

On the Dark Side of Secured Debt

Javier F. Navas¹

This Draft: October 16, 2007

JEL Classification: C15, G13, G31, G32

Keywords: Secured debt, Underinvestment, Asset substitution

¹I thank Isabel Abínzano and John J. McConnell for their valuable suggestions. The usual caveat applies. Universidad Pablo de Olavide, Ctra. de Utrera, Km. 1, 41013 Sevilla, Spain. Phone: +34 954348551. Fax: +34 954348353. E-mail: jfernav@upo.es

On the Dark Side of Secured Debt

Abstract

It is widely accepted that secured debt can increase the value of the firm by reducing the agency costs of debt. Many studies have shown that secured debt alleviates the underinvestment problem and prevents asset substitution. In this paper we show that secured debt can also produce the opposite effects. Using the valuation framework of Stulz and Johnson (1985) we provide numerical examples where junior secured debt creates or worsens the underinvestment problem while senior secured debt creates or worsens the overinvestment problem. The overall conclusion, though, is not that firms should avoid using secured debt, as other positive effects of secured debt can more than compensate the results reported here.

JEL Classification: C15, G13, G31, G32

Keywords: Secured debt, Underinvestment, Asset substitution

This Draft: October 16, 2007

1 Introduction

A loan agreement is said to be secured when the borrower offers an asset to the lender as collateral. If the borrower defaults, the lender can take control of the asset and sell it. The reasons for using secured debt are diverse. For example, Scott (1977) argues that collateral increases firm value by diminishing the funds available to pay legal damages in bankruptcy.

Another recognized benefit of secured debt is the reduction of the conflict of interests between debtholders and equityholders relative to the firm's investment policy. When there is risky debt outstanding and managers make investment decisions that maximize stockholder wealth rather than firm value, managers will tend to avoid low-risk positive-NPV projects in which equity value decreases. This is the underinvestment problem analyzed by Myers (1977). Additionally, managers will tend to accept high-risk negative-NPV projects in which the value of equity increases. This is known as the overinvestment or asset substitution problem discussed by Jensen and Meckling (1976). A number of authors, including Smith and Warner (1979), Jackson and Kronman (1979) and Stulz and Johnson (1985, p. 513), argue that secured debt can prevent asset substitution, while others, such as Stulz and Johnson (1985), demonstrate that secured debt reduces the underinvestment problem since the firm will reject fewer low-risk profitable projects.¹ One empirical implication of this theory is that firms with more growth options should use more secured debt. This is supported by the findings of Chen, Yeo, and Ho (1998) but not by the results of Barclay and Smith (1995).

A third benefit of secured debt is the reduction of information asymmetries between borrowers and lenders. Chan and Kanatas (1985) and Besanko and Thakor (1987), among others, argue that collateral allows a firm to signal the high quality of a project

¹Other ways to reduce the underinvestment problem are by using debt whose maturity can be extended by stockholders (see Franks and Torous, 1989) and by hedging with derivatives (see Gay and Nam, 1998).

to a lender. However, Barclay and Smith (1995) and Chen, Yeo, and Ho (1998) find little support for this hypothesis. More recently, Gonas, Highfield and Mullineaux (2004) obtain that, in their sample, firms with S&P rating tend to secure loans less often than firms not rated by S&P, and that larger firms are less likely to enter a secured loan agreement, which is consistent with the idea that banks require collateral when there is asymmetric information.

Obviously, there are also costs associated with the use of collateral since secured debt 1) requires security registration, 2) restricts asset usage, and 3) increases the need for valuing and monitoring the asset pledged. In this paper we provide some new insights into these costs. Drawing on theoretical work by Merton (1974) and Stulz and Johnson (1985), we argue that secured debt can bring about a decrease in firm value not recognized before. We distinguish between junior and senior debt and we present numerical examples where junior secured debt aggravates the underinvestment problem while senior secured debt worsens the overinvestment problem (relative to unsecured debt financing). Moreover, we show that junior (senior) secured debt produces underinvestment (overinvestment) in situations where those problems do not exist with unsecured debt financing.

The rest of the paper is organized as follows. Next section introduces the notation and briefly reviews the underinvestment and overinvestment problems. In Section 3 we present some numerical examples to illustrate our point. Section 4 summarizes and concludes the paper.

2 Pricing corporate liabilities for different financing choices

Our valuation framework follows Stulz and Johnson (1985) and is similar to that of Merton (1974). We consider a firm which owns one productive asset with value $A(t)$ at time $t > 0$. The current value is assumed to be A_0 . The firm is initially financed by unsecured debt DU and equity E . The debt consists of a single zero-coupon bond with face value F and maturity T . Default occurs if, at time T , the value of the firm's asset is smaller than the face value of the bond. In this case, the asset value is transferred to the bondholders. Other basic assumptions are:

- A1. Markets are perfect: there are no taxes, no transaction costs, and no limits in short sales. Moreover, trading takes place continuously in time.
- A2. The instantaneous riskless interest rate r is constant.
- A3. The value of the firm is independent of the capital structure of the firm (i.e., the Modigliani-Miller's Proposition I holds).
- A4. In the event of default, priority rules are strictly enforced.
- A5. Markets are complete. Thus, there exists an equivalent "risk-neutral" probability measure under which discounted asset values are martingales.
- A6. The asset price under the risk-neutral measure is governed by the following stochastic differential equation:

$$\frac{dA(t)}{A(t)} = rdt + \sigma_A dZ_A$$

where σ_A is a non-negative constant and Z is a standard Wiener process. We assume that the asset does not pay or receive any cash flow.

In this framework, the values of debt and equity at maturity are given by

$$\begin{aligned}
DU(T) &= \min \{F; A(T)\} \\
&= F - \max \{0; F - A(T)\} \\
E(T) &= \max \{A(T) - F; 0\}
\end{aligned}$$

Hence, before maturity we can write

$$(1) \quad DU(t) = Fe^{-rT} - P(A(t); F; \sigma_A)$$

$$(2) \quad E(t) = C(A(t); F; \sigma_A)$$

where $C(S(t), X, \sigma)$ and $P(S(t), X, \sigma)$ denote, respectively, European call and put options on $S(t)$ with exercise price X when the volatility of asset returns is given by σ . We omit the expiration date and the risk-free rate for easy of notation. Since $A(t)$ is lognormally distributed, the expressions (1) and (2) can be computed with the Black-Scholes-Merton formula.

We now suppose that the firm has the opportunity to invest the amount I in asset B with current value B_0 . The value of the asset at time $t > 0$ is given by the risk-neutral process

$$\frac{dB(t)}{B(t)} = rdt + \sigma_B dZ_B$$

where σ_B is a non-negative constant, and Z_B is a standard Wiener process with instantaneous correlation coefficient ρ_{AB} with Z_A .

The project can be financed using 1) unsecured debt, 2) junior secured debt, or 3) senior secured debt. We assume that secured debt is collateralized by asset B . When

the project is financed with junior secured debt, old unsecured debtholders have a prior claim on asset A . With senior secured debt financing, both unsecured and secured debtholders have the same claim on asset A . We next value the debt and equity of the firm in the three cases.

2.1 Unsecured Debt Financing

In this section we assume that the investment project, with value I at $t = 0$, is financed with new unsecured debt DU_2 with face value G . Denote the value of the original unsecured debt by DU_1 and the value of equity by EU . Then we can write at maturity

$$\begin{aligned} DU_1(T) &= F - \max\left\{F - \frac{F}{F+G}[A(T) + B(T)]; 0\right\} \\ DU_2(T) &= G - \max\left\{G - \frac{G}{F+G}[A(T) + B(T)]; 0\right\} \\ EU(T) &= \max\{A(T) + B(T) - (F + G); 0\} \end{aligned}$$

So that at time $t < T$ we have

$$(3) \quad DU_1(t) = Fe^{-rT} - P\left(\frac{F}{F+G}[A(t) + B(t)]; F; \sigma\right)$$

$$(4) \quad DU_2(t) = Ge^{-rT} - P\left(\frac{G}{F+G}[A(t) + B(t)]; G; \sigma\right)$$

$$(5) \quad EU(t) = C(A(t) + B(t); F + G; \sigma)$$

where

$$\begin{aligned} \sigma^2 &= x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2x_A x_B \sigma_{AB} \\ x_A &= \frac{A_0}{A_0 + B_0} \\ x_B &= \frac{B_0}{A_0 + B_0} \end{aligned}$$

Notice that these call and put values cannot be computed using the Black-Scholes-Merton formula. Since $A(t)$ and $B(t)$ are lognormal, there is no analytical expression for the distribution of $A(t) + B(t)$. Thus, we must employ numerical methods to price the debt and equity. Another complication is that the face value of the new debt G is unknown. It must be computed recursively by making the current value of the new debt equal to the initial investment of the project, that is $DU_2(0) = I$.

2.2 Junior Secured Debt Financing

Now, the investment project is financed with junior secured debt which is collateralized with asset B and has a promised payoff of K at maturity. As unsecured debt has greater priority on asset A than junior secured debt the claim of junior secured debtholders on $A(T)$ is $\max\{A(T) - F; 0\}$. Thereby, we have the following payoffs at maturity

$$\begin{aligned}
DJ(T) &= \min\{K; B(T) + \max\{A(T) - F; 0\}\} \\
&= K - \max\{K + F - \max\{A(T) + B(T); B(T) + F\}; 0\} \\
DUJ(T) &= \min\{F; A(T) + \max\{A(T) + B(T) - K; 0\}\} \\
&= F - \max\{F + K - \max\{A(T) + B(T); A(T) + K\}; 0\} \\
EJ(T) &= \max\{0; A(T) + B(T) - F - K\}
\end{aligned}$$

where DJ , DUJ , and EJ indicate the values of junior secured debt, unsecured debt, and equity, respectively, when the new asset is financed with junior secured debt.

At time $0 \leq t < T$ the values of the contingent claims are given by

$$(6) \quad DJ(t) = Ke^{-rT} - \text{PMAx}(A(t) + B(t); B(t) + Fe^{-rT}; F + K; \sigma_1; \sigma_2; \rho_{12})$$

$$(7) \quad DUJ(t) = Fe^{-rT} - \text{PMAx}(A(t) + B(t); A(t) + Ke^{-rT}; F + K; \sigma_1; \sigma_3; \rho_{13})$$

$$(8) \quad EJ(t) = C(A(t) + B(t); F + K; \sigma)$$

where σ is defined as before and

$$\begin{aligned}\sigma_1 &= \sigma \\ \sigma_2 &= \frac{B_0}{B_0 + Fe^{-rT}}\sigma_B \\ \sigma_3 &= \frac{A_0}{A_0 + Ke^{-rT}}\sigma_A \\ \rho_{12} &= \frac{1}{\sigma_1\sigma_2} \left(\frac{B_0}{B_0 + Fe^{-rT}} \right) (x_A\sigma_{AB} + x_B\sigma_B^2) \\ \rho_{13} &= \frac{1}{\sigma_1\sigma_3} \left(\frac{A_0}{A_0 + Ke^{-rT}} \right) (x_A\sigma_A^2 + x_B\sigma_{AB})\end{aligned}$$

Here, $\text{PMAX}(S_1(t); S_2(t); X; \sigma_1; \sigma_2; \rho_{12})$ represents the value of a European put option with strike X on the maximum of two assets $S_1(t)$, $S_2(t)$, with volatilities σ_1 and σ_2 and correlation coefficient ρ_{12} . As before, the expressions (6), (7), and (8) must be computed numerically. The face value of the junior secured debt is related with the initial investment through the equation $DUJ(0) = I$.

2.3 Senior Secured Debt Financing

Now the project is financed with senior secured debt, with face value L , and is collateralized with asset B . Senior secured creditors rank equally with unsecured creditors for their claims on asset A . Accordingly, their claim on A at maturity is

$$\frac{\max\{L - B(T); 0\}}{\max\{L - B(T); 0\} + F} A(T)$$

Thus, the payoffs at maturity are

$$\begin{aligned}DS(T) &= \min \left\{ L; B(T) + \left(\frac{\max\{L - B(T); 0\}}{\max\{L - B(T); 0\} + F} \right) A(T) \right\} \\ &= L - \max \{ L - \max\{Q(T), B(T)\}; 0 \}\end{aligned}$$

$$\begin{aligned}
DUS(T) &= \min \left\{ F; \max \left\{ A(T) + B(T) - L; \frac{FA(T)}{F + L - B(T)} \right\} \right\} \\
&= F - \max \{ F + L - \max \{ S_1(T); S_2(T) \}; 0 \} \\
ES(T) &= \max \{ 0; A(T) + B(T) - F - L \}
\end{aligned}$$

where DS , DUS , and ES stand for the values of senior secured debt, unsecured debt, and equity, respectively. In these expressions, the following definitions are used

$$\begin{aligned}
Q(T) &= \frac{LB(T) - B^2(T) + B(T)F + LA(T) - B(T)A(T)}{L - B(T) + F} \\
S_1(T) &= A(T) + B(T) \\
S_2(T) &= \frac{FA(T)}{F + L - B(T)} + L
\end{aligned}$$

Prior to maturity we can write

$$(9) \quad DS(t) = Le^{-rt} - \text{PMAX}(Q(t); B(t); L; \sigma_Q, \sigma_B; \rho_{QB})$$

$$(10) \quad DUS(t) = Fe^{-rt} - \text{PMAX}(S_1(t); S_2(t); F + L; \sigma_{S_1}, \sigma_{S_2}; \rho_{S_1S_2})$$

$$(11) \quad ES(t) = C(A(t) + B(t); F + L; \sigma)$$

where σ_Q , ρ_{QB} , σ_{S_1} , σ_{S_2} , and $\rho_{S_1S_2}$ are given in the Appendix.

As before, the distributions of some of the underlying assets in (9), (10), and (11) are unknown and numerical methods must be used. The face value of the new debt is obtained from $DUS(0) = I$.

3 Secured debt and investment decisions

In this section we analyze numerically how secured debt affects the investment decisions of the firm. We assume that managers act on behalf of the equityholders. In the presence

of risky debt this produces conflicts of interests between debtholders and equityholders as maximizing equity value is not be equivalent to maximizing firm value. An investment project will be taken when the value of equity increases and rejected when equity value decreases, regardless of the NPV.

We compare the value of equity when the firm does not take the investment project with the value of equity when the project is financed with unsecured, junior secured, or senior secured debt. That is, we compare E with EU , EJ and ES . To compute EU , EJ , and ES we use Monte Carlo simulation. In order to obtain reliable results we need to use a large number of simulations. Concretely, we generate 1,000,000 (500,000 plus 500,000 antithetic) paths.

Our first analysis draws upon Stulz and Johnson (1985, Table 1). They consider a firm which already owns two assets A and B whose values are lognormally distributed. The firm is financed with a) unsecured debt and junior secured debt and b) unsecured debt and senior secured debt. The face values of debt are given. Stulz and Johnson price junior and senior debt and calculate the values of the security provision.²

In their table, Stulz and Johnson perform a comparative statics analysis of debt values. In row 11, the current values of the assets are $A_0 = 50$ and $B_0 = 50$. The two assets have the same standard deviation, $\sigma_A = \sigma_B = 0.20$. The correlation coefficient between returns is $\rho_{AB} = -0.50$. The face value of unsecured debt (F) is 1,000 and is equal to the face value of secured debt. Other parameters are $r = 0.15$ and $T = 20$ years. With these data they obtain that the value of junior secured debt is 38.9 and the value of senior secured debt 43.4.

The setting of Stulz and Johnson is slightly different to ours. To study how the issuance of secured debt affects investment decisions we assume that the firm owns initially only asset A and that asset B is financed with the proceeds of new debt issues.

²Obtained as the price of the secured bond minus the price of an identical unsecured bond.

Hence, the net present value of the investment project is computed as B_0 minus the current value of the new debt. This means that, in the previous example, the NPV of the project is 11.1 when financed with junior secured debt and 6.6 when financed with senior secured debt. To make our point more clear, we take as given the NPV of the project and we set the face value of the new debt so that the firm raises enough money to take the project. Figure 1 shows the increment in equity value when asset B is financed with each alternative.

[Insert Figure 1 about here]

When the NPV of the project is positive but the increment in equity value is negative, the firm underinvests as it will reject profitable projects. This is indicated in the graphic (fourth quadrant). When the NPV of the project is negative and the increment of equity is positive, the firm overinvests as it will take unprofitable projects. This is also marked in the figure (second quadrant).

We see that when asset B is financed with unsecured debt, the firm underinvests unless the NPV of the project is greater than 3. Surprisingly, with junior secured debt the underinvestment is more severe as the decrease in equity value is higher and the firm will reject projects with higher NPVs. To the best of our knowledge, this is the first time that it is shown that secured debt can decrease the value of the firm by worsening the agency costs of debt. Using the data of Stulz and Johnson (1985, Table 1, row 11), this happens when the NPV of the project is between 0 and 4.9.

Figure 1 also shows that there is no overinvestment with unsecured debt financing. That is, when the required investment is higher than B_0 and the firm issues unsecured debt, the project is rejected as equity value will decrease. However, we see that senior secured debt produces overinvestment. When the NPV of the project is between 0 and -2.5 and the project is financed with senior secured debt, the project is accepted since

the value of equity increases. Thus, senior secured debt will destroy firm value.

To further illustrate our point, we study in detail more typical examples in Table 1. The underinvestment problem is more likely to appear when there is a large proportion on risky debt outstanding, when the risk of the new asset is smaller than the risk of the existing assets of the firm, and when the NPV of the project is small. Accordingly, in the second column of Table 1 we assume that $A_0 = 400$, $\sigma_A = 0.50$, $F = 2,400$, $T = 5$, and $r = 0.10$. We suppose that the firm can take a low-risk project ($\sigma_B = 0.10$) which has a current value $B_0 = 1,000$ by investing $I = 900$. The correlation coefficient is $\rho_{AB} = 0.5$

[Insert Table 1 about here]

We use the Merton (1974) model to compute the value of equity and unsecured debt before taking the project, and we obtain $E = 47.26$ and $DU = 352.74$. Since the NPV is positive the firm should take the project. However, the value of equity decreases to 17.39 with unsecured debt financing. This is due to the significant reduction in the volatility of the firm (from 0.50 to 0.18899). Consequently, equityholders will be unwilling to accept the project and the value of the firm will not increase. Notice that should the project be taken unsecured debtholders will benefit from the whole NPV of the project plus the reduction in shareholders' wealth. As a consequence, the value of the original unsecured debt will go up to 482.61. New unsecured debtholders will receive debt with a face value of 4,473.13 paying for it 900 (the fair market price).

If the firm issues junior secured debt to finance the project, the firm raises the same amount of money but the face value of the new debt is much smaller (1,609.59). Now the value of equity goes up to 62.48 and the project is taken, so that the underinvestment problem disappears. Relative to unsecured debt financing, we observe that junior secured debt transfers wealth from unsecured debtholders to equityholders. The value of equity increases (from 17.39 to 62.48) while the value of the old debt decreases (from 482.61 to

437.52).

With senior secured debt the value of equity goes up to 64.39, the project is accepted and there is no underinvestment.

To know the magnitude of the underinvestment problem and to which extent secured debt palliates it, we plot in Figure 2 the increment in the value of equity of the three financing choices as a function of the NPV of the project.

[Insert Figure 2 about here]

We see that with unsecured debt the underinvestment problem is severe and takes place unless the NPV of the project is greater than 355. Junior secured debt practically avoids the underinvestment, and senior secured debt eliminates the underinvestment but produces overinvestment.

One interesting question is how large must the standard deviation of the project be to avoid the underinvestment problem. We study this question in Figure 3.

[Insert Figure 3 about here]

In the figure we maintain constant the NPV of the project and we change σ_B . We see that, with unsecured debt financing, there exists underinvestment unless the standard deviation of the returns of asset B exceeds 0.31. We also see that there is no underinvestment for any σ_B if the project is financed with junior or senior secured debt. Interestingly, junior secured debt makes this profitable project less attractive to shareholders than unsecured debt when the risk of the new asset exceeds the risk of the existing one.

We now study a typical case of overinvestment in column 3 of Table 1. The overinvestment will be more likely when the risk of the new asset is greater than the risk

of the existing asset. Thus, we suppose that $\sigma_A = 0.30$ and $\sigma_B = 0.50$. The required investment is now $I = 1,100$ and the project has negative NPV. The other parameter values are as in column 2. With these data the initial value of the equity is $E = 5.07$ whereas the value of unsecured debt is $DU = 394.93$. Despite the negative NPV of the project, with unsecured debt financing the value of equity goes up to 45.89. This is due to the increment in the total risk of the firm, which goes up from 0.30 to 0.40683. Thus, managers acting on behalf of shareholders will take the project and the value of the company for the original investors will decrease by 100. With junior secured debt the incentives to take this unprofitable project are smaller but they still exist as equity value increases from 5.07 to 8.62. However, senior secured debt exacerbates the overinvestment problem because it increases the value of equity to 99.52. The value of unsecured debt decreases from 394.93 to 200.48 which reflects the amount lost by investing in the project plus the increment in the value of equity value.

Those results are shown graphically in Figure 4 where we change the initial investment so that the NPV goes from -400 to 400. We see that there is no underinvestment in any of the cases, but there is serious overinvestment for unsecured and senior secured debt financing.

[Insert Figure 4 about here]

Finally in Figure 5 we present a case where there is (practically) not underinvestment or overinvestment with unsecured debt financing but secured debt creates those problems. Concretely, we see that junior secured debt produces underinvestment unless the NPV of the project is sufficiently high and that senior secured debt causes overinvestment for all the (negative) NPVs considered.

[Insert Figure 5 about here]

4 Summary and Conclusions

A number of authors claim that secured debt increases the value of the firm by reducing the agency costs of debt. Many have shown that secured debt reduces the problems of underinvestment and asset substitution. Building on the classical model of Merton (1994) and following the work of Stulz and Johnson (1985) we have studied the investment decisions of a firm with risky debt outstanding when managers maximize shareholders wealth instead of firm value. For lognormal asset prices we have shown that secured debt financing can not only aggravate the underinvestment and overinvestment problems but also produce them in cases where these problems did not exist.

Our results are consistent with the findings of Barclay and Smith (1995) who did not find support for the idea that secured debt reduces the agency costs of debt. Our results do not imply that firms should not use secured debt, since these results relate to only one side effect of secured debt. Rather, our findings suggest that competing theories might play a more important role explaining the prevalent use of this type of debt³ than what was previously thought.

³See, for example, Leeth and Scott (1989), Chen, Yeo, and Ho (1998), and Linn and Stock (2005).

Appendix

Values of σ_Q and ρ_{QB} in expression (9).

$$\sigma_Q^2 = \sigma_C^2 + \sigma_D^2 - 2\sigma_{CD}$$

$$\begin{aligned} \sigma_C^2 = & \left(x_A^C\right)^2 \sigma_A^2 + \left(x_B^C\right)^2 \sigma_B^2 + \left(x_E^C\right)^2 \sigma_E^2 + \left(x_F^C\right)^2 \sigma_F^2 + \\ & + 2x_A^C x_B^C \sigma_{AB} - 2x_A^C x_E^C \sigma_{AE} - 2x_A^C x_F^C \sigma_{AF} - 2x_B^C x_E^C \sigma_{BE} - 2x_B^C x_F^C \sigma_{BF} - 2x_E^C x_F^C \sigma_{EF} \end{aligned}$$

$$\sigma_D^2 = \left(x_B^D\right)^2 \sigma_B^2$$

$$\sigma_E^2 = \sigma_A^2 + \sigma_B^2 + 2\sigma_{AB}$$

$$\sigma_F^2 = 4\sigma_B^2$$

$$\sigma_{CD} = x_B^D \left(x_A^C \sigma_{AB} + x_B^C \sigma_B^2 + x_E^C \sigma_{EB} + 2x_F^C \sigma_B^2 \right)$$

$$\sigma_{AE} = \sigma_A^2 + \sigma_{AB}$$

$$\sigma_{AF} = 2\sigma_{AB}$$

$$\sigma_{BE} = \sigma_B^2 + \sigma_{AB}$$

$$\sigma_{BF} = 2\sigma_B^2$$

$$\sigma_{EF} = 2(\sigma_{AB} + \sigma_B^2)$$

where

$$\begin{aligned} x_B^D &= \frac{-B_0}{(F+L)e^{-rT} - B_0} \\ x_A^C &= \frac{LA_0}{LA_0 + B_0(L+F)e^{-rT} - B_0A_0 - B_0^2} \\ x_B^C &= \frac{(L+F)B_0e^{-rT}}{LA_0 + B_0(F+L)e^{-rT} - B_0A_0 - B_0^2} \\ x_E^C &= \frac{-B_0A_0}{LA_0 + B_0(F+L)e^{-rT} - B_0A_0 - B_0^2} \\ x_F^C &= \frac{-B_0^2}{LA_0 + B_0(F+L)e^{-rT} - B_0A_0 - B_0^2} \end{aligned}$$

Values of σ_{S1} , σ_{S2} , and ρ_{S1S2} in expression (10).

$$\begin{aligned}\rho_{QB} &= \frac{1}{\sigma_Q\sigma_B} \left(x_A^C\sigma_{AB} + x_B^C\sigma_B^2 + x_E^C\sigma_{BE} + x_F^C\sigma_{BF} - x_B^D\sigma_B^2 \right) \\ \sigma_{S1} &= \sigma \\ \sigma_{S2}^2 &= x_Z^2\sigma_Z^2 \\ \rho_{S1S2} &= \frac{1}{\sigma_{S1}\sigma_{S2}} \left(x_Ax_Z \left(\sigma_A^2 - x_B^D\sigma_{AB} \right) + x_Bx_Z \left(\sigma_{AB} - x_B^D\sigma_B^2 \right) \right)\end{aligned}$$

where

$$\begin{aligned}\sigma_Z^2 &= \sigma_A^2 + \left(x_B^D \right)^2 \sigma_B^2 - 2x_B^D\sigma_{AB} \\ x_Z &= \frac{C_0/D_0}{C_0/D_0 + Le^{-rT}} \\ x_B^D &= \frac{-B_0}{(F+L)e^{-rT} - B_0}.\end{aligned}$$

References

- Barclay, M. J. and C. W. Smith, Jr. “The priority structure of corporate liabilities.” *Journal of Finance*, 50 (1995), 899–917.
- Besanko, D. and A. Thakor. “Collateral and rationing: Sorting equilibria in monopolistic and competitive credit markets.” *International Economic Review*, 28 (1987), 671–689.
- Black, F. and M. Scholes. “The pricing of options and corporate liabilities.” *Journal of Political Economy*, 81 (1973), 637–659.
- Chan, Y. and G. Kanatas. “Asymmetric valuations and the role of collateral in loan agreements.” *Journal of Money, Credit, and banking*, 17 (1985), 84–95.
- Chen, S., G. H. H. Yeo, and K. W. Ho. “Further evidence on the determinants of secured debt versus unsecured loans.” *Journal of Business Finance & Accounting*, 25 (1998), 371–385.
- Franks, J. R. and W. N. Torous. “An empirical investigation of U.S. firms in reorganization.” *Journal of Finance*, 44 (1989), 747–769.
- Gay, G. D. and J. Nam. “The underinvestment problem and corporate derivatives use.” *Financial Management*, 27 (1998), 53–69.
- Gonas, J. S., M. J. Highfield, and D. J. Mullineaux. “The determinants of secured loans.” Unpublished Manuscript, (2004).
- James, C. “The use of loan sales and standby letters of credit by commercial banks.” *Journal of Monetary Economics*, 22 (1988), 395–422.

Jensen, M. C. and W. H. Meckling. “Theory of the firm: Managerial behavior, agency costs and ownership structure.” *Journal of Financial Economics*, 3 (1976), 305–360.

Leeth, J. D. and J. A. Scott. “The incidence of secured debt: Evidence from the small business community.” *Journal of Financial and Quantitative Analysis*, 24 (1989), 379–394.

Linn S. C. and D. R. Stock. “The impact of junior debt issuance on senior unsecured debt’s risk premiums.” *Journal of Banking and Finance*, 29 (2005), 1585–1609.

Merton, R. C. “On the pricing of corporate debt: The risk structure of interest rates.” *Journal of Finance*, 29 (1974), 442–470.

Myers, S. “Determinants of corporate borrowing.” *Journal of Financial Economics*, 5 (1977), 147–175.

Smith, C. W. and J. B. Warner. “On financial contracting: An analysis of bond covenants.” *Journal of Financial Economics*, 7 (1979), 117–161.

Stulz, R. M. and H. Johnson. “An analysis of secured debt.” *Journal of Financial Economics*, 14 (1985), 501–521.

Table 1: Numerical examples of the underinvestment and overinvestment problems.

Variable	Underinvestment problem	Overinvestment problem
A_0	400	400
σ_A	0.50	0.30
r	0.10	0.10
T	5	5
F	2400	2400
E	47.26	5.07
DU	352.74	394.93
B_0	1000	1000
σ_B	0.10	0.50
ρ_{AB}	0.50	0.50
I	900	1100
NPV	100	-100
σ_V	0.18899	0.40683
EU	17.39	45.89
DU_1	482.61	254.11
DU_2	900	1100
G	4473.13	10392.57
EJ	62.48	8.62
DUJ	437.52	291.38
DJ	900	1100
K	1609.59	25576.48
ES	64.39	99.52
DUS	435.61	200.48
DS	900	1100
L	1586.28	5786.09

The current values of the new debt are set equal to the initial investment I . E denotes the value of equity when the firm does not take the investment project. EU , EJ , and ES are the values of equity when the project is financed with unsecured, junior secured, and senior secured debt, respectively. DU_1 , DUJ , and DUS are the values of the original unsecured debt when the project is financed with unsecured, junior secured, and senior secured debt, respectively.

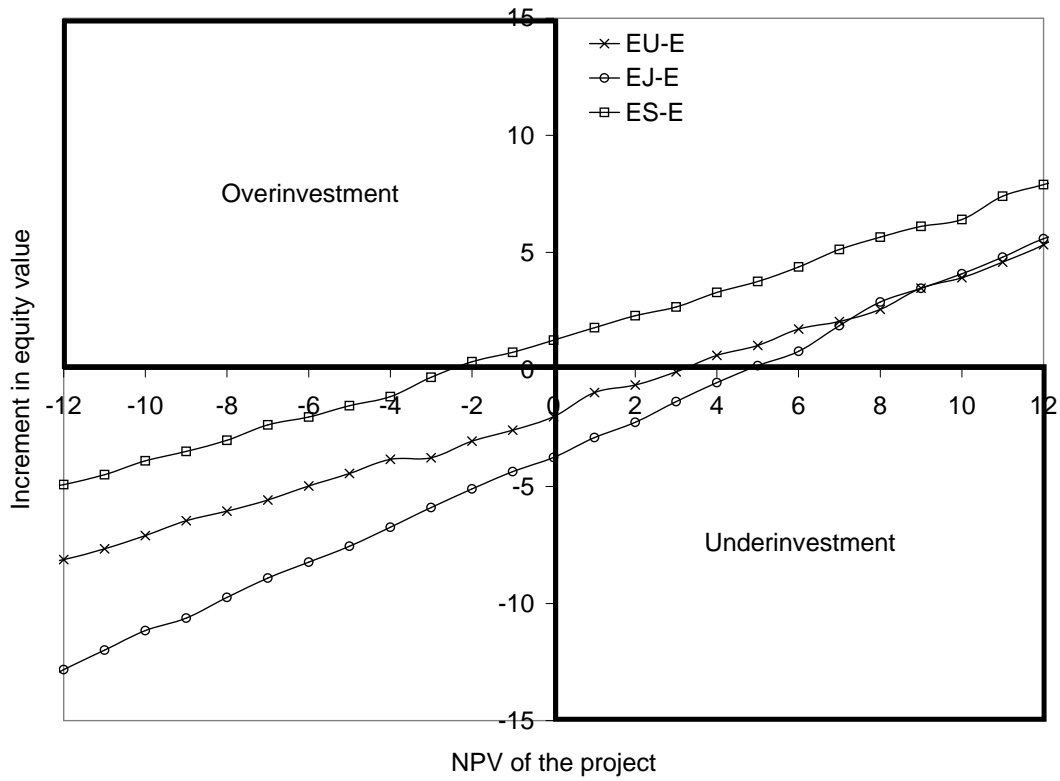


Figure 1: Example based on Stulz and Johnson (1985, Table 1, row 11). The figure plots the increment in the value of equity for unsecured and secured debt financing as a function of the NPV of the project. The values of the parameters are: $A_0 = 50$, $B_0 = 50$, $\sigma_A = 0.2$, $\sigma_B = 0.2$, $\rho_{AB} = -0.5$, $FD_1 = 1,000$, $T = 20$, and $r = 0.15$. E represents the value of equity before taking the investment project, while EU , EJ , and ES refer to the value of equity when the project is taken and financed with unsecured, junior secured, and senior secured debt, respectively.

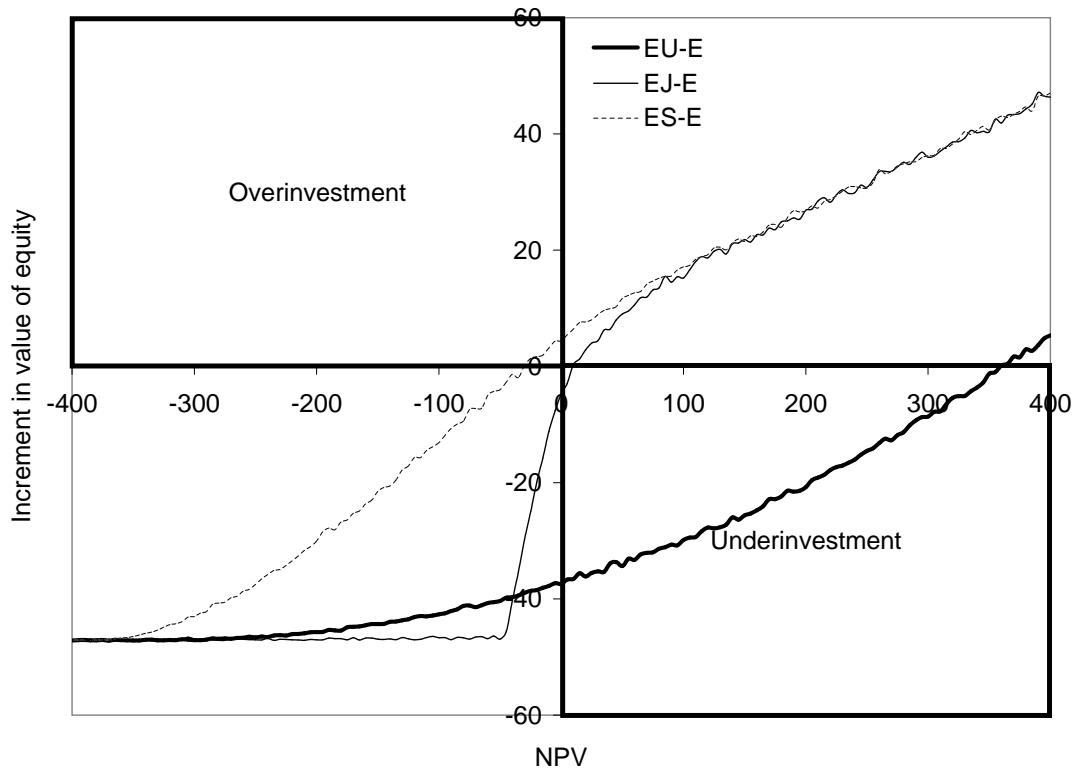


Figure 2: A typical case of underinvestment with unsecured debt financing. The figure shows the increment in the value of equity for unsecured and secured debt financing as a function of the NPV of the project. The values of the parameters are: $A_0 = 400$, $B_0 = 1000$, $\sigma_A = 0.5$, $\sigma_B = 0.1$, $\rho_{AB} = 0.5$, $FD_1 = 2400$, $T = 5$, and $r = 0.1$. E represents the value of equity before taking the investment project, while EU , EJ , and ES refer to the value of equity when the project is taken and financed with unsecured, junior secured, and senior secured debt, respectively.

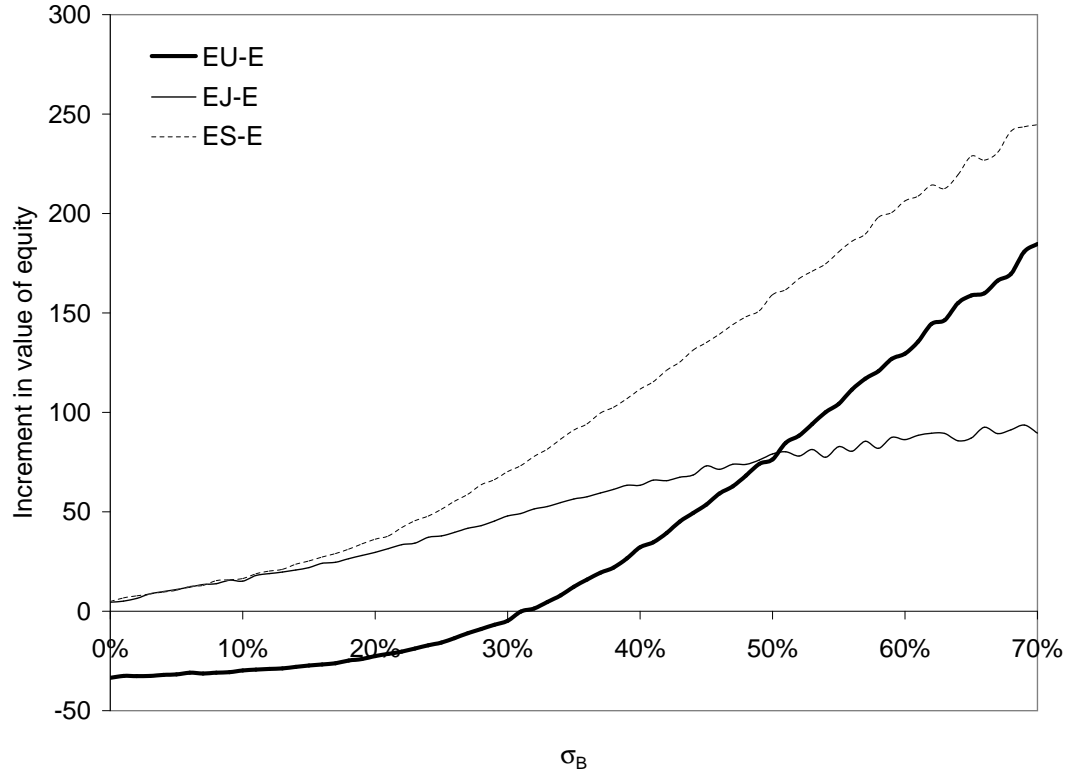


Figure 3: Effect of the risk of the new project on the investment decisions of the firm. The figure displays the increment in the value of equity for unsecured and secured debt financing as a function of the risk of the project. The values of the parameters are: $A_0 = 400$, $B_0 = 1000$, $I = 900$, $\sigma_A = 0.5$, $\rho_{AB} = 0.5$, $FD_1 = 2400$, $T = 5$, and $r = 0.1$. E represents the value of equity before taking the investment project, while EU , EJ , and ES refer to the value of equity when the project is taken and financed with unsecured, junior secured, and senior secured debt, respectively.

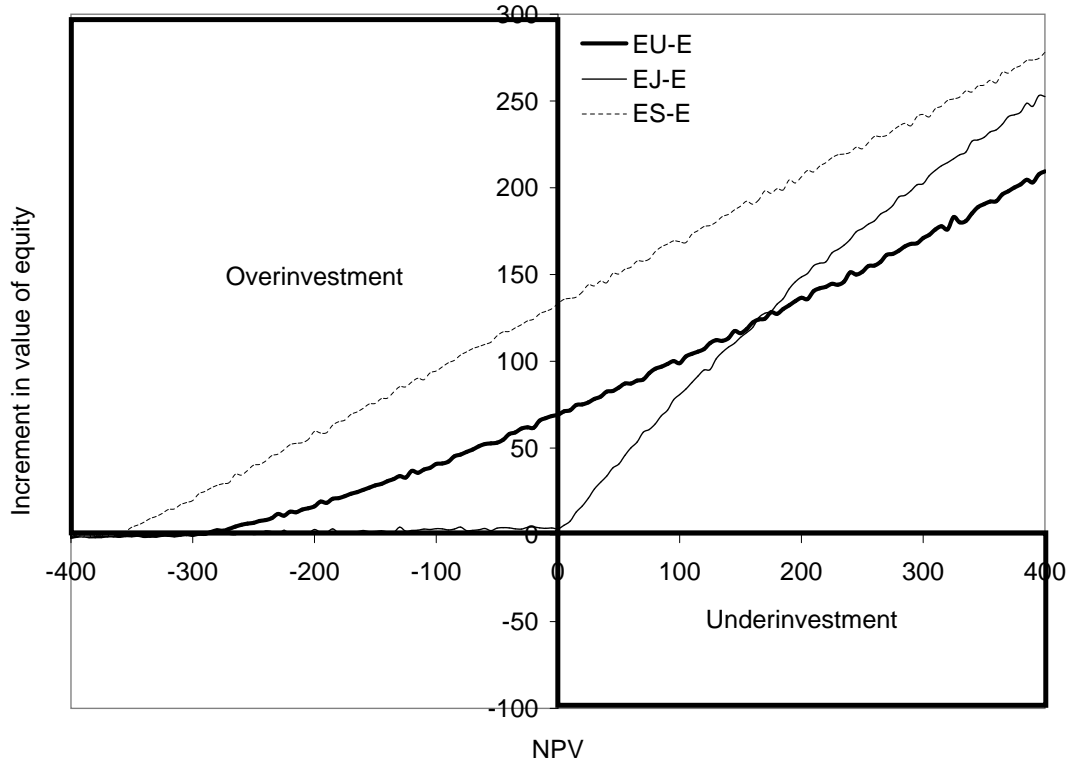


Figure 4: A typical case of overinvestment with unsecured debt financing. The figure plots the increment in the value of equity for unsecured and secured debt financing as a function of the NPV of the project. The values of the parameters are: $A_0 = 400$, $B_0 = 1000$, $\sigma_A = 0.3$, $\sigma_B = 0.5$, $\rho_{AB} = 0.5$, $FD_1 = 2400$, $T = 5$, and $r = 0.1$. E represents the value of equity before taking the investment project, while EU , EJ , and ES refer to the value of equity when the project is taken and financed with unsecured, junior secured, and senior secured debt, respectively.

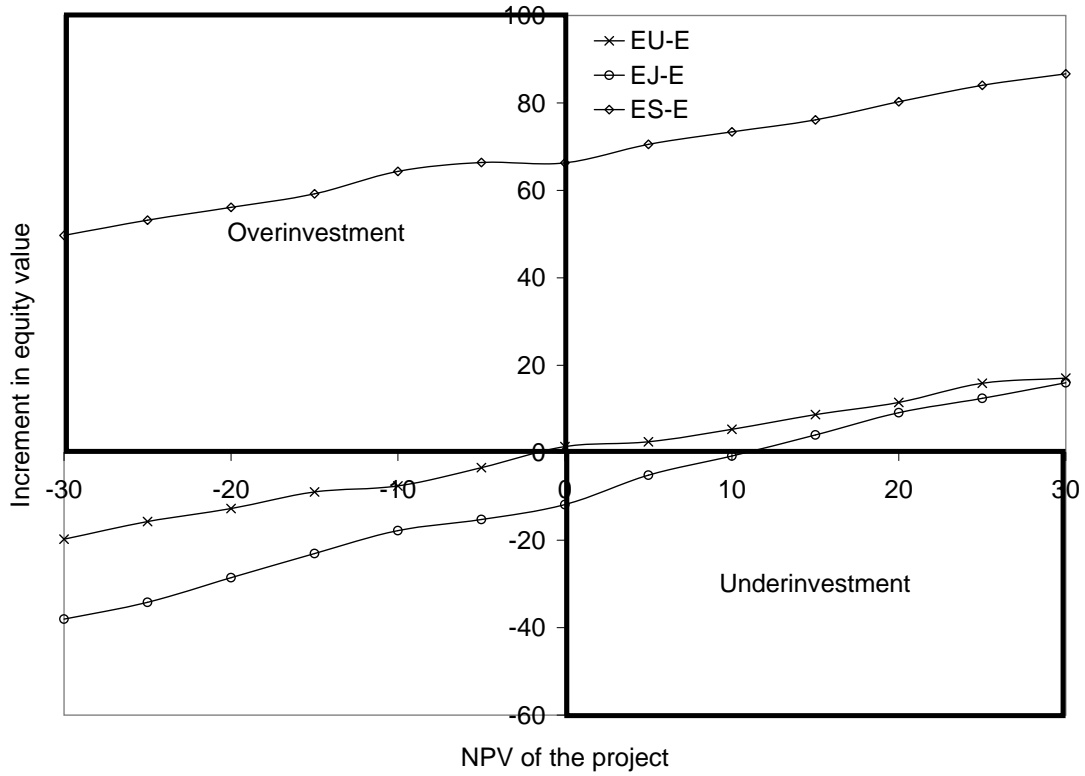


Figure 5: No investment distortions with unsecured debt financing. The figure shows the increment in the value of equity for unsecured and secured debt financing as a function of the NPV of the project. The values of the parameters are: $A_0 = 1600$, $B_0 = 1600$, $\sigma_A = 0.3$, $\sigma_B = 0.2$, $\rho_{AB} = 0.5$, $FD_1 = 2400$, $T = 5$, and $r = 0.1$. E represents the value of equity before taking the investment project, while EU , EJ , and ES refer to the value of equity when the project is taken and financed with unsecured, junior secured, and senior secured debt, respectively.