $\sigma(t)$ defines the volatility of the short rate, and $a(t)$ defines the relative volatilities of long and short rates. The functions $a(t)$ and $\sigma(t)$ can be chosen to match the current prices of a set of caps, bond options, or swaptions. As Hull and White (1996) point out, this approach will be useful only if the future term structure of volatilities is likely to be similar to the initial one. They recommend keeping a and σ constant, and using the model

$$dr = (\theta(t) - ar)\, dt + \sigma dz,$$

which is the one studied in this paper. For this model, Hull and White (1990) derive closed-form solutions for the price of bonds and bond options. These solutions can be used to obtain the values of a and σ that minimize the sum of squared pricing errors for caps, or European bond options.

To value other derivatives for which there are not closed-form expressions, Hull and White (1994) build a trinomial tree where the function $\theta(t)$ is determined iteratively as the tree is built.

The valuation of options on discount bonds enables us to price interest-rate caps. A cap is a set of caplets, each of which caps the interest rate on a floating-rate loan at a fixed rate (the strike) during a given time interval. The value of the caplet at its maturity is the difference (if greater than 0) between the current market interest rate and the caplet exercise price applied to a notional amount. Then, it is easy to show that the cap is equivalent to a set of put options on discount bonds (see Jarrow and Turnbull (1996) or Hull (1997) for details).

Swaptions are options on interest-rate swaps that give the holder the right to enter into a given swap at a future date. There are two types of swaptions. A pay fixed, receive floating swaption gives its holder the right to exchange fixed-rate for floating-rate payments. Since the floating leg of the swap is always worth par, this swaption can be viewed as an option on the fixed leg with a par strike. At maturity, one would exercise this swaption when the fixed leg of the swap is worth less than its floating leg. Consequently, this swaption is equivalent to a put option on a bond that pays a coupon equal to the fixed rate of the swap, with strike equal to the principal of the swap. Analogously, a pay floating, receive fixed swaption can be treated as a call option on a coupon bond (see again Jarrow and Turnbull (1996), or Hull (1997)). To value these European options on coupon-bearing bonds with the Vasicek and CIR models, we use the results of Jamshidian (1989). He shows that an option on a coupon bond is equivalent to a portfolio of options on discount bonds of different maturities with different exercise prices. In the HW model, we value the swaptions directly on the trinomial tree.

2 Estimation and Implementation of the Models

In this section, we estimate the parameters of the interest rate process in the Vasicek and CIR models using both time-series and cross-sectional data. We then choose arbitrarily a date (June 30, 1997) in our sample period and we price different securities with these models, as well as with the HW model fitted to the market yield curve.

To avoid the large fluctuations of one-day and one-week money market rates, we take the one-month Madrid Interbank Offer Rate (MIBOR) as a proxy for the instantaneous interest rate (similar approach is followed by Ball and Torous (1999), Nowman (1997) and Bühler, Uhrig, Walter, and Weber (1999), among others). We use a sample[1] of daily rates for the period from 01/02/1996 through 02/09/1998 (524 observations). Statistics describing the data are provided in Table 1.

To estimate the models from historical data, we follow Nowman (1997) and use the Gaussian estimation method. We first express equation (1) in a discrete-time setting as

$$r_t = e^{-\kappa \Delta t} r_{t-1} + \theta \left(1 - e^{-\kappa \Delta t}\right) + \eta_t \qquad \text{(t=1, 2} \ldots \text{T)} \tag{2}$$

where Δt is the time interval, and η_t satisfies the conditions

$$\begin{aligned} E\left(\eta_t\right) &= 0, \\ E\left(\eta_s \eta_t\right) &= 0, \qquad (s \neq t), \\ E\left(\eta_t^2\right) &= \frac{\sigma^2 r_{t-1}^{2\delta}}{2\kappa}\left(1 - e^{-2\kappa \Delta t}\right). \end{aligned}$$

We then obtain the parameter estimates maximizing[2] the Gaussian log-likelihood function of the process (2), given by

$$L(\kappa, \theta, \sigma, \delta) = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln E\left(\eta_t^2\right) - \frac{1}{2}\frac{\left[r_t - e^{-\kappa \Delta t} r_{t-1} - \theta\left(1 - e^{-\kappa \Delta t}\right)\right]^2}{E\left(\eta_t^2\right)}. \tag{3}$$

This expression is the exact log-likelihood function for the Vasicek process, but only an approximation of it for the CIR process, since in this case the distribution of the error term is non-central χ^2. However, the approximation is valid when $\Delta t \to 0$ (see Brown and Schaefer (1996))[3].

Table 2 shows the parameter estimates for the Vasicek, CIR, and unrestricted mean reverting processes. t-statistics are provided in parentheses. We see that all the parameters are significant at the 95% level, and that the mean reversion parameters are similar in the three models. The long-term mean, θ, of the short rate is 4.2299% in the Vasicek model, 4.1863% in the CIR model, and 4.2122% in the unrestricted model. Interestingly, as we deduct from Table 1, these long-term means are not reached during the sample period. This is not surprising, given that, during this period, interest rates in Spain decreased systematically due to the convergence process of economic variables in the European Union. The speed of adjustment, κ, of r to its mean is 0.948140 in the Vasicek model, 0.929799 in the CIR model, and 0.944595 in the unrestricted model, which implies a mean half-life[4] of 0.7312, 0.7455, and 0.7338 years respectively.

The estimates of the parameter σ are 0.007027, 0.026211, and 0.813706 for the Vasicek, CIR, and unrestricted models, respectively.

The unrestricted estimate of δ is 1.7692 and highly significant with a t-statistic of 11.59, indicating that the volatility of the short rate is highly sensitive to the level of the interest rate (similar results are found by Chan, Karolyi, Longstaff, and Sanders (1992) and Nowman (1997) for U.S. one-month Treasury-Bill yields, and by Uhrig and Walter (1996) for German money market rates). Figure 1 plots the conditional volatility implied by the Vasicek, CIR and unrestricted models estimated from time series (denoted as VAST, CIRT, and URMR, respectively). The figure also shows the actual volatility of the short rate, computed as the absolute value of the day-to-day change in the one-month MIBOR rate. Notice that the volatility decreases with the level of the spot rate.

Since the interest rate is not a tradeable security, to price interest-rate claims with the Vasicek and CIR models we must estimate the market price of short-term interest-rate risk, λ. We estimate λ from daily cross-sections of cap volatilities and swap rates, using the estimates of κ, θ, and σ previously obtained. The data are the average across different brokers[5] of mid-market volatility quotes at 5 p.m. for at-the-money caps and of mid-market swap rates, with maturities of 1, 2, 3, 4, 5, 7, and 10 years (14 data points per day). The strikes of the caps are defined as the spot swap rates of the same maturity. The sample period is the same as that of the MIBOR rates. Using these data, we compute market and model prices of the caps and model prices of the fixed leg of the swaps. We then minimize the sum of squared relative cap and swap pricing errors (SSRE). For computational reasons, we minimize the SSRE with respect to model prices[6], i.e.

$$\min_{\lambda}\left(\sum_{i=1}^{7}\left(\frac{CMP_i - CP_i}{CP_i}\right)^2 + \sum_{i=1}^{7}\left(\frac{1-SP_i}{SP_i}\right)^2\right), \tag{4}$$

where CMP_i, CP_i, and SP_i stands for cap market price, cap model price, and swap model price respectively, and $i = 1, 2, \ldots 7$ represents the seven maturities of the caps and swaps traded on the market. In expression (4) we have used the fact that the fixed leg of the swap must be worth par.

Figure 2 plots the estimates of λ for the Vasicek and CIR models. We see that the parameter is unstable through time in both models, and ranges from -8.8011 to -2.1666 in the Vasicek model and from -0.5909 to -0.2698 in the CIR model. Notice that the risk premium remains positive (λ negative), and monotonically decreasing, as interest rates fall, in both models during the sample period. The parameter estimates of λ on 06/30/1997 are -3.7626 and -0.3870, for the Vasicek and CIR models, respectively.

We also estimate these models cross-sectionally. We center our attention on the Vasicek model since, as we shall see later, the CIR model behaves similarly in our sample. We denote this model as VASC. We first risk-neutralize the interest rate process

$$\begin{aligned} dr &= \kappa\left(\theta - r - \lambda\sigma\right)dt + \sigma d\hat{z}, \\ &= \kappa\left(\hat{\theta} - r\right)dt + \sigma d\hat{z}, \end{aligned}$$

where $\hat{\theta} = \theta - \frac{\lambda}{\kappa}\sigma$ (risk-neutral long-term mean of the instantaneous interest rate), and

$\hat{z}$ is a Wiener process under the risk-neutral probability measure $\widehat{Q}$. We then estimate the three risk-neutral parameters of the model, $\kappa, \hat{\theta}$, and σ, as well as the unobserved instantaneous interest rate, r, from daily cross-sections of swap rates and cap volatilities.

Figure 3 displays the estimates of the three risk-adjusted parameters of the VASC model. The parameter estimates are highly unstable through time. The figure also plots the instantaneous interest rate implied by the VASC model versus the one-month MIBOR (both continually compounded) used as a proxy in the time-series estimation of the model. Notice that the implied short-term rate is significantly smaller than the one-month MIBOR during the entire sample period (the average spread is 141 basis points). However, in Figure 4 we see that the conditional volatility implied by these estimates is very reasonable, decreasing with the level of the spot rate. The parameter estimates on June 30, 1997 are $\kappa = 0.014988, \hat{\theta} = 0.335109, \sigma = 0.011953$, and $r = 0.041663$. Thus, under the risk-neutral measure, the instantaneous interest rate is moving towards a long-term mean of 33.5109% at a very slow speed (mean half-life = 46.25 years).

To implement the HW model, we construct every day the zero-coupon yield curve from 1-day, 1-week, 1-month, 3-month, 6-month, and 12-month MIBOR rates and 2, 3, 4, 5, 7, and 10-year swap rates using a bootstrapping technique. We employ quadratic interpolation to compute the interest rates at other points of the curve. We then build trinomial trees for the short-term interest rate process with time intervals of 0.1 years and use the forward induction technique to make the tree consistent with the current term structure of interest-rate swap yields (see Hull and White (1994)). Every day we construct two trees. In the first one, we assume that $\theta(t) = 0$ and that the initial value of r is zero. In the second tree, we displace every node at time $i\Delta t$ by the amount α_i to make the tree match the market yield curve. Figure 5 displays α_i on June 30, 1997. The corresponding trees up to 1.2 years are shown in Figure 6. We see that negative interest rates do appear on the tree, although with a very low probability (for example, the probability of reaching node (6,-6) is 0.000122). Finally, we calibrate the parameters a and σ of the model daily using cross-sections of cap volatilities. As can be seen in Figure 7 the parameter estimates are relatively stable through time. As before, the conditional volatility of the spot short rate implied by the model is consistent with the market data (see Figure 4). The parameter estimates of a and σ on 06/30/1997 are 0.010024 and 0.009846, respectively.

Before pricing options, we analyze the valuation of discount bonds on 06/30/1997. Figure 8 displays the market yield curve on 06/30/1997, as well as the yield curves implied from the VAST, the CIRT, and the VASC models, given that the one-month MIBOR is 5.3152%, and the instantaneous risk-neutral rate is 4.1663% (both instantaneously compounded). The VAST and CIRT models clearly overestimate the curve. They are consistent with the current short-term interest rate by construction[7], but they do not fit the rest of the curve. For example, the 3-year spot rate is overestimated by 150 and 129 basis points by the VAST and CIRT models, respectively. The VASC model clearly underestimates the curve for short maturities, but it fits the curve much better than the other two models for medium- and long-term maturities. For instance, the model underestimates the 3-year rate by only 24 basis points. The yields on a consol bond, given by $r_\infty = \theta - \sigma\frac{\lambda}{\kappa} - \frac{1}{2}\frac{\sigma^2}{\kappa^2}$ in the Vasicek model and $r_\infty = \frac{2\kappa\theta}{(\gamma+\kappa+\lambda)^2}$ in the CIR model, are 7.0200% and 7.1631% in the VAST and CIRT models, respectively. In the VASC model, the yield curve increases up to 9.6427% (for a time to maturity of 73 years), and then it decreases very slowly towards 1.7611%.

The models considered price short maturity bonds accurately, but they misprice the rest of the bonds. The price difference is greater for the VAST and CIRT models and for medium-term maturities. For example, the VAST and CIRT models underprice the 6-year bond by 6.29% and 5.73%, respectively. However, the VASC model overprices it by 1.62%. As we shall see later, this mispricing will have a significant effect on the valuation of options.

Figure 8 also plots the term structure of spot rate volatilities, given by

$$\sigma(t,T) = \frac{\sigma}{\kappa(T-t)}\left(1 - e^{-\kappa(T-t)}\right),$$

in the Vasicek model and by

$$\sigma(t,T) = \frac{\sigma\sqrt{(r)}}{T-t}B(t,T),$$

in the CIR model, where

$$B(t,T) = \frac{2\left(\exp\left(\gamma(T-t)\right)-1\right)}{(\gamma+\kappa+\lambda)\left(\exp\left(\gamma(T-t)\right)-1\right)+2\gamma}.$$

We observe that the volatilities of long-term rates in the VAST and CIRT models are substantially lower than those in the VASC and HW models.

Panel A of Table 3 presents the pricing of a call option on a 10-year discount bond with a face value of \$100 on 06/30/1997 for different exercise prices and maturities. To determine the moneyness of the options, forward bond prices are also given on the table. We see that the VAST and CIRT option values are very similar, and that, as expected, they are lower than the VASC and HW prices. This is especially true for ATM and OTM options. For example, the price of a 2-year option with exercise price \$60 for the VAST and CIRT models is 0, while for the VASC and the HW model is \$2.68 and \$1.85, respectively.

Panel B of Table 3 shows the valuation, on 06/30/1997, of a call option on a 5-year bond with a face value of \$100 and a coupon of 10% per year paid semiannually. We see that, again, the VAST and CIRT models value the option very similarly, both underpricing it relative to the VASC and HW models. The relative “errors” are greater than for the 10-year discount bond option, which is consistent with the greater underpricing of the medium-term discount bonds. As before, the price difference is greater for ATM and OTM options. For example, the model prices given by the VAST, CIRT, VASC and HW models for a 1-year call with exercise price \$110 are \$0.51, \$0.98, \$6.19, and \$5.57, respectively.

Since caps are portfolios of put options on discount bonds, we expect the VAST and CIRT models to overprice them relative to the VASC and HW models on 06/30/97. Panel C of Table 3 shows this overpricing, which is greatest for at-the-money options (cap rate equal to the 6-month MIBOR, 5.1902%). The relative difference in prices is very large (higher than 100%) for short-term near-the-money caps. For instance, the price of a 1-year cap with exercise price 5.0% is \$0.94, \$0.77, 0.08, and \$0.11 for the VAST, CIRT, VASC and HW models respectively. We see that the VAST and CIRT models produce similar option prices.

In Panel D of Table 3 we price a pay fixed, receive floating European swaption for different option maturities and strikes (fixed swap rates). The underlying swap matures in 5 years, its principal is \$100, and interest rate payments are made every 6 months. As in the case of the caps, we expect the VASC and HW swaption prices to be lower than those of the Vasicek and CIR models. The table confirms our expectation. The difference in theoretical prices is higher for short-term at-the-money swaptions (fixed swap rate equal to the 3-, 6-, and 12-month forward swap rates: 5.4821%, 5.5993%, and 5.7954% respectively). For example, the VAST and CIRT swaption prices for a maturity of 3 months and a strike of 6% are \$3.40 and \$2.83, respectively, while that in the VASC and the HW models are just \$0.15, and \$0.13, respectively.

To summarize, our results indicate that there can be substantial price differences between the models for ATM and OTM options. What we do not know yet is which model best describes the market prices of the options. In the next sections, we study the ability of the models to price interest-rate caps and swaptions in the Spanish market.

3 Pricing Interest-Rate Caps on the MIBOR

As indicated earlier, our sample of caps consists of mid-market volatility quotes for at-the-money caps (the strike is set equal to the market swap rate of the same maturity). From these quotes, we compute cap market prices using the Black (1976) formula.

Figure 9 shows theoretical and market cap prices during the sample period for the VAST and CIRT models. We see that both models yield similar results: a very poor pricing of the 1-year cap, and a slightly better valuation of longer maturity caps. Table 4, Panel A, shows the Mean Absolute Percentage Errors (MAPE), defined as

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|CMP_i - CP_i|}{CP_i}, \tag{5}$$

where CMP_i and CP_i were previously defined.

In the VAST and CIRT models, the error is very large for 1-year caps (MAPE = 1,114.0% and 868.4.0%, respectively), although it decreases for longer maturities. A possible explanation of this pattern is that the one-month MIBOR seems to systematically overestimate the instantaneous interest rate (recall Figure 3), producing the overpricing of short-term caps. This overpricing is reduced for long maturity caps, as the volatility of spot rates implied by the models typically seems to decrease too fast (recall the term structure of volatilities on June 30, 1997, shown in Figure 8). The overall MAPE for the sample of caps is 263.5% for the VAST model and 202.3% for the CIRT model, indicating that these interest-rate models estimated from time-series data are not useful for pricing interest-rate derivatives in our sample.

Figure 10 plots market and model cap prices for the VASC and HW models. We see that the VASC model describes the market prices of caps remarkably well, especially for the 1- and 5-year caps. The HW model also price the caps accurately (except for the 1-year cap). In Table 4, Panel A, we have that the MAPE for the 1-year cap is just 3.1% in the VASC model, while it jumps to 30.6% in the HW model. The overall MAPE is 4.0% in the VASC model, more than 65 times lower than the MAPE of the VAST model. In the HW model, the overall MAPE is 8.2%, which is similar to the error reported in other studies (for example, Pelsser (1997) finds an average relative error of 12.32% for a sample of caps and floors in the US market on 03/15/1994).

4 Pricing Swaptions

We use a sample[8] of mid-market volatility quotes at 5 p.m. for 6- and 12-month at-the-money swaptions on 1-, 2-, 3-, 4-, 5-, 7-, and 10-year swaps. The strikes of the options are the 6- and 12-month forward swap rates, which we have to compute daily from the swap yield curve. The sample period is March 22, 1996 to June 26, 1997 (314 observations).

We obtain market prices using again Black's formula. Figures 11 and 12 show the pricing of 6- and 12-month swaptions, respectively, on 1-, 5-, and 10-year swaps. As before, the VAST and CIRT models largely misprice the options. Table 4, Panel B, reports that the overall MAPEs for the 6-month swaption are 278.9% and 210.7% for the VAST and CIRT models, respectively, while that for the 12-month swaption the MAPEs are 154.8% and 109.5%, respectively. As in the sample of caps, the CIRT model performs slightly better than the VAST model.

Since the VASC and HW models have not been calibrated to swaption prices, we expect the overall pricing error to increase with respect to the sample of caps. Table 4, Panel B, shows that the overall MAPE does indeed increase substantially. We see that the VASC model prices swaptions on short-maturity swaps much better than the HW model. For example, the MAPEs for the 6-, and 12-month swaptions on 1-year swap are 10.7% and 12.1% in the VASC model, respectively, while that in the HW model the MAPEs are 76.1% and 63.6%, respectively. For swaptions on long-maturity swaps, the HW model is superior to the VASC model in terms of both MAPE and the dispersion of the pricing error. However, the overall MAPE of the VASC model is lower than that of the HW model: 22.9% and 39.4%, respectively, for the 6-month swaption, and 15.4% and 32.7%, respectively, for the 12-month swaption.

5 Summary and Conclusions

From a practical point of view, it is appealing to use the Vasicek and CIR models to price interest-rate derivatives because they conduce to closed-form expressions for the prices of bonds and bond options. These models describe accurately the evolution of the short-term interest rate, but they will not be consistent (in general) with the market yield curve. Consequently, these models can have difficulties in pricing claims that depend on interest rates at different points of the curve. One way to avoid this problem is to build a model based on the evolution of the entire term structure. Another way is to use time-dependent parameters to make the model consistent with the market data.

In this paper, we value interest-rate claims with the Vasicek, CIR and HW models and we study the effect that the initial matching of the term structure has on model prices. We also examine whether the Vasicek and CIR models estimated from the same data set produce similar results.

We use Spanish one-month interbank deposit rates, swap rates, and implied volatilities of ATM caps to estimate the parameters of the models. We then choose a day in our sample, and we price bonds, bond options, caps, and swaptions. The results indicate that the Vasicek and CIR models estimated from historical data (VAST and CIRT, respectively) produce similar prices for interest-rate derivatives. However, when the models are estimated cross-sectionally, option prices change substantially (in some cases more than 100%), and they are close to those of the HW model.

To study the performance of the models, we use two samples. For the sample of caps, the Mean Absolute Relative Error (MAPE) of the VAST and CIRT models is 263.5% and 202.3%, respectively, while that for the Vasicek model estimated cross-sectionally (VASC) and the HW models, the MAPE dramatically drops to 4.0% and 8.2%, respectively. For the sample of swaptions, the errors of the VAST and CIRT models remain large, while the errors of VASC and the HW models increase significantly (22.9% and 39.4% for 6-month swaptions, and 15.4% and 32.7% for 12-month swaptions, respectively).

A possible explanation of the bad performance of the VAST and CIRT models is that the choice of the one-month MIBOR as a proxy for the instantaneous interest rate is not adequate. Another possibility is the existence of some degree of inefficiency in the Spanish fixed-income OTC market. With the HW model, we obtain a perfect fit to the yield curve and a reasonably good calibration to cap market prices. However, the model still misprices swaptions, reflecting the difficulty that one-factor models have to describe the volatility structure of the market. Finally, the VASC model, despite not being consistent with the market yield curve, prices caps and swaptions more accurately than the HW model. These results suggest that it is the combination of the matching of the term structure of interest rates and the term structure of volatilities what is important to price interest-rate derivatives.

Notes

[1] I am grateful to Fermin Alvarez of BSCH for kindly providing the MIBOR rates, swap rates and cap volatilities and to José Antonio Soler for comments on market issues.

[2] We employ the FORTRAN routine MINIM for function minimization using the simplex method.

[3] Since we use daily data, in our case $\Delta t = 1/250$.

[4] Defined as the time that the short rate needs to achieve the halfway between the current level and the long-run mean θ. It is computed as $\ln(2)/\kappa$.

[5] Capital Markets, Intercapital, and Euro Brokers.

[6] Minimizing the SSRE with respect to market prices produces similar, but more unstable parameter estimates. As before, we use the routine MINIM.

[7] The instantaneous interest rate in the Vasicek and CIR models is equal to the one-month MIBOR.

[8] I thank Juan Carlos Garcia Céspedes of Argentaria for kindly providing the swaptions data.

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Table 1: Descriptive statistics of the one-month MIBOR.

Mean	0.06458
Standard Deviation	0.01263
Minimum	0.04791
Maximum	0.09257
Skewness	0.51170
Kurtosis	-0.86844
ρ_1	0.99937
ρ_2	0.99878
ρ_3	0.99819

The data are daily one-month Madrid Interbank Offer Rates for the period from 01/02/1996 through 02/09/1998 (524 observations). The autocorrelation coefficient of order i is denoted as ρ_i.

Table 2: Gaussian estimates of the mean-reverting one-factor model for the instantaneous interest rate.

Model	κ	θ	σ	δ	Log-Likelihood
Vasicek	0.948140	0.042299	0.007027	0.0	3295.7479
	(2.6908)	(4.1297)	(34.7842)		
CIR	0.929799	0.041863	0.026211	0.5	3329.1499
	(2.3919)	(4.3530)	(32.3204)		
Unrestricted	0.944595	0.042122	0.813706	1.769248	3364.9351
	(4.1689)	(12.9337)	(2.3668)	(11.5876)	

The data are daily one-month interbank Spanish deposit rates (MIBOR) for the period from 01/02/1996 through 02/09/1998 (524 observations). The continuous-time model is $dr = \kappa\left(\theta - r\right)dt + \sigma r^{\delta} dz$. Gaussian estimates with t-statistics in parentheses are presented for each model. The estimates (expressed in yearly basis) are obtained maximizing the Gaussian log-likelihood function of the discrete-time version of the process

$$r_t = e^{-\kappa\Delta t} r_{t-1} + \theta\left(1 - e^{-\kappa\Delta t}\right) + \eta_t,$$

where $\Delta t = \frac{1}{250}$, and η_t satisfies the conditions

$$\begin{aligned} E\left(\eta_t\right) &= 0, \\ E\left(\eta_s \eta_t\right) &= 0, (s \neq t), \\ E\left(\eta_t^2\right) &= \frac{\sigma^2 r_{t-1}^{2\delta}}{2\kappa}\left(1 - e^{-2\kappa\Delta t}\right). \end{aligned}$$

We obtain the parameter estimates maximizing the Gaussian log-likelihood function of the discrete process, given by

$$L(\kappa, \theta, \sigma, \delta) = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln E\left(\eta_t^2\right) - \frac{1}{2}\frac{\left[r_t - e^{-\kappa\Delta t} r_{t-1} - \theta\left(1 - e^{-\kappa\Delta t}\right)\right]^2}{E\left(\eta_t^2\right)}.$$

Table 3:

Panel A. Prices ($) on 06/30/1997 of a European call option on a 10-year discount bond with a face value of $100.

Option Maturity (years)	Model	Forward Bond Price	Exercise Price					
			50	60	70	80	90	100
2	VAST	58.08	6.35	0.00	0.00	0.00	0.00	0.00
	CIRT	58.30	6.22	0.00	0.00	0.00	0.00	0.00
	VASC	63.29	9.22	2.68	0.42	0.00	0.00	0.00
	HW	61.72	8.25	1.85	0.14	0.00	0.00	0.00
4	VAST	66.82	12.04	4.36	0.00	0.00	0.00	0.00
	CIRT	67.22	11.84	4.10	0.00	0.00	0.00	0.00
	VASC	72.47	13.70	6.32	1.81	0.38	0.00	0.00
	HW	71.31	12.89	5.43	1.15	0.11	0.00	0.00
6	VAST	76.86	17.06	10.38	3.70	0.00	0.00	0.00
	CIRT	77.40	16.92	10.20	3.48	0.00	0.00	0.00
	VASC	81.90	18.39	11.17	4.75	1.12	0.30	0.00
	HW	81.09	17.77	10.65	4.14	0.71	0.05	0.00
8	VAST	88.24	21.43	15.63	9.82	4.01	0.00	0.00
	CIRT	88.62	21.38	15.55	9.72	3.89	0.00	0.00
	VASC	91.22	22.98	16.65	10.32	4.17	0.53	0.00
	HW	90.62	22.57	16.40	10.22	4.14	0.39	0.00
10	VAST	100.00	25.24	20.19	15.14	10.09	5.04	0.00
	CIRT	100.00	25.27	20.21	15.16	10.11	5.05	0.00
	VASC	100.00	27.31	21.85	16.39	10.92	5.46	0.00
	HW	100.00	26.71	21.37	16.02	10.68	5.34	0.00

Panel B. Prices on 06/30/1997 of a European call option on a 5-year bond with a face value of $100 and a coupon of 10% per year paid semiannually.

Option Maturity (years)	Model	Exercise Price					
		60	70	80	90	100	110
1	VAST	47.60	38.17	28.75	19.32	9.90	0.51
	CIRT	48.15	38.71	29.26	19.82	10.38	0.98
	VASC	53.90	44.33	34.75	25.18	15.61	6.19
	HW	53.04	43.53	34.01	24.50	14.99	5.57
2	VAST	42.23	33.41	24.58	15.76	6.93	0.00
	CIRT	42.62	33.76	24.90	16.03	7.17	0.00
	VASC	47.35	38.23	29.11	19.99	10.87	2.77
	HW	46.69	37.63	28.56	19.50	10.44	2.22
3	VAST	37.35	29.11	20.87	12.63	4.39	0.00
	CIRT	37.62	29.34	21.05	12.76	4.47	0.00
	VASC	41.35	32.69	24.03	15.37	6.74	0.64
	HW	40.89	32.29	23.69	15.09	6.50	0.37
4	VAST	32.85	25.16	17.48	9.79	2.10	0.00
	CIRT	33.06	25.32	17.49	9.85	2.11	0.00
	VASC	35.87	27.68	19.49	11.30	3.15	0.00
	HW	35.70	27.59	19.49	11.38	3.27	0.00
5	VAST	30.10	22.57	15.05	7.52	0.00	0.00
	CIRT	30.30	22.73	15.15	7.57	0.00	0.00
	VASC	32.41	24.31	16.21	8.10	0.00	0.00
	HW	33.54	25.15	16.77	8.38	0.00	0.00

Panel C. Prices on 06/30/1997 of a cap on the 6-month MIBOR when the principal is $100, interest payments are made every 6 months, and the cap rate is compounded semiannually.

Life of the Cap (years)	Model	Cap Rate (%)					
		3.0	3.5	4.0	4.5	5.0	5.5
1	VAST	2.86	2.38	1.90	1.45	0.94	0.47
	CIRT	2.69	2.21	1.73	1.25	0.77	0.29
	VASC	1.39	0.91	0.47	0.18	0.08	0.03
	HW	1.46	0.98	0.55	0.26	0.11	0.03
2	VAST	6.19	5.25	4.32	3.40	2.47	1.54
	CIRT	5.78	4.85	3.92	2.99	2.06	1.14
	VASC	3.17	2.27	1.46	0.85	0.51	0.28
	HW	3.25	2.34	1.52	0.90	0.49	0.23
3	VAST	9.51	8.16	6.81	5.47	4.12	2.77
	CIRT	8.98	7.63	6.28	4.93	3.58	2.23
	VASC	5.25	3.96	2.80	1.87	1.27	0.82
	HW	5.31	4.00	2.80	1.85	1.16	0.68
4	VAST	12.69	10.95	9.22	7.48	5.74	4.00
	CIRT	12.13	10.39	8.64	6.90	5.15	3.42
	VASC	7.56	5.90	4.40	3.17	2.29	1.60
	HW	7.76	6.07	4.50	3.22	2.22	1.47
5	VAST	15.69	13.59	11.49	9.39	7.28	5.18
	CIRT	15.16	13.05	10.94	8.83	6.72	4.61
	VASC	10.04	8.03	6.21	4.68	3.52	2.59
	HW	10.33	8.28	6.37	4.76	3.46	2.45

Panel D. Prices on 06/30/1997 of a pay fixed, receive floating European swaption on a 5-year interest rate swap when the principal is $100 and interest payments are made every 6 months.

Life of Swaption (months)	Model	Fixed Swap Rate (%)				
		4.0	4.5	5.0	5.5	6.0
3	VAST	11.66	9.60	7.53	5.46	3.40
	CIRT	11.14	9.07	6.99	4.91	2.83
	VASC	5.66	3.61	1.81	0.65	0.15
	HW	6.15	4.03	2.11	0.72	0.13
6	VAST	11.73	9.71	7.68	5.64	3.61
	CIRT	11.28	9.23	7.19	5.15	3.10
	VASC	6.18	4.12	2.44	1.22	0.49
	HW	6.35	4.30	2.45	1.07	0.35
12	VAST	11.73	9.75	7.79	5.83	3.86
	CIRT	11.40	9.43	7.45	5.47	3.50
	VASC	6.87	5.02	3.42	2.14	1.22
	HW	7.04	5.11	3.39	2.00	1.03

VAST and CIRT denote the Vasicek (1977) and Cox, Ingersoll, and Ross (1985) models, respectively, estimated from time-series data. VASC represents the Vasicek model estimated cross-sectionally, and HW stands for the Hull and White (1990) model. The parameters of the true interest rate processes are estimated from a sample of Spanish 1-month interbank deposit rates (MIBOR) for the period 01/02/1996-02/09/1998. They are $\kappa = 0.948140, \theta = 0.042299, \sigma = 0.007027$ for the VAST model, and $\kappa = 0.929799, \theta = 0.041863, \sigma = 0.026211$ for the CIRT model. The market price of risk, λ, is estimated from a cross-section of Spanish swap rates and cap volatilities on 06/30/1997, resulting in $\lambda = -3.762620$ and $\lambda = -0.387043$ for the VAST and CIRT models respectively. The VASC model is estimated from a cross-section of Spanish swap rates and cap volatilities on 06/30/1997, resulting in $\kappa = 0.01498, \hat{\theta} = 0.335109$, and $\sigma = 0.011953$. The instantaneous interest rate implied by the VASC model is $r = 4.1663\%$. The HW model is fitted to the term structure of Spanish swap yields on 06/30/1997 and the parameters a and σ are estimated from a cross-section of Spanish cap volatilities on the same day, obtaining 0.010024 and 0.009846 respectively. The 1-month MIBOR and 6-month MIBOR are 5.3270%, and 5.1902%, respectively. The 3-, 6-, and 12-month forward swap rates are 5.4821%, 5.5993% and 5.7954% respectively. The year is assumed to have 360 days.

Table 4:

Panel A. Mean Absolute Percentage Errors (%) for a sample of Spanish caps.

Life of the cap (years)	VAST	CIRT	VASC	HW
1	1,114.0	868.4	3.1	30.6
2	338.5	255.6	6.8	1.7
3	189.0	139.6	5.0	5.1
4	110.1	78.6	2.9	6.5
5	62.1	41.9	3.1	6.7
7	9.6	8.2	3.4	4.4
10	21.0	23.7	3.6	2.5
Overall mean	263.5	202.3	4.0	8.2

Panel B. Mean Absolute Percentage Errors (%) for a sample of Spanish swaptions.

Life of the swap (years)	6-Month swaptions				12-Month swaptions			
	VAST	CIRT	VASC	HW	VAST	CIRT	VASC	HW
1	413.6	310.9	10.7	76.1	255.2	170.4	12.1	63.6
2	367.5	263.1	12.7	47.1	226.3	151.1	11.4	37.6
3	339.1	244.1	15.1	35.1	193.8	131.5	9.6	27.4
4	296.8	218.3	20.0	29.0	161.3	113.0	11.8	21.9
5	251.1	190.2	26.1	26.2	128.9	93.6	15.8	19.1
7	172.0	141.0	34.8	34.9	76.0	61.4	21.9	35.8
10	112.0	107.5	41.2	27.7	42.1	45.2	25.5	23.8
Overall mean	278.9	210.7	22.9	39.4	154.8	109.5	15.4	32.7

In the Vasicek and CIR models implemented using time series (VAST and CIRT respectively), the parameters of the true interest rate processes are estimated from a sample of 1-month MIBOR rates for the period from 01/02/1996 to 02/09/1998. They are $\kappa = 0.948140, \theta = 0.042299, \sigma = 0.007027$ for the VAST model, and $\kappa = 0.929799, \theta = 0.041863, \sigma = 0.026211$ for the CIRT model. The market price of risk, λ, is estimated from a cross-section of swap rates and cap volatilities. In the Vasicek model implemented cross-sectionally (VASC), the risk neutral parameters are estimated from cross-sections of swap rates and cap volatilities. The HW model is daily fitted to the term structure of swap yields and the parameters a and σ are estimated from a cross-section of cap volatilities on the same day. The cap sample covers the same period as the MIBOR sample, while the swaption sample covers the period from 3/22/1996 to 6/27/1997.

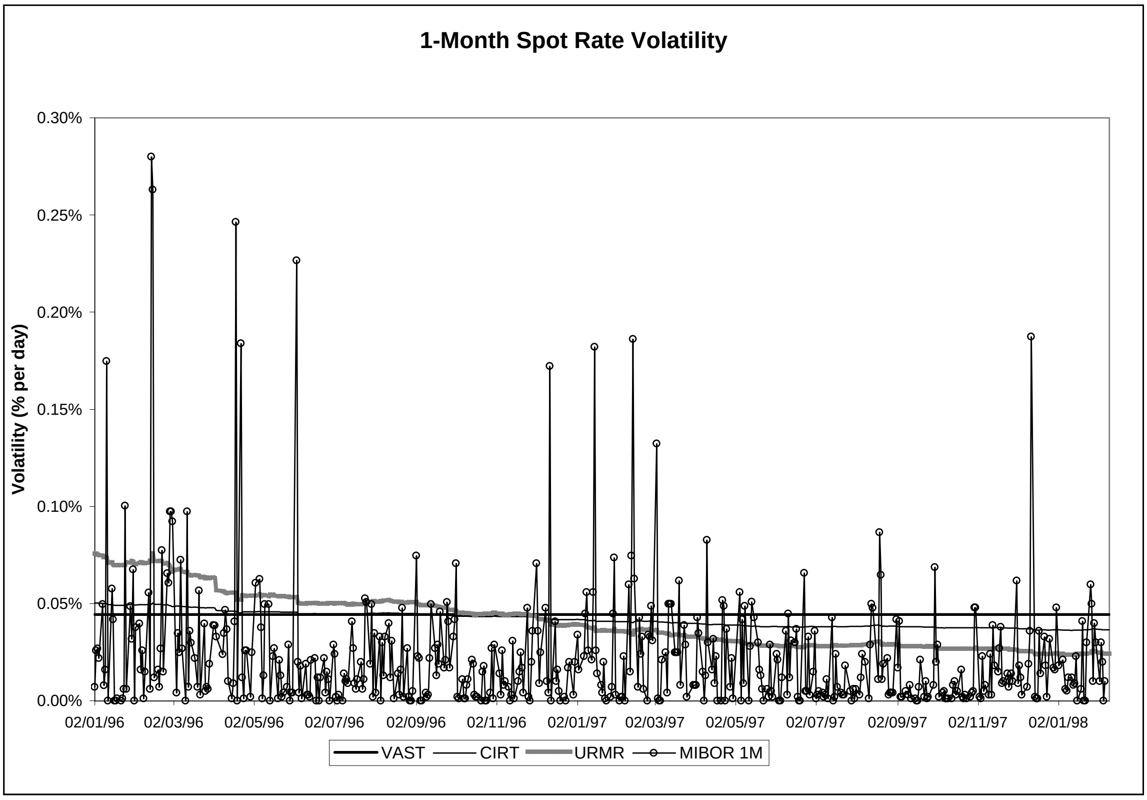


Figure 1: One-month spot rate volatility in the VAST, CIRT, and unrestricted mean reversion (URMR) models. The volatility of the one-month MIBOR is computed as the absolute value of the day-to-day change in the spot rate.

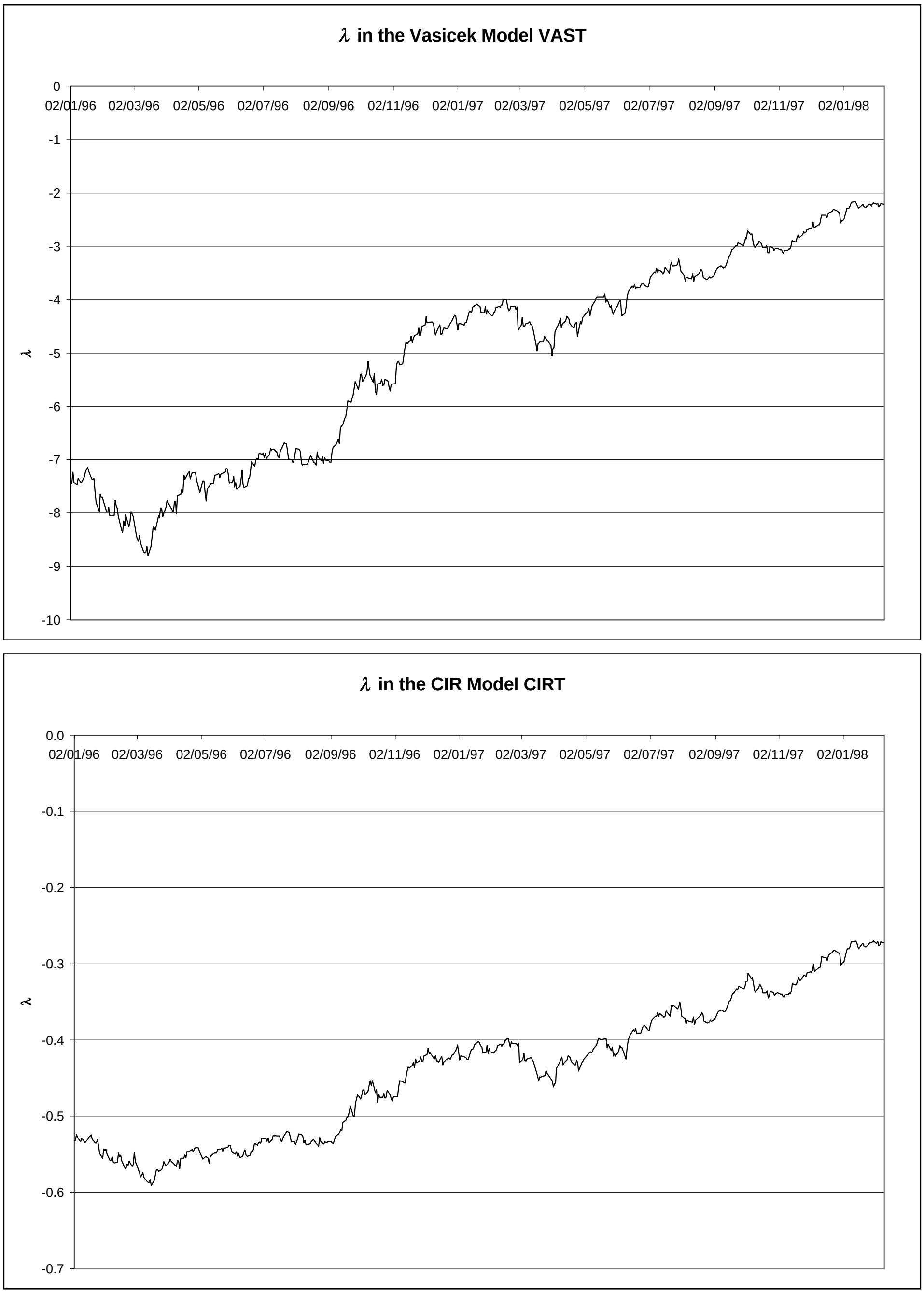
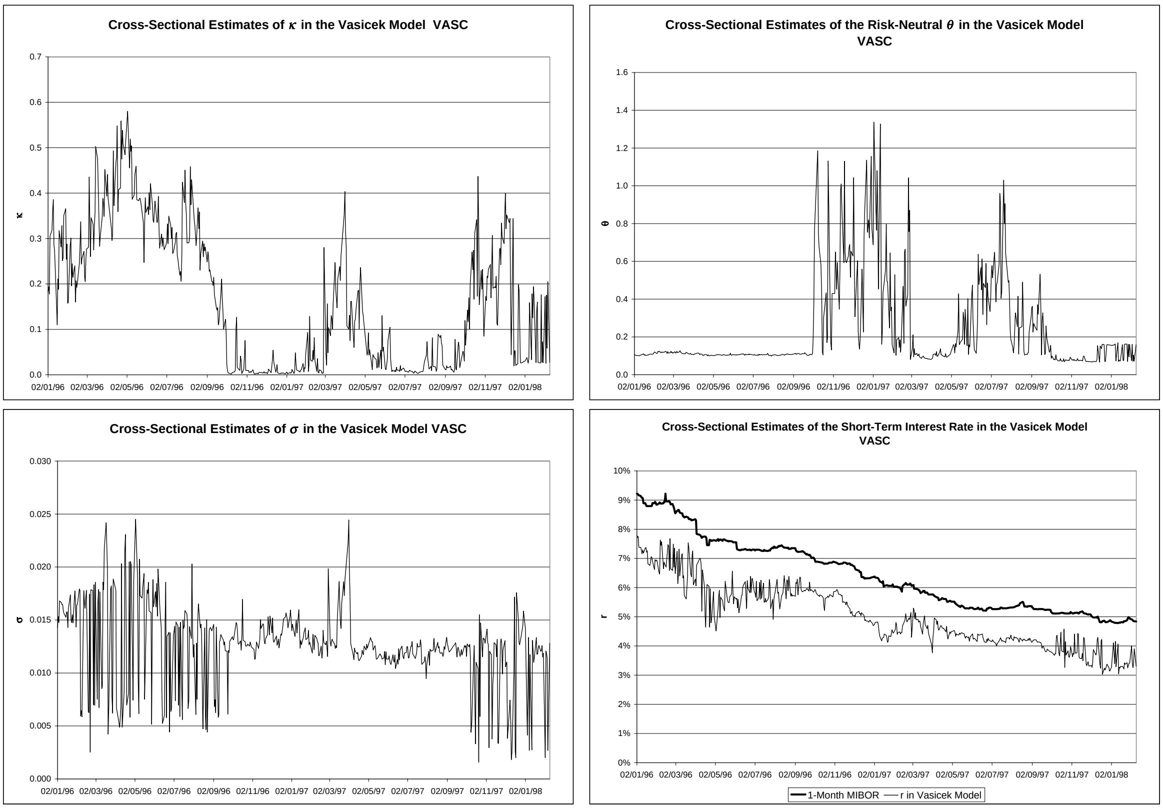


Figure 2: Cross-sectional estimates of λ in the VAST and CIRT models. The parameters of the interest rate process are estimated from time-series data, and the market price of risk, λ, is estimated from cross-sections of swap rates and cap volatilities.



Cross-Sectional Estimates of κ in the Vasicek Model VASC

Cross-Sectional Estimates of the Risk-Neutral θ in the Vasicek Model VASC

Cross-Sectional Estimates of σ in the Vasicek Model VASC

Cross-Sectional Estimates of the Short-Term Interest Rate in the Vasicek Model VASC

Figure 3: Cross-sectional estimation of the parameters of the VASC model using swap rates and cap volatilities.

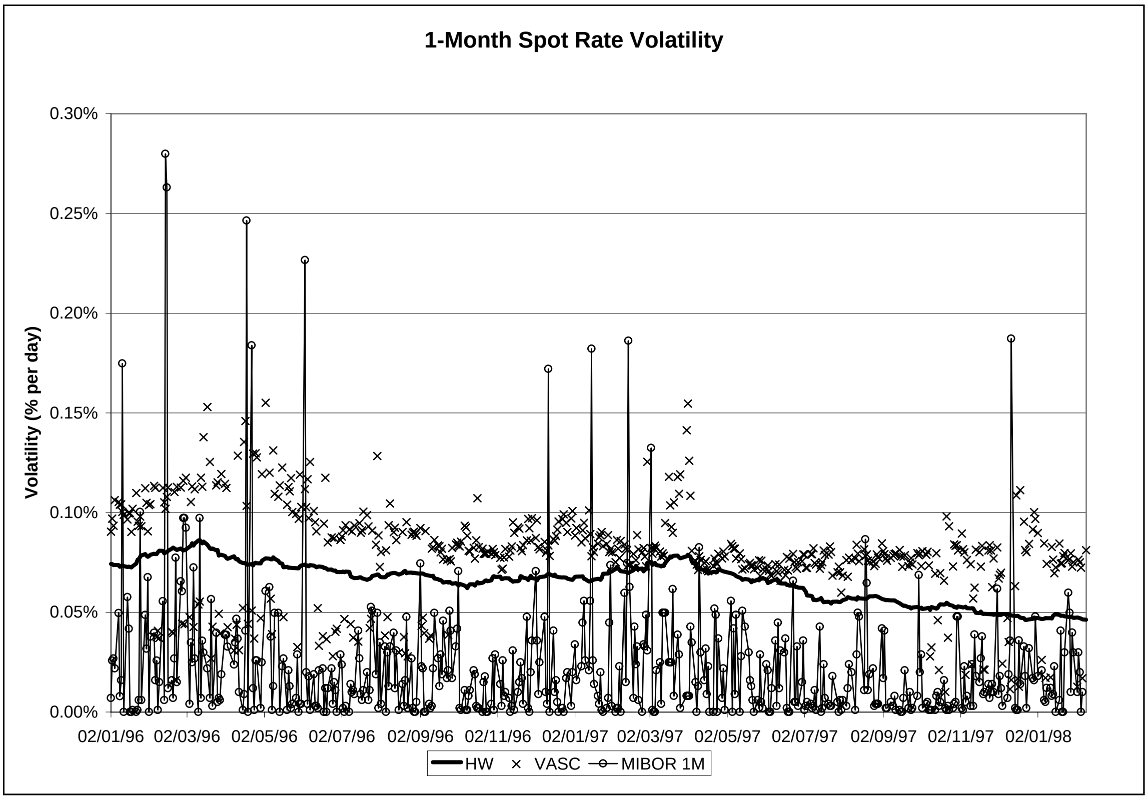


Figure 4: One-month spot rate volatility in the VASC and HW models. The volatility of the one-month MIBOR is computed as the absolute value of the day-to-day change in the spot rate.

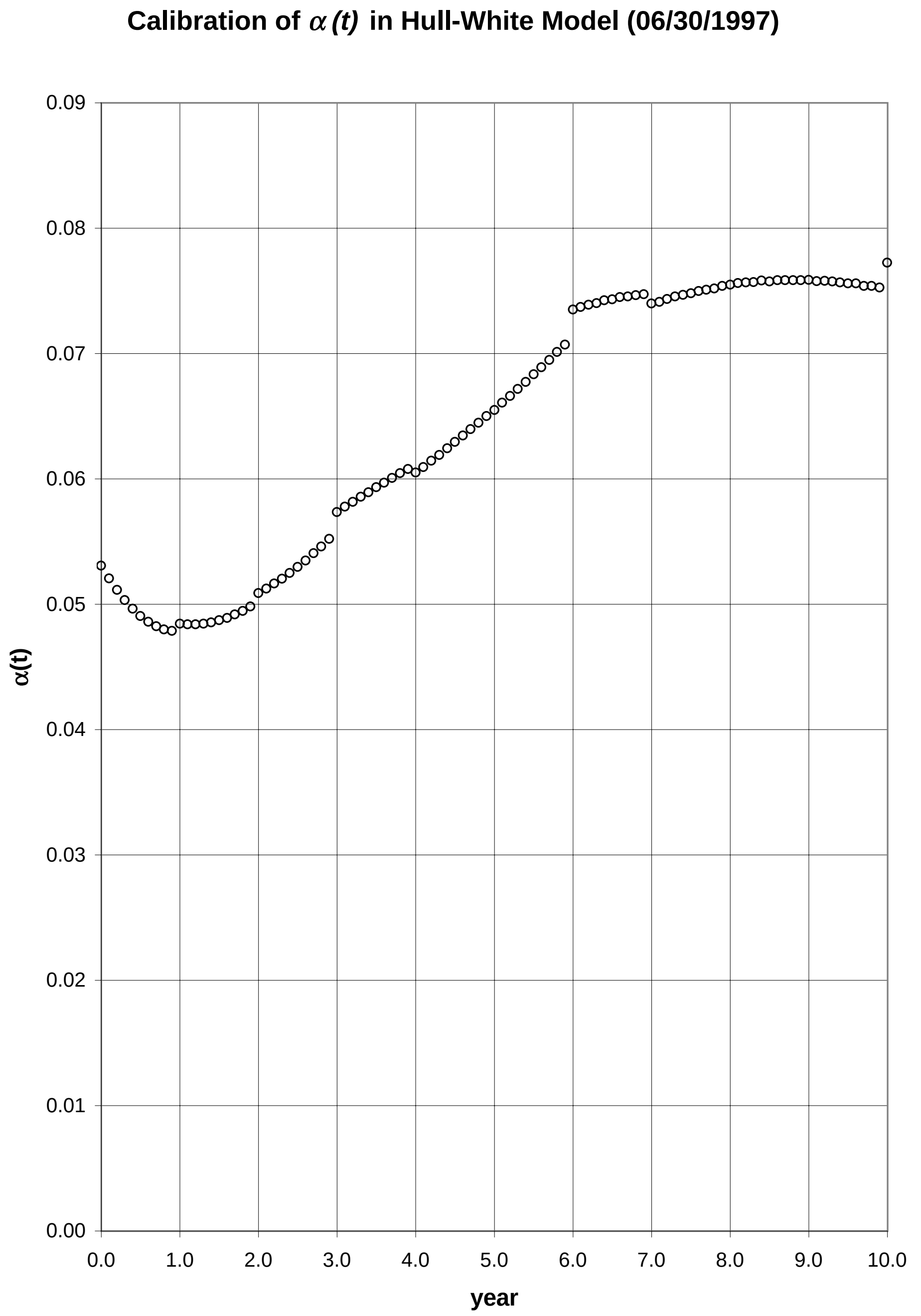


Figure 5: Calibration of α_i in the Hull-White model on 06/30/1997. The model is fitted to the term structure of interest-rate swap yields.

Hull-White Tree Assuming θ(t)=0 (06/30/1997)

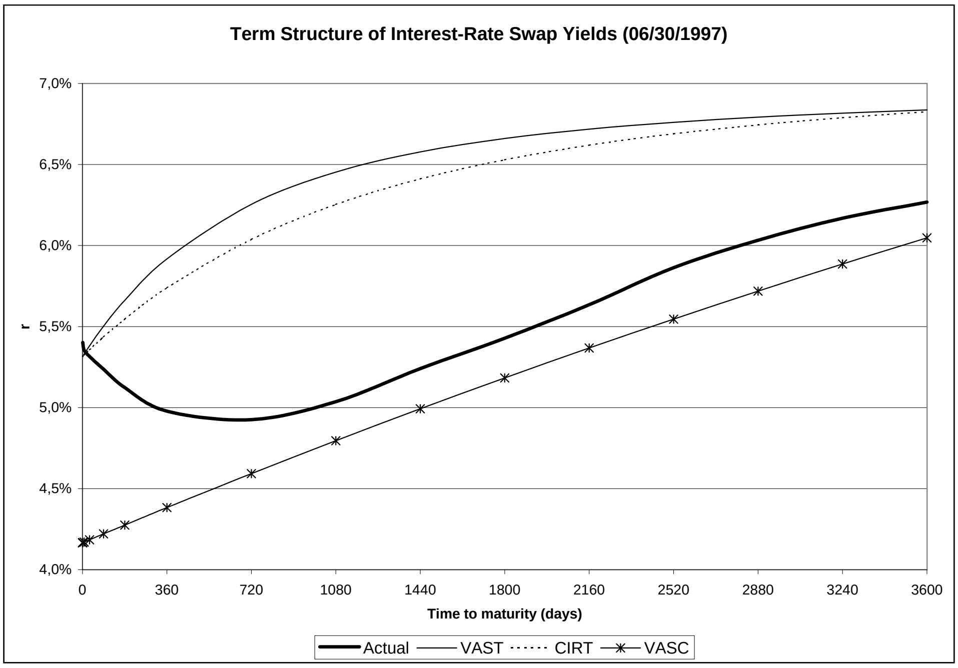
10											0.0923	0.0923	0.0923
9										0.0831	0.0831	0.0831	0.0831
8									0.0738	0.0738	0.0738	0.0738	0.0738
7								0.0646	0.0646	0.0646	0.0646	0.0646	0.0646
6							0.0554	0.0554	0.0554	0.0554	0.0554	0.0554	0.0554
5						0.0461	0.0461	0.0461	0.0461	0.0461	0.0461	0.0461	0.0461
4					0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369	0.0369
3				0.0277	0.0277	0.0277	0.0277	0.0277	0.0277	0.0277	0.0277	0.0277	0.0277
2			0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185	0.0185
1		0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092	0.0092
0	0	0	0	0	0	0	0	0	0	0	0	0	0
-1		-0.0092	-0.0092	-0.0092	-0.0092	-0.0092	-0.0092	-0.0092	-0.0092	-0.0092	-0.0092	-0.0092	-0.0092
-2			-0.0185	-0.0185	-0.0185	-0.0185	-0.0185	-0.0185	-0.0185	-0.0185	-0.0185	-0.0185	-0.0185
-3				-0.0277	-0.0277	-0.0277	-0.0277	-0.0277	-0.0277	-0.0277	-0.0277	-0.0277	-0.0277
-4					-0.0369	-0.0369	-0.0369	-0.0369	-0.0369	-0.0369	-0.0369	-0.0369	-0.0369
-5						-0.0461	-0.0461	-0.0461	-0.0461	-0.0461	-0.0461	-0.0461	-0.0461
-6							-0.0554	-0.0554	-0.0554	-0.0554	-0.0554	-0.0554	-0.0554
-7								-0.0646	-0.0646	-0.0646	-0.0646	-0.0646	-0.0646
-8									-0.0738	-0.0738	-0.0738	-0.0738	-0.0738
-9										-0.0831	-0.0831	-0.0831	-0.0831
-10											-0.0923	-0.0923	-0.0923
Δt	0	1	2	3	4	5	6	7	8	9	10	11	12

Hull-White Tree Fitted to Yield Curve (06/30/1997)

α(t)	0.0531	0.0520	0.0511	0.0503	0.0496	0.0491	0.0486	0.0482	0.0480	0.0479	0.0484	0.0484	0.0484
10											0.1407	0.1407	0.1407
9										0.1309	0.1315	0.1314	0.1315
8									0.1218	0.1217	0.1223	0.1222	0.1222
7								0.1128	0.1126	0.1125	0.1130	0.1130	0.1130
6							0.1040	0.1036	0.1034	0.1032	0.1038	0.1038	0.1038
5						0.0952	0.0947	0.0944	0.0941	0.0940	0.0946	0.0945	0.0945
4					0.0865	0.0860	0.0855	0.0851	0.0849	0.0848	0.0853	0.0853	0.0853
3				0.0780	0.0773	0.0767	0.0763	0.0759	0.0757	0.0755	0.0761	0.0761	0.0761
2			0.0696	0.0688	0.0681	0.0675	0.0670	0.0667	0.0664	0.0663	0.0669	0.0669	0.0669
1		0.0613	0.0604	0.0596	0.0589	0.0583	0.0578	0.0575	0.0572	0.0571	0.0577	0.0576	0.0576
0	0.0531	0.0520	0.0511	0.0503	0.0496	0.0491	0.0486	0.0482	0.0480	0.0479	0.0484	0.0484	0.0484
-1		0.0428	0.0419	0.0411	0.0404	0.0398	0.0394	0.0390	0.0388	0.0386	0.0392	0.0392	0.0392
-2			0.0327	0.0319	0.0312	0.0306	0.0301	0.0298	0.0295	0.0294	0.0300	0.0299	0.0299
-3				0.0226	0.0220	0.0214	0.0209	0.0206	0.0203	0.0202	0.0208	0.0207	0.0207
-4					0.0127	0.0121	0.0117	0.0113	0.0111	0.0110	0.0115	0.0115	0.0115
-5						0.0029	0.0024	0.0021	0.0018	0.0017	0.0023	0.0023	0.0023
-6							-0.0068	-0.0071	-0.0074	-0.0075	-0.0069	-0.0070	-0.0070
-7								-0.0164	-0.0166	-0.0167	-0.0162	-0.0162	-0.0162
-8									-0.0258	-0.0260	-0.0254	-0.0254	-0.0254
-9										-0.0352	-0.0346	-0.0347	-0.0346
-10											-0.0438	-0.0439	-0.0439
Δt	0	1	2	3	4	5	6	7	8	9	10	11	12

Figure 6: Hull-White trinomial trees up to 1.2 years on 30/06/1997.

Figure 7: Cross-sectional estimation of the parameters of the Hull-White model using swap rates and cap volatilities.



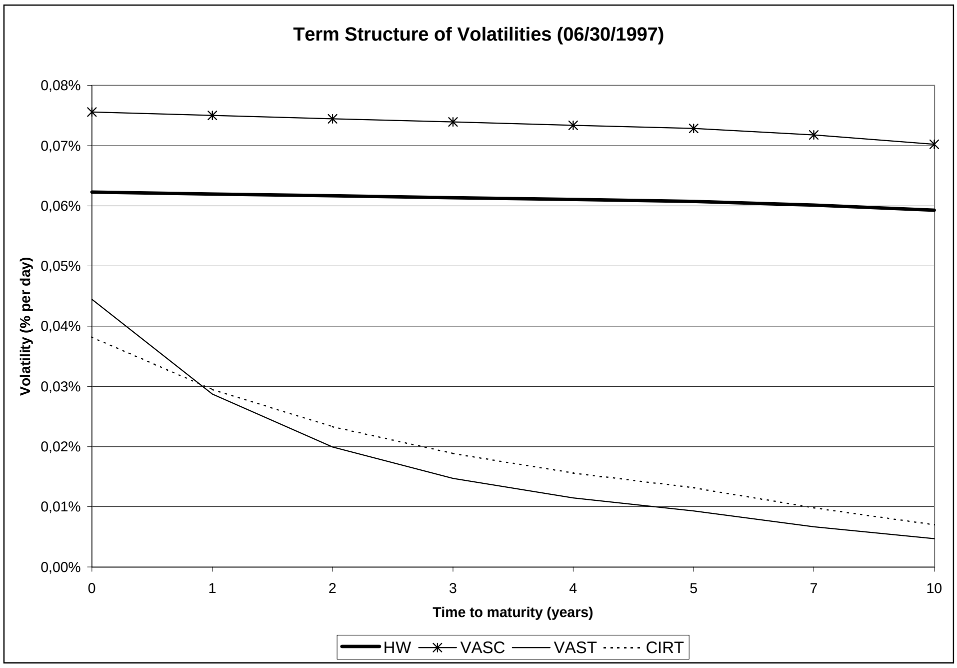


Figure 8: Term structure of interest-rate swap yields and volatilities on 06/30/1997.

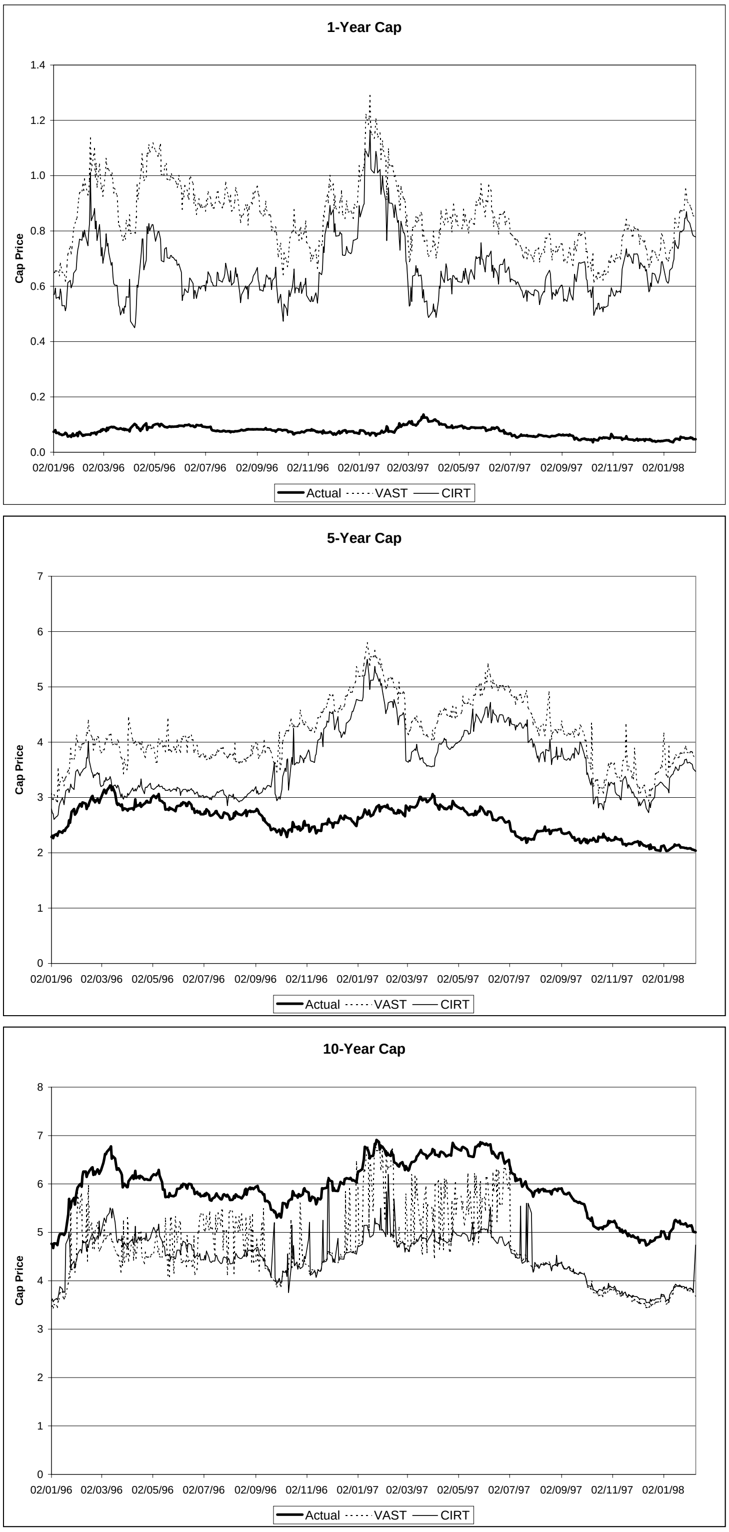


Figure 9: Cap prices for the VAST and CIRT models. The parameters of the interest rate process are estimated from time-series data, and the market price of risk, λ is estimated from a cross-section of swap rates and cap volatilities.

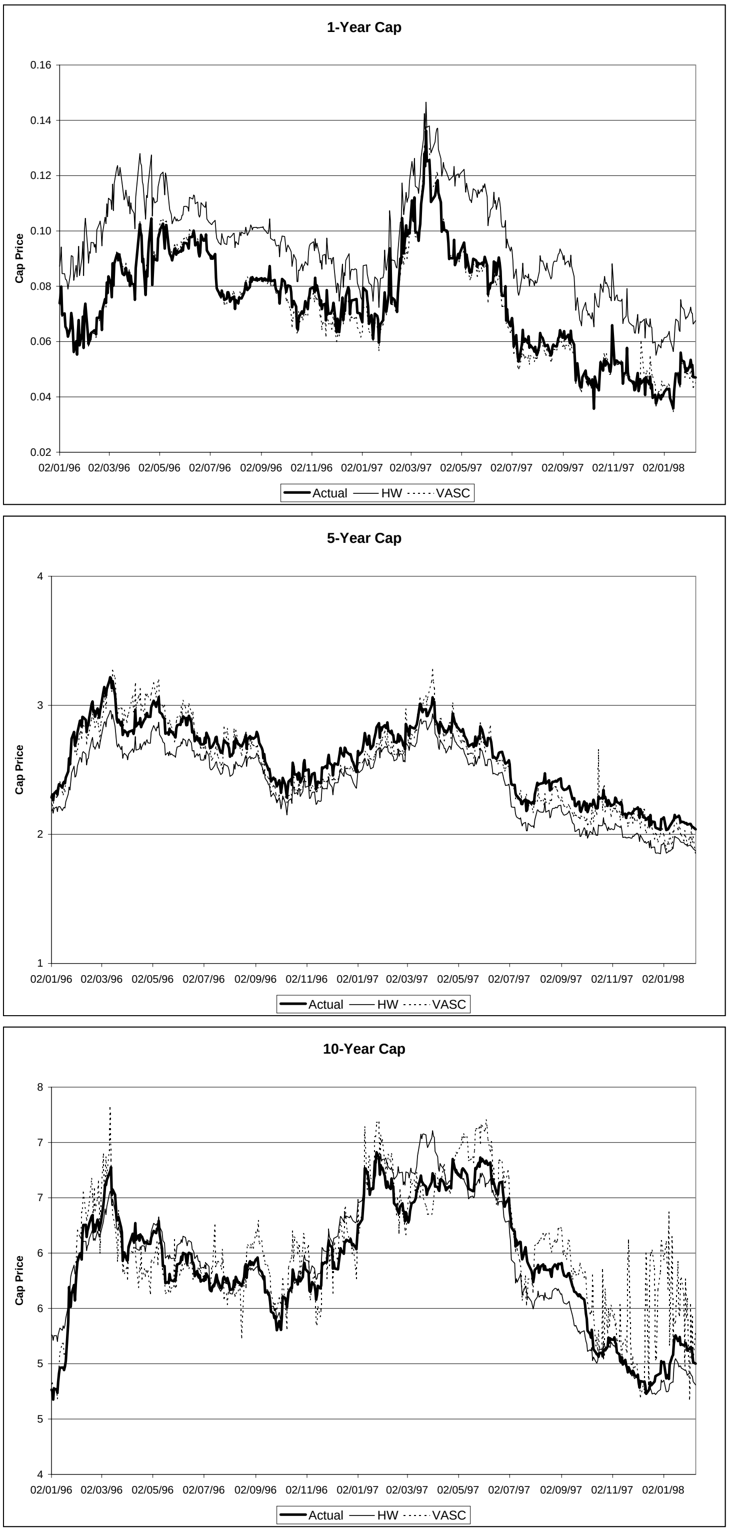


Figure 10: Cap prices for the Hull-White and VASC models. The former model is fitted to the swap yield curve and calibrated to cap market prices, while the latter model is estimated cross-sectionally using swap rates and cap volatilities.

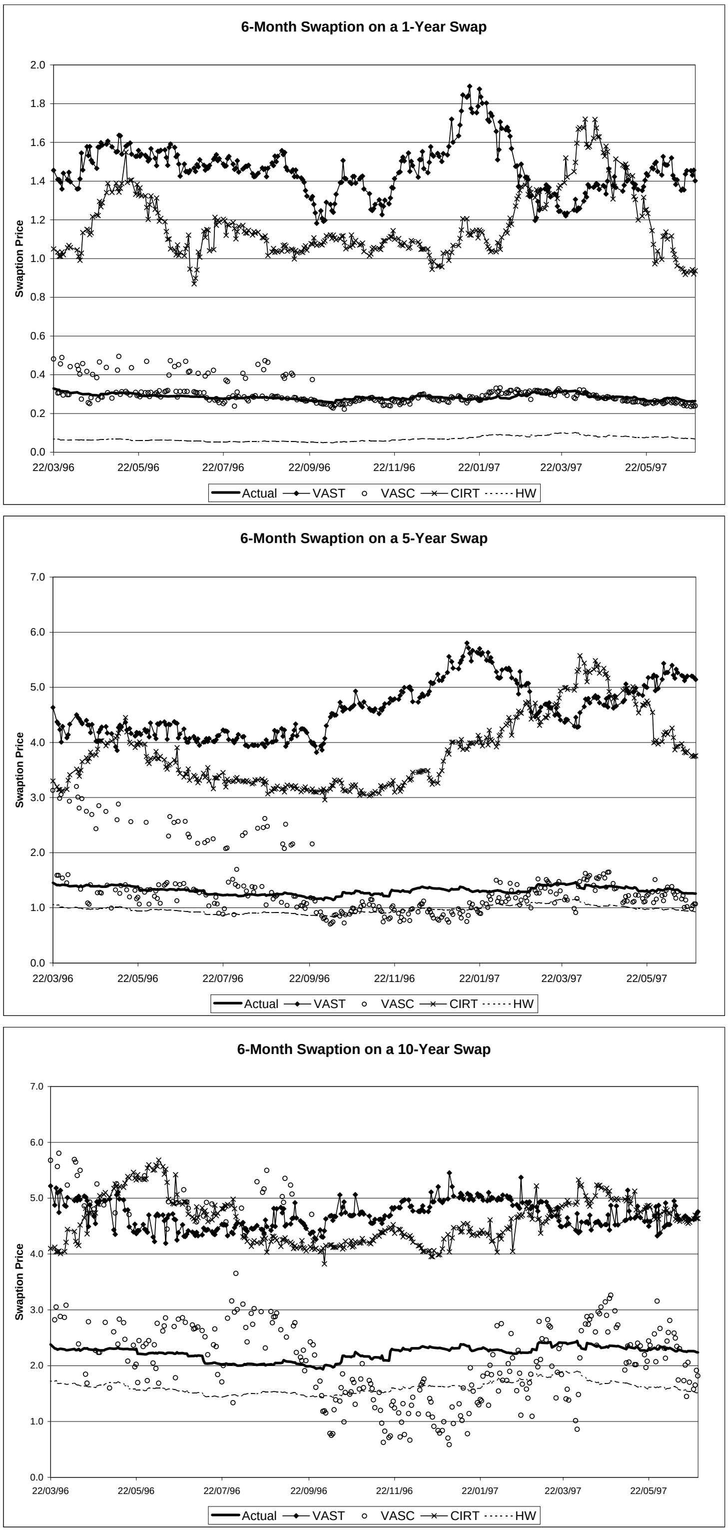


Figure 11: Six-Month Swaption Prices.

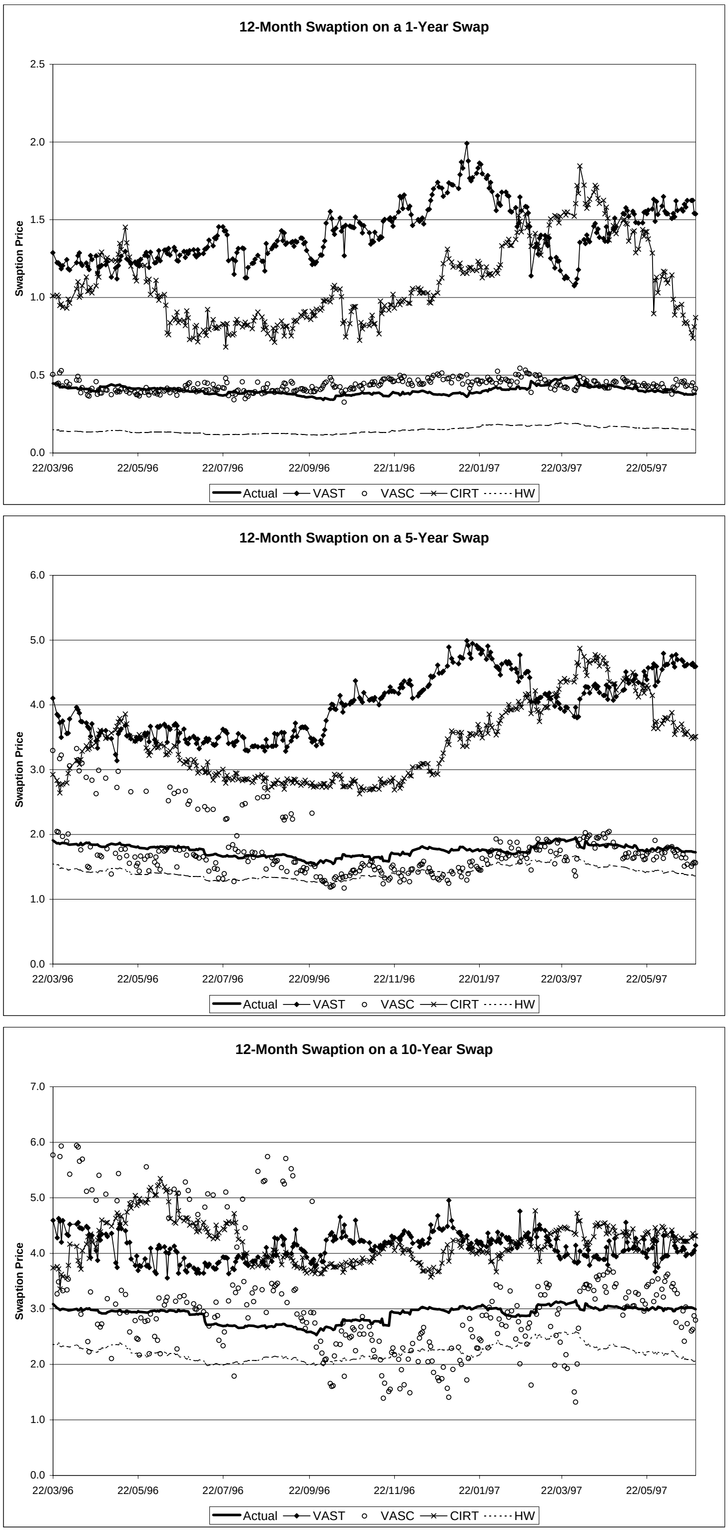


Figure 12: Twelve-Month Swaption Prices.