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Multiattribute Utility Functions*

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JEL Classification numbers: C61, Q12

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Function, Simulation, Agriculture



**Department of Economics**

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# A DUALITY PROCEDURE TO ELICIT NONLINEAR MULTIATTRIBUTE UTILITY FUNCTIONS

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## **Abstract:**

The practical implementation of the Multiattribute Utility Theory is limited, partly for the lack of operative methods to elicit the parameters of the Multiattribute Utility Function, particularly when this function is not linear. As a consequence, most studies are restricted to linear specifications, which are easier to estimate and to interpret. We propose an indirect method to elicit the parameters of a nonlinear utility function to be compatible with the actual behaviour of decision makers, rather than with their answers to direct surveys. The idea rests on approaching the parameter estimation problem as a dual of the decision problem and making the observed decisions to be compatible with a rational decision making process.

**Keywords:** Multiple-Criteria Analysis, Multi-Attribute Utility Function, Simulation, Agriculture

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## 1. INTRODUCTION

The Multiattribute Utility Theory (MAUT) provides a conceptual framework that allows linking Multicriteria Decision Making (MCDM) to economics and decision theory by defining a Multiattribute Utility Function (MAUF). This utility function comprises all the relevant attributes to be optimized by the decision maker, subject to all the constraints of the problem (see Keeney and Raiffa 1976 for a classic reference).

Once the existence of a MAUF is accepted, the practical implementation of this approach faces at least two technical difficulties. First, the mathematical specification for the MAUF must be chosen, and second, the parameters of this function need to be elicited by some estimation or calibration procedure. Actually, both problems are strongly connected in practice, because the availability of an elicitation procedure strongly determines the selection of a specific function.

Assume there are  $n$  relevant attributes and the preferences of the decision maker for these attributes are represented by the monoattribute utility functions  $u_i$  ( $i=1, \dots, n$ ). Keeney (1974) and Keeney and Raiffa (1976) demonstrated that, if the attributes are mutually utility independent, then the MAUF  $U(u_1, \dots, u_n)$  can be expressed as follows:

$$U(u_1, \dots, u_n) = \sum_{i=1}^n k_i u_i + k \sum_{\substack{i=1 \\ j>i}}^n k_i k_j u_i u_j + k^2 \sum_{\substack{i=1 \\ j>i \\ l>j}}^n k_i k_j k_l u_i u_j u_l + \dots + k^{n-1} k_1 k_2 \dots k_n u_1 u_2 \dots u_n$$

where  $U$  and  $u_1, \dots, u_n$  are normalized to be bounded between zero (for the worst possible value) and one (for the best possible value) and  $k$  is a scaling constant that

must satisfy the normalizing constraint  $1+k = \prod_{i=1}^n (1+k k_i)$ . If  $\sum_{i=1}^n k_i \neq 1$  and, as a

consequence,  $k \neq 0$ , then the general utility function proposed by Keeney and Raiffa can be expressed in the following multiplicative form:

$$kU(u_1, \dots, u_n) + 1 = \prod_{i=1}^n [k k_i u_i + 1] \quad [1]$$

On the other hand, if  $\sum_{i=1}^n k_i = 1$ , then  $k = 0$  and the Keeney and Raiffa function

collapses to the following linear form:

$$U(u_1, \dots, u_n) = \sum_{i=1}^n k_i u_i \quad [2]$$

where  $n-1$  parameters need to be elicited and the  $n$ -th one can be calculated from the condition  $\sum_{i=1}^n k_i = 1$ .

The conventional way to elicit the parameters of the MAUF in applied studies is to use face-to-face surveys in order to get direct information from decision makers about the weight attached to each criterion in the decision making process (see Tiwari et al., 1999, Linares and Romero, 2000, Prato and Hajkowicz, 2001). On the other hand, Sumpsi et al. (1997) proposed a non-interactive method to elicit the weights given by farmers to each criterion, so that these weights are “compatible not with the answers of the farmers to artificial questionnaires but compatible to the actual behavior which they follow” (p. 65). These weights can be understood as sensible estimates for the parameters  $k_1, \dots, k_n$  in a linear MAUF as [2] (see Sumpsi et al., 1997, Gómez-Limón and Berbel, 1999, Berbel and Gómez-Limón, 2000, or Gómez-Limón and Riesgo, 2004).

Specification [2] can be understood as a limiting case of [1], implying that [2] is more restrictive or, equivalently, that [1] is more general and flexible, so that it could be potentially more accurate in some real situations. Nevertheless, specification [2] is chosen much more often than [1] for obvious technical reasons: the linear structure of [2] makes the interpretation of the parameters much more apparent and, therefore, it is easier for the decision makers to reveal their preferences as measured by these parameters in a survey. Furthermore, the linear structure of [2] allows the natural use of an indirect linear elicitation method such as the one proposed by Sumpsi et al. (1997) without the need of interactive surveys<sup>1</sup>.

To the best of our knowledge, there is no equivalent (non interactive) method to elicit the parameters of a nonlinear function as [1] and the only available procedures require

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<sup>1</sup> As a further argument to use a linear MAUF, some authors have claimed that, in some cases, it seems to represent a reasonably close approximation to a hypothetical real utility function (Edwards, 1977; Farmer, 1987; Huirne and Hardaker, 1998; Amador et al., 1998). As a matter of fact, a linear expression can be considered as a good local approximation to a nonlinear one, so that, if the environment under which the decision making process takes place is very stable and close to a initial observed situation, the linear approach is likely to be accurate enough. Nevertheless, an estimated linear function will not be probably very suitable to reproduce decisions made under a changing environment.

direct surveys. Furthermore, the nonlinear structure makes the interpretation of the parameters more obscure so that the questions in the surveys need to be more artificial, typically involving lotteries rather than the values of the criteria themselves (see, for example, Herath, 1981, Herath et al., 1982, Le Galès et al., 2002).

In this paper, we propose a non interactive method to elicit the parameters of nonlinear utility functions starting from the structure of the problem and from the observed behavior of the decision makers. From an analytical point of view, the method consists of writing the problem of determining the values of the parameters, given the observed decision, as a dual problem of that of making the optimal decision, given the values of the parameters. From a conceptual point of view, the idea is to make the observed decision to be consistent with a rational decision making process by finding an expression of the utility function that reaches its maximum at the observed point. We use the fact that a rational decision maker will always choose an efficient solution, so that we can restrict the feasible set to an auxiliary set given just by the efficient solutions. When the efficient set is not fully known, an operative approximation is needed. Our proposal to elicit the parameters of the MAUF is independent of the method used for this approximation, but we present an application in which the efficient set is approximated by means of a simple linear procedure combining the elements of the payoff matrix. This procedure gives satisfactory results for our case study, but more sophisticated methods can be applied if required.

Section 2 presents the problem to be solved and the method proposed to solve it. Section 3 offers an application for a Spanish agricultural system in which we compare the simulation ability of both the linear and the multiplicative form of the function proposed by Keeney and Raiffa (1976). We come up with the result that, in most cases, the multiplicative specification provides a better approximation to the observed behavior of farmers. Section 4 presents the main conclusions and some discussion.

## **2. METHODOLOGY**

### **2.1 The problem**

The main idea of our proposal is to make observed decisions to be consistent with a rational decision making process. To illustrate this idea, assume that a decision maker

has a vector  $x$  of decision variables and two criteria over which his preferences are represented by the mono-attribute utility functions  $u_1(x)$ ,  $u_2(x)$ . Let us postulate the existence of a multiattribute utility function:

$$U[u_1(x), u_2(x)] \quad [3]$$

which is partially unknown. To focus on the proposed method to elicit  $U$ , assume that  $u_1(x)$  and  $u_2(x)$  are fully known. For the decision maker the problem is to choose the value of  $x$  to maximize [3] subject to  $x \in \Omega$ , where  $\Omega$  is the feasible set for the decision variables in  $x$ . Figure 1 shows an example where the feasible set, in terms of  $u_1$  and  $u_2$ , is given by the polygon  $ABCDE$ . The figure also shows the map of indifference curves of the decision maker (those combinations providing a fixed value of function  $U$ ). It is important to stress that, the decision maker being rational, the optimal solution will belong to the efficient set which in this example is represented by segment  $AB$ . Specifically, the optimal decision is located at point  $P^*$ , where an indifference curve (that one as far as possible from the origin) is tangent to the efficient set. Using the fact that the solution necessarily belongs to the efficient set, we can represent the decision problem as the following auxiliary problem:

$$\begin{aligned} \max_{u_1, u_2} \quad & U(u_1, u_2) \\ \text{s.t.} \quad & (u_1, u_2) \in AB \end{aligned} \quad [4]$$

where the feasible set is replaced by the efficient set. This simplification is both theoretically sound and operationally convenient for our methodology. Moreover, in [4] the decision variables are  $u_1$  and  $u_2$  rather than  $x$ , which is an innocuous change of variable if the mono-attribute utility functions are known.

The elicitation problem can be stated in the following terms: we can observe the decision actually made (in the example, point  $P^*$ ) and, typically, we also know the feasible set, from which we can construct, or at least approximate, the efficient set. Using this information we need to find a function such that the tangency condition holds exactly at the observed point  $P^*$ . If we postulate a specific parametric expression for  $U$ , the problem can be seen as finding the value of the parameters in this expression in such a way that the tangency conditions are satisfied at  $P^*$ .

## 2.2. A simple example

Assume the efficient set is given by the equation  $u_1 + 1.5 u_2 = 1.9$  and, by construction, the mono-attribute utility functions are bounded so that  $0 \leq u_1, u_2 \leq 1$ . Let us postulate a multiplicative multiattribute utility function of the Cobb-Douglas type, as in Stam and Duarte-Silva (2003):

$$U(u_1, u_2) = (u_1)^{w_1} (u_2)^{w_2} \quad [5]$$

where  $w_1, w_2$  are unknown parameters to be elicited. Assume, furthermore, that we can observe the decisions made by the decision-maker and these decisions provide the values  $u_1 = 0.7$ ,  $u_2 = 0.8$ , which can be understood as the solution for the problem of maximizing [5] subject to  $u_1 + 1.5 u_2 = 1.9$ . From the first order conditions of this problem, we get  $u_1/u_2 = 1.5 w_1/w_2$  and, using the observed values for  $u_1, u_2$ , we can conclude that  $w_2 = 12/7 w_1$ . Finally, using the common normalization  $w_1 + w_2 = 1$ , we get the estimates  $w_1 = 7/19$ ,  $w_2 = 12/19$ .

### 2.3 Determining the efficient set and the reference point: a simple linear approach

The proposed procedure has two main steps: first, determining the efficient set and the reference point, and second, finding the values of the parameters such that the tangency conditions meet exactly at the reference point. This section elaborates on the first part.

In practice, it may well be the case that decision makers are not fully efficient, so that their decisions may not belong to the efficient set (for example, point  $P$  in Figure 1). Since an inefficient decision cannot be reconciled with a rational decision making process, we propose to project the observed point on the efficient frontier by finding that efficient point as close as possible to the observed one. For example, in Figure 1, point  $P$  is projected on  $P^*$ . We can interpret the distance between both points as an error made by the decision maker. We label  $P^*$  as *reference point*, and it is taken as a surrogate of  $P$ . If the observed point is efficient, then the reference point is the observed point itself.

## INSERT FIGURE 1

If the knowledge of the problem allows us to specify an analytical expression for the efficient set, this can be used as the “landing surface” for the utility function. Otherwise, some approximation technique for the efficient set is needed. In this section, we propose a simple linear method which is used in the application presented below (section 3), but the rest of the elicitation procedure is independent of the approach followed to construct the efficient set. Our proposal is to approximate the efficient set by the hyperplane connecting the elements of the payoff matrix. As noted by André et al. (2004), these elements turn out to be efficient if properly constructed, and we claim that combining them can provide a good enough approximation in some cases. Specifically, it seems to work rather well for our case study but more precise approximations can be made (at the cost of a higher computational burden) if needed<sup>2</sup>. In Annex A we present in detail how to compute the elements of the payoff matrix to ensure that they are efficient. Moreover, we propose to express those elements in terms of (mono-attribute) utilities. Although this step is not crucially needed, it is convenient for operational purposes: by working with utilities we eliminate any problem of heterogeneity between units of measurement, because all  $u_i(x)$ 's are normalized by construction (typically between 0 -for the worst value- and 1 -for the best value). Furthermore, we do not need to distinguish between “more is better” or “less is better” attributes, because “more” is always better when dealing with utilities.

Assume the feasible set is given the polygon  $ABCDEFG$  shown in Figure 2. In this example, the set of efficient solutions is given by  $BCD$ . The linear convex combinations of the points of the payoff matrix are given by the hyperplane  $BD$ . Concerning the reliability of this approximation, in some cases (such as the example in Figure 1) a linear combination of the elements of the payoff matrix provides exactly the efficient set, so that there is no approximation error. In other cases (such as the example in Figure 2), some approximation error can be made<sup>3</sup>.

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<sup>2</sup> Other classic methods to obtain the efficient set are the *constraint* method, the *weighting* method or the *multicriterion Simplex* method (Romero and Rehman 1989, pp. 71-74, for a brief introduction). See also Evans (1984) for an overview.

<sup>3</sup> We could also get the paradoxical situations that, if the decision maker is not fully efficient (so that the observed point is below the efficient set), projecting on the linear combinations of the payoff matrix provides a better approximation to the real observed behaviour than projecting on the true efficient set. For example, if the observed point is  $P$  in Figure 2, the projection ( $P^*$ ) on the linear approximation of the payoff matrix can be closer to reality than the projection ( $P^{**}$ ) on the real efficient set.



## INSERT FIGURE 2

Once we have constructed the payoff matrix (in terms of utilities), we need to determine the reference point. If the observed point belongs to the hyperplane  $BD$ , then it should be taken as the reference point itself. Otherwise, it needs to be projected on the (approximation of the) efficient set. Following Sumpsi et al. (1997), this can be done by solving the following system of  $n + 1$  equations:

$$\begin{aligned} \sum_{j=1}^n \omega_j u_{ij} &= u_i, & i = 1, \dots, n \\ \sum_{j=1}^n \omega_j &= 1 \end{aligned} \quad [6]$$

where  $u_i$  is the observed value of the  $i$ -th criterion and  $u_{ij}$  is the  $ij$ -th element of the payoff matrix. If a positive solution exists, then the observed point is a linear convex combination of the payoff matrix (in terms of Figure 2, it belongs to  $BD$ ) and it can be taken as the reference point. Otherwise, we need to project the observed point on the hyperplane connecting the points of the payoff matrix. We propose to do this by finding the closest point (according to the Euclidean metric) by solving the following goal programming problem:

$$\begin{aligned} \min \sum_i^n (n_i + p_i)^2 \\ \text{s.t. } \omega_1 u_{i1} + \dots + \omega_q u_{iq} + n_i - p_i &= u_i \\ \omega_i, n_i, p_i &\geq 0 \\ \omega_1 + \dots + \omega_q &= 1 \end{aligned} \quad [7] \quad i = 1, \dots, n$$

where  $n_i$  ( $p_i$ ) is the negative (positive) deviation variable from the observed value  $u_i$ .

The reference point is then constructed as  $u^* = \sum_{i=1}^n \omega_i \bar{u}_j$  where  $\bar{u}_j$  is the  $j$ -th column of the payoff matrix.

Finally, we need to find an analytical expression for (the approximation of) the efficient set, i.e. an equation like

$$F(u_1, \dots, u_n) = 0, \quad [8]$$

to be satisfied by all the elements in the efficient set. We follow the simplest approach which is to estimate a linear function<sup>4</sup>:

$$F(u_1, \dots, u_n) = \beta_0 + u_1 + \beta_2 u_2 + \dots + \beta_n u_n = 0 \quad [9]$$

that needs to be met by the  $q$  columns of the payoff matrix and from which the parameters  $\beta$  can be calculated by standard linear methods.

## 2.4 Eliciting the parameters of the utility function: a dual approach

As shown in section 2.1, the problem of the decision maker can be expressed as deciding the values of  $u_1, \dots, u_n$  to

$$\begin{aligned} \max U(u_1, \dots, u_n, \gamma) \\ \text{s.t. } F(u_1, \dots, u_n) = 0 \end{aligned} \quad [10]$$

where  $F(u_1, \dots, u_n) = 0$  represents the efficient set and we postulate a parametric multiattribute utility function  $U(u_1, \dots, u_n, \gamma)$ ,  $\gamma$  being a vector of parameters to be elicited. Assume also that the function is concave so that the first order conditions of problem [10] provide a maximum. Manipulating these first order conditions we get the following system of  $n-1$  equations:

$$\begin{aligned} \frac{\partial U(u_1, \dots, u_n, \gamma) / \partial u_2}{\partial U(u_1, \dots, u_n, \gamma) / \partial u_1} &= \frac{\partial F(u_1, \dots, u_n) / \partial u_2}{\partial F(u_1, \dots, u_n) / \partial u_1} \\ \dots & \\ \frac{\partial U(u_1, \dots, u_n, \gamma) / \partial u_n}{\partial U(u_1, \dots, u_n, \gamma) / \partial u_1} &= \frac{\partial F(u_1, \dots, u_n) / \partial u_n}{\partial F(u_1, \dots, u_n) / \partial u_1} \end{aligned} \quad [11]$$

where, if the linear approximation [9] is used,  $\partial F(u_1, \dots, u_n) / \partial u_i = \beta_i$ . By substituting the reference values (those obtained from  $P^*$ ) of  $u_1, \dots, u_n$  in [11] we get a system of equations where the parameters  $\gamma$  are unknowns. This is the key system to be solved in order to elicit the values of the parameters. Typically, we need to solve the system [11] including some normalization constraint and/or some restriction on the values of the parameters for the utility function to have desirable properties (for example, the parameters being nonnegative and smaller than one, etc).

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<sup>4</sup> Since the generic equation of a  $q$ -dimensional hyperplane has  $n+1$  parameters ( $\beta_0, \beta_1, \dots, \beta_n$ ) we can arbitrarily normalize one of them to be equal to 1. We choose  $\beta_1 = 1$  for computational convenience: since we need later on to compute the ratios of the  $\beta$ 's, fixing the denominator to be always equal to one can avoid part of the numerical errors due to rounding.

If we represent these constraints as  $\gamma \in \Theta$ , where  $\Theta$  is the feasible set for the parameters, the resulting system is  $\{[11], \gamma \in \Theta\}$ . We can find three cases:

1.- The easiest case happens when there is a unique feasible solution for the system, as illustrated in the example shown in section 2.2. Then this solution provides the elicited parameter values.

2.- If the system  $\{[11], \gamma \in \Theta\}$  is unfeasible, we can conclude that the reference point (observed or surrogate) cannot be explained as the result of a decision making process with the postulated utility function<sup>5</sup>. Nevertheless, we can understand it as an approximation by solving the following goal programming problem:

$$\begin{aligned} \min_{\gamma} \quad & \sum_{i=2}^n (n_i + p_i)^2 \\ \text{s.t.} \quad & \\ & \frac{\partial U(u_1, \dots, u_n, \gamma) / \partial u_i}{\partial U(u_1, \dots, u_n, \gamma) / \partial u_1} + n_i - p_i = \frac{\partial F(u_1, \dots, u_n) / \partial u_i}{\partial F(u_1, \dots, u_n) / \partial u_1} \quad i = 2, \dots, n \\ & \gamma \in \Theta \end{aligned} \quad [12]$$

3.- The most interesting case is that in which there are multiple solutions, which typically happens when there are more parameters to be elicited than conditions to be satisfied by these parameters in the system  $\{[11], \gamma \in \Theta\}$ . To deal with this case, we propose to formulate the parameter elicitation problem as a dual problem of [10]. We do this by taking advantage of the general formulation of duality proposed by Johri (1993 and 1994). Consider problem [10] as the *primal problem*, which can be formulated as:

$$\max_{(u_1, \dots, u_n, \gamma) \in \Gamma} U(u_1, \dots, u_n, \gamma) \quad [13]$$

$\Gamma \equiv \{(u_1, \dots, u_n, \gamma) / F(u_1, \dots, u_n) = 0, \gamma = \gamma^*\}$  being the feasible set for  $(u_1, \dots, u_n, \gamma)$ , where the value of  $\gamma$  is fixed and denoted as  $\gamma^*$  (since the decision maker is assumed to take it as given). Nevertheless, we include  $\gamma$  as a (trivial) decision variable to fit the problem into Johri's setting. The Johri's *general dual problem* can be expressed as:

$$\min_{\Delta \supseteq \Gamma} \left\{ \max_{(u_1, \dots, u_n, \gamma) \in \Gamma} U(u_1, \dots, u_n, \gamma) \right\} \quad [14]$$

<sup>5</sup> For example, assume that, in the numerical example shown in section 2.2, we have the additional constraint  $w_1 \geq 0.5$ . Since the only combination of parameters that guarantee tangency in the observed point is  $w_1 = 7/19$ ,  $w_2 = 12/19$ , we have an infeasible problem.

where the minimization is carried out over all the sets  $\Delta$  which include  $\Gamma$ . Given the particular nature of our problem, we have more relevant information which we can include, as a constraint, in order to tighten the feasible set and pin down the solution. By hypothesis, we know the solution of [10] in terms of  $(u_1, \dots, u_n)$  –which we label as  $(u_1^*, \dots, u_n^*)$ . If we include this information, by constraining  $(u_1, \dots, u_n)$  to be equal to  $(u_1^*, \dots, u_n^*)$ , the resulting *restricted dual problem* collapses to decide just  $\gamma$ . Moreover, we need to guarantee that the value of  $\gamma$  is such that  $(u_1^*, \dots, u_n^*)$  maximizes  $U(u_1, \dots, u_n, \gamma)$ . In an operational way, we can do it by including the optimality conditions [11]. Furthermore, any feasibility constraint  $\gamma \in \Theta$  on the parameter values should also be included. Since the constraint  $F(u_1^*, \dots, u_n^*) = 0$  holds by construction, it does not need to be explicitly imposed. Summing up, we propose to solve the following problem in order to elicit the values of the parameters:

$$\begin{aligned}
 & \min U(u_1^*, \dots, u_n^*, \gamma) \\
 & s.t. \quad \frac{\partial U(u_1, \dots, u_n, \gamma) / \partial u_i}{\partial U(u_1, \dots, u_n, \gamma) / \partial u_1} = \frac{\partial F(u_1, \dots, u_n) / \partial u_i}{\partial F(u_1, \dots, u_n) / \partial u_1} \quad i = 2, \dots, n \\
 & \quad \gamma \in \Theta
 \end{aligned} \tag{15}$$

Figure 3 shows a flow chart summarizing the proposed method. Note that case 1 (single solution) can be seen as a particular case of case 3 (multiple solutions), so that, in practice, it is enough to solve [15] and, if we are in case 1, i.e., the feasible set contains a single point, that point will trivially be the solution of [15].

### INSERT FIGURE 3

### 3. AN APPLICATION TO AGRICULTURAL ECONOMICS

A number of authors have pointed out that, contrary to the usual assumption in conventional economics, farmers are not only concerned with the maximization of profit, but other attributes such as risk, management complexity, leisure time, indebtedness, etc., are also involved in farmers' decision making. See Gasson (1973), Smith and Capstick (1976) or Cary and Holmes (1982). More recently Willock et al. (1999), Solano et al. (2001) and Bergevoet et al. (2004) have also stressed this point.

Since farmers take their decisions trying to simultaneously optimize a range of conflicting objectives, both the MCDM paradigm and the multiattribute utility theory seem to be relevant in this context. In this section we present an application to agricultural economics in order to test the multiplicative expression [1] as compared to the linear one [2] and to check if the former can provide some better performance (i.e., better ability to reproduce the observed behavior) in some cases.

### 3.1. Case study

The case study is a sample of 22 average farmers from the Douro basin in northern Spain. This basin is the greatest of Spanish rivers, with a surface of 78.954 km<sup>2</sup>. The climate is warm Mediterranean<sup>6</sup>, with long cold winters and short warm summers. The average rainfall ranges between 400 and 500 millimeters per year. The most important crops in this area are strongly dependent on CAP (Common Agricultural Policy) subsidies and low value-added crops. In an average year, the main activities are winter cereals (30%), maize (25%), sugar beet (15%), alfalfa (10%), sunflowers (5%) and other minor crops (15%). All the data used to feed the models were obtained both from official statistics and from a survey developed in the area under study during the 2000-01 agricultural year. For more information on the survey and other elements of the case study see Gómez-Limón and Riesgo (2004).

### 3.2. Mathematical model

To simulate the farmers' decision-making process under the MAUT framework, we construct a mathematical model where farmers decide the value of their decision variables, being limited by certain constraints, in order to achieve various objectives:

*Decision variables.* Each farmer has a vector  $x$  of decision variables  $x_h$ , where  $x_h$  measures the amount of land devoted to every particular crop, including winter cereals, maize, sugar beet, sunflowers, alfalfa, beans, potatoes and set-aside<sup>7</sup>.

*Constraints.* We identify the following constraints as applied to each farmer:

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<sup>6</sup> Papadakis classification (1965)

<sup>7</sup> Specifically,  $x_1$  is (amount of land devoted to) winter cereals,  $x_2$  is maize,  $x_3$  is sugar beet,  $x_4$  is alfalfa,  $x_5$  is potatoes,  $x_6$  is sunflowers,  $x_7$  is beans,  $x_8$  is set-aside



- **Land constraint.** The sum of all crops must be equal to the total surface available to each farmer (denoted as *sup*):

$$\sum_{h=1}^8 x_h = sup$$

- **CAP constraints.** To fulfill the CAP requirements, we included 20% of set-aside for cereal, oilseed and protein crops (COP crops). Any land devoted to set-aside greater than this percentage is excluded of EU subsidies, and this is taken as an invalid option in the model:

$$\text{Maximum set-aside: } x_8 \leq 20\% \cdot (x_1 + x_2 + x_6)$$

On the other hand, the CAP force farmers to withdraw at least the 10% of the land devoted to COP crops to obtain compensatory payments. This withdrawal is made in irrigated and non irrigated lands bearing in mind both theoretical yields. A good estimation of the set-aside in irrigated land is the observed data in the period under study:

$$\text{Minimum set-aside: } x_8 \geq \text{observed withdraw}$$

Furthermore, because of the quota, sugar beet is limited for each farmer to the maximum area in the period studied:

$$\text{Sugar beet quota: } x_3 \leq \text{maximum sugar beet}$$

- **Agronomic constraints.** For rotational conditions, land devoted to alfalfa have to rest before cultivating again this crop:

$$x_4 \leq \frac{p}{p+q} \cdot sup$$

where *p* is the number of years during the crop is on the land (4 for alfalfa) and *q* is the number of years off the land (3 for alfalfa).

- **Market constraints.** Alfalfa and potatoes are the only perishable crops considered. We limited their surface to the maximum observed in the period 1993-2000:

$$\begin{aligned} x_4 &\leq \text{observed area} \\ x_5 &\leq \text{observed area} \end{aligned}$$

**Objectives.** After the survey developed in the area, we concluded that farmers take the following objectives into account:

- **Maximization of total gross margin (TGM)**, as a proxy of profit since, in the short run, the availability of structural productive factors (land, machinery, etc.) cannot be changed and financial viability of farms basically depends on gross margin. TGM data

are obtained from the average crop margins in a time series of seven years (1993/1994 to 1999/2000) in constant 2000 euros. According to this TGM can be calculate as follows:  $TGM = \sum_h GM_h \cdot x_h$ , where  $GM_h$  represents the gross margin per unit of crop  $h$ .

- *Minimization of risk (VAR)*. As noted by several authors (for example Just and Pope, 1979, Young, 1979, and Gómez-Limón et al., 2003), farmers typically have a marked risk aversion, so that risk is an important factor in agricultural activity. Following the conventional Markowitz (1952) approach, risk is measured by the variance of  $TGM$ :  $VAR = x_h' [Cov] \cdot x_h$ , where  $[Cov]$  is the variance-covariance matrix of the crop gross margins obtained from different crops, during the seven-year period.
- *Minimization of working capital (K)*. This objective represents the aim of reducing the level of indebtedness. In order to model this objective we divided the year into months, differentiating in this way the periods of cropping activities (capital immobilization) and sales (income). In month  $m$ , the working capital ( $NWK_m$ ) is the sum of the working capital for the present month ( $\sum_h WK_{hm} \cdot x_h$ ) and the working capital from the previous month ( $NWK_{m-1}$ ), whenever sales are less than the capital immobilization. Mathematically:

$$NWK_m - \sum_h WK_{hm} \cdot x_h - NWK_{m-1} \geq 0 \quad \forall m$$

where  $WK_{hm}$  is the working capital per unit of crop  $h$  in month  $m$ .

The aim of a farmer is to minimize the maximum working capital ( $K$ ), which can be represented as minimizing the maximum  $NWK_m$  calculated for twelve months. To do that we use the *minimax* method, and therefore we introduce twelve new equations:

$$NWK_m \leq K \quad \forall m$$

In order to can test the ability of the MAUT approach to reproduce farmers' behavior using both an additive and a multiplicative MAUF specification. we performed the following experiment:

- 1.- Taking into account the observed vector of decision variables of each farmer and the constraints of the problem, we elicit the parameters (weights) of a linear MAUF [2] using the approach developed by Sumpsi et al. (1997).

- 2.- We simulate the farmers' behavior (decision variables) by maximizing the linear utility function (as estimated in the first step) subject to the constraints of the problem.
- 3.- Compare the simulated decisions (obtained in the second step) with the observed ones for each farmer.
- 4.- Repeat steps 1, 2 and 3 with the multiplicative specification [1] of the MAUF.
- 5.- Compare the performance of the linear and the multiplicative specifications to replicate observed behavior.

### 3.3. Results

We applied the procedure described above to each representative farmer in our sample. In most cases we came up with the result that the simulation ability of the multiplicative MAUF is better than the linear one. For further clarification, we present all the intermediate steps of our experiment in a representative case.

#### ***Results for the additive MAUF***

Firstly we obtain a payoff matrix (in terms of utilities) with efficient solutions for all the columns, as explained in Annex A. The results are displayed in Table 1, in which we have included an additional column to show the real observed values.

#### INSERT TABLE 1

Using the data in Table 1, and following Sumpsi et al. (1997) we estimate the weights of the different objectives solving problem [7], which takes the following form:

$$\begin{aligned}
 & \min_{\omega_i, n_i, p_i} \sum_{i=1}^3 (n_i + p_i)^2 \\
 & s.t. \quad 1 \cdot \omega_1 + 0.101 \cdot \omega_2 + 0 \cdot \omega_3 + n_1 - p_1 = 0.459 \\
 & \quad \quad 0 \cdot \omega_1 + 1 \cdot \omega_2 + 0.942 \cdot \omega_3 + n_2 - p_2 = 0.755 \\
 & \quad \quad 0 \cdot \omega_1 + 0.843 \cdot \omega_2 + 1 \cdot \omega_3 + n_3 - p_3 = 0.562 \\
 & \quad \quad \omega_1 + \omega_2 + \omega_3 = 1 \\
 & \quad \quad \omega_i, n_i, p_i \geq 0 \quad i=1, \dots, 3
 \end{aligned}
 \tag{16}$$



Solving this mathematical program, we obtain that the weight given by the analyzed farmer to *TGM* maximization is  $\omega_1=31.9\%$  and the weight of risk minimization is  $\omega_2=68.1\%$ . On the other hand, minimization of *K* does not appear to be taken into account by the farmer in his decision-making process ( $\omega_3=0$ ). Using the estimated weights for each objective, and taking  $k_i=\omega_i$  ( $i=1,2,3$ ) we get the following algebraic expression of the additive utility function [2]:

$$U = 0.319 \cdot \left( \frac{TGM - 20,826.58}{52,337.46 - 20,826.58} \right) + 0.681 \cdot \left( \frac{VAR - 81,704,050.27}{9,285,839.22 - 81,704,050.27} \right) + 0 \cdot \left( \frac{K - 56,297.02}{4,264.06 - 56,297.02} \right)$$

which can be simplified to

$$U = 10.12 \cdot TGM - 0.094 \cdot VAR \tag{17}$$

Afterwards, we simulate the farmer's behaviour by finding the values of the decision variables that maximize [17]. The results are shown in Table 2.

### INSERT TABLE 2

#### **Results for the multiplicative MAUF**

Firstly, we obtain the equation of the hyperplane [9] that connects all the points of the payoff matrix. Forcing all the columns of the payoff matrix to satisfy [9], we have a three-equation system with the following solution:

$$-1 + u(MBT) + 0.272 \cdot u(VAR) + 0.744 \cdot u(K) = 0 \tag{18}$$

Using the  $\omega_i$ 's obtained in [16], we get the reference point as a linear combination of the elements of the payoff matrix,  $P^* = (0.388, 0.681, 0.574)$ , which by construction satisfies [9]. Finally, we elicit the parameters  $k_i$ 's and  $k$  solving [15] which, in this case, takes the form:

$$\min_{k_1, k_2, k_3, k} U(u_1, \dots, u_3) \equiv \frac{1}{k} \left\{ \prod_{i=1}^3 [kk_i u_i + 1] - 1 \right\}$$

s.t.

$$\begin{aligned} \frac{\partial U(u_1, u_2, u_3, k, k_1, k_2, k_3)}{\partial u_2} \Big|_{P^*} &= 0.272 \\ \frac{\partial U(u_1, u_2, u_3, k, k_1, k_2, k_3)}{\partial u_1} \Big|_{P^*} & \\ \frac{\partial U(u_1, u_2, u_3, k, k_1, k_2, k_3)}{\partial u_3} \Big|_{P^*} &= 0.744 \\ \frac{\partial U(u_1, u_2, u_3, k, k_1, k_2, k_3)}{\partial u_1} \Big|_{P^*} & \\ k_1, k_2, k_3 \geq 0, \quad 1 + k &= \prod_{i=1}^n (1 + kk_i) \end{aligned} \quad [19]$$

where  $\Big|_{P^*}$  means that the associated expression is evaluated in the reference point  $P^*$ . The result of [16] is  $k_1 = 0.359$ ,  $k_2 = 0.087$ ,  $k_3 = 0.274$ ,  $k = 1.665$ . This gives the following multiplicative utility function:

$$\begin{aligned} U &= 0.359 \cdot \left( \frac{TGM - 20826.58}{52337.46 - 20826.58} \right) + 0.087 \cdot \left( \frac{VAR - 81,704,050.27}{9,285,839.22 - 81,704,050.27} \right) + 0.274 \cdot \left( \frac{K - 56,297.02}{4,264.06 - 56,297.02} \right) + \\ &+ 1.665 \cdot \left\{ \begin{aligned} &\left[ 0.359 \cdot 0.087 \cdot \left( \frac{TGM - 20826.58}{52337.46 - 20826.58} \right) \cdot \left( \frac{VAR - 81,704,050.27}{9,285,839.22 - 81,704,050.27} \right) \right] + \\ &\left[ 0.359 \cdot 0.274 \cdot \left( \frac{TGM - 20826.58}{52337.46 - 20826.58} \right) \cdot \left( \frac{K - 56,297.02}{4,264.06 - 56,297.02} \right) \right] + \\ &\left[ 0.087 \cdot 0.274 \cdot \left( \frac{VAR - 81,704,050.27}{9,285,839.22 - 81,704,050.27} \right) \cdot \left( \frac{K - 56,297.02}{4,264.06 - 56,297.02} \right) \right] \end{aligned} \right\} + \\ &+ (1.665)^2 \cdot \left\{ 0.359 \cdot 0.087 \cdot 0.274 \cdot \left( \frac{TGM - 20826.58}{52337.46 - 20826.58} \right) \cdot \left( \frac{VAR - 81,704,050.27}{9,285,839.22 - 81,704,050.27} \right) \cdot \left( \frac{K - 56,297.02}{4,264.06 - 56,297.02} \right) \right\} \end{aligned}$$

which can be simplified to get:

$$\begin{aligned} U &= 1.14 \cdot 10^{23} \cdot TGM - 1.20 \cdot 10^{19} \cdot VAR - 5.26 \cdot 10^{22} \cdot K + 5.23 \cdot TGM \cdot VAR + 9.99 \cdot 10^7 \cdot TGM \cdot K + \\ &+ 1.11 \cdot VAR \cdot K + 1.74 \cdot 10^{-29} \cdot TGM \cdot VAR \cdot K \end{aligned} \quad [20]$$

The last stage of the exercise is to simulate the farmer's behaviour by maximizing [20] subject to all the constraints of the model. The results are shown in Table 2, together with those from the linear utility function. In order to compare the performance of both approaches, we use a common validation approach by comparing the simulated and observed values of the surface devoted to each individual crop and calculating the sum

of the absolute value of all the deviations as a percentage of total surface (Qureshi *et al.*, 1999). In Table 2 we can observe that the deviation from the simulated to the observed behaviour is lower when we use the multiplicative specification (34.70%) than when we use the additive function (67.28%), meaning that the multiplicative utility function allows a better approximation to the farmer's decision making process. Although we just display the results for a representative farmer, the multiplicative approach turns out to be superior to the linear one for most of the cases in the sample.

#### 4. CONCLUSIONS AND DISCUSSION

In this paper we have developed a non-interactive method based on duality to elicit the parameters of a nonlinear utility function to be compatible with the actual behaviour of the decision makers. The main idea is to make the observed decisions to be consistent with a rational decision making process by finding an expression of the utility function that makes the observed decision to be (approximately) optimal. The information needed to apply this method is (some approximation to) the efficient set and the observed decisions. Moreover, we need to postulate a specific parametric expression for the multiattribute utility function.

To assess the gain from using a multiplicative rather than additive utility function, we have developed an application in the field of agricultural economics. In this case study, we aim at reproducing the actual decisions of farmers using both the additive and the multiplicative specification suggested by Keeney and Raiffa (1976). Since the additive utility function can be taken as a limiting case of the multiplicative one, it can be reasonably expected that (at least in some cases) the second will be more effective to simulate observed decisions. This intuition is confirmed in our empirical application, since the multiplicative approach turns out to be superior to the additive one.

Some remarks should be made about the conditions for the suggested method to work successfully. This approach rests on the duality relationship between the elicitation problem [15] and problem [10], which is taken as a surrogate for the real problem of the decision maker. Henceforth, the performance of the elicitation procedure crucially depends on how accurate is [10] to approximate the real problem of the decision maker. In this sense, the method rests on the assumption that the decision maker is rational so that he always makes efficient decisions. If the observed point is close

enough to the efficient set and we have a good enough approximation to the efficient set, then the elicitation procedure developed here will provide a good approximation to the observed decisions by construction. On the other hand, inefficient decisions cannot be reconciled with a rational decision making process, so that the proposed method will be less successful to replicate observed decisions the less efficient these decisions are.

Our elicitation procedure uses, as an input, an expression for the efficient set and, if the observed point is not efficient, a projection of the observed point on the efficient set is also needed. We have illustrated a simple linear procedure by combining the elements of the payoff matrix, which provides a good enough approximation for our case study but the elicitation procedure is also compatible with other (perhaps more sophisticated) methods. For more complex problems the efficient set will probably be more complex as well, so that it may be impossible to find an analytical expression for the whole efficient set. Note that we only need an expression for *the relevant part* of the efficient set, i.e. that where the reference point belongs. We suggest the following procedure for more complex problems: 1) find a discrete approximation to the efficient set (a number of efficient points). 2) Project the observed unit on the efficient set with a DEA method, taking the efficient points as decision making units. This provides  $n$  peer units for the observed point. 3) Find the equation of the hyperplane connecting these  $n$  points.

Finally, we can observe that in general a large number of utility functions could reproduce a single observed decision. Then, some additional information should be included to select a specific expression for the MAUF. Our procedure uses the tangency condition which has to be satisfied in interior solutions, so that corner solutions are treated as interior. To illustrate this, assume the problem is that represented in Figure 4, where  $ABCDE$  is the feasible set. The observed point  $P$  is projected on the corner point  $A$ . This solution can be rationalized by different utility functions. We show three possibilities associated with three different indifference curves, labelled 1, 2 and 3. By construction, our method picks up the MAUF associated to indifference curve 1, which is tangent to the efficient set  $AB$  precisely at point  $A$ .

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## ANNEX A: Computing the (efficient) payoff matrix

If there are  $n$  criteria and  $f_i(x)$  denotes the value of the  $i$ -th criterion ( $i=1, \dots, n$ ) depending on the decision variables, the first element of the first column in the payoff matrix can be obtained by solving the problem:

$$\begin{aligned} \max f_1(x) \\ \text{s.t. } x \in \Omega \end{aligned} \quad [A-1]$$

The optimal value  $f_1(x)$  resulting from [A-1], denoted as  $f_1^* \equiv f_{11}$ , is the first entry of the payoff matrix. To obtain the other entries of the first column, we substitute  $\arg \max f_1(x)$  in  $f_i(x)$ , for  $i=2, \dots, n$ . The rest of the columns are obtained by implementing similar calculations, i.e., the generic element  $f_{ij}$  is obtained by plugging  $\arg \max f_j(x)$  in  $f_i(x)$ . In some cases, the payoff matrix could not be unique, that is, problem [A-1] could have alternative optimal solutions, and some of them could be inefficient. For example, assume there are only 2 objectives and the feasible set is represented by  $ABCDEFG$  in Figure 2. Then, when optimizing objective 1, we could obtain any point on the segment  $AB$ <sup>8</sup>, but it is convenient to choose  $B$ , which is efficient, while  $A$  is not. For this purpose, we solve the following lexicographic problem for every objective  $i$ :

$$\begin{aligned} \text{Lex max } \left\{ f_i(x), \sum_{j \neq i} \alpha_j f_j(x) \right\} \\ \text{s.t.: } x \in \Omega \end{aligned} \quad [A-2]$$

meaning that objective  $i$  is maximized and, if some alternative optima exist, then an arbitrary linear combination of the other objectives (with  $\alpha_j > 0$ ,  $\forall j \neq i$ )<sup>9</sup> is optimized without worsening the performance of objective  $i$ . By solving  $q$  problems like [A-2], we obtain efficient solutions for all the columns of the payoff matrix.

<sup>8</sup> More specifically, when using a simplex algorithm, we could obtain either  $A$  or  $B$ .

<sup>9</sup> Any set of positive values of  $\alpha_j$  provide an efficient solution. We propose to use  $\alpha_j = \frac{u_j}{\sum_{j=1}^n u_j}$ , i.e., the

value of the observed (mono) utility of attribute  $j$  with respect to (the sum of) all mono-attribute utilities, as defined below.

Moreover, we transform all the (optimal and observed) objectives by substituting into the mono-attribute utility functions, so that the resulting values can be taken as utilities. In our application, we select the usual mono-attribute utility transformation  $u_i(x) \equiv u_i(f_i(x)) = (f_i(x) - f_{*i}) / (f_i^* - f_{*i})$  where  $f_i^*$  ( $f_{*i}$ ) denotes the best (worst) value that the  $i$ -th attribute achieves in all the columns of the payoff matrix.



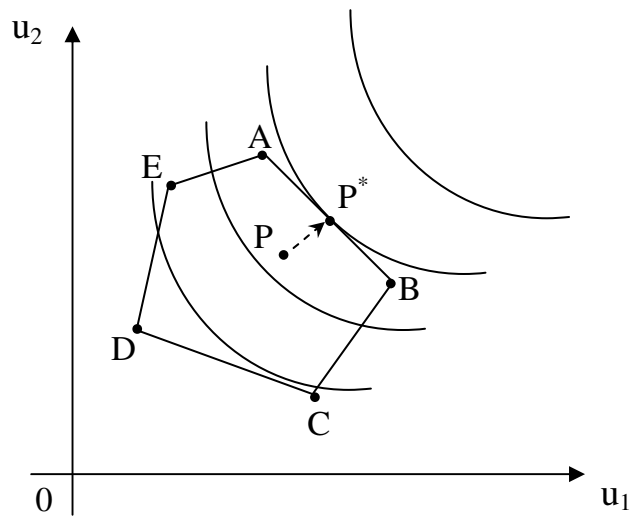


FIGURE 1

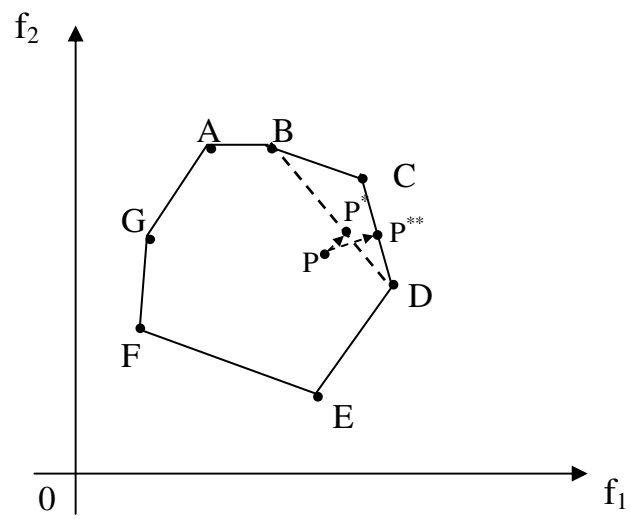


FIGURE 2

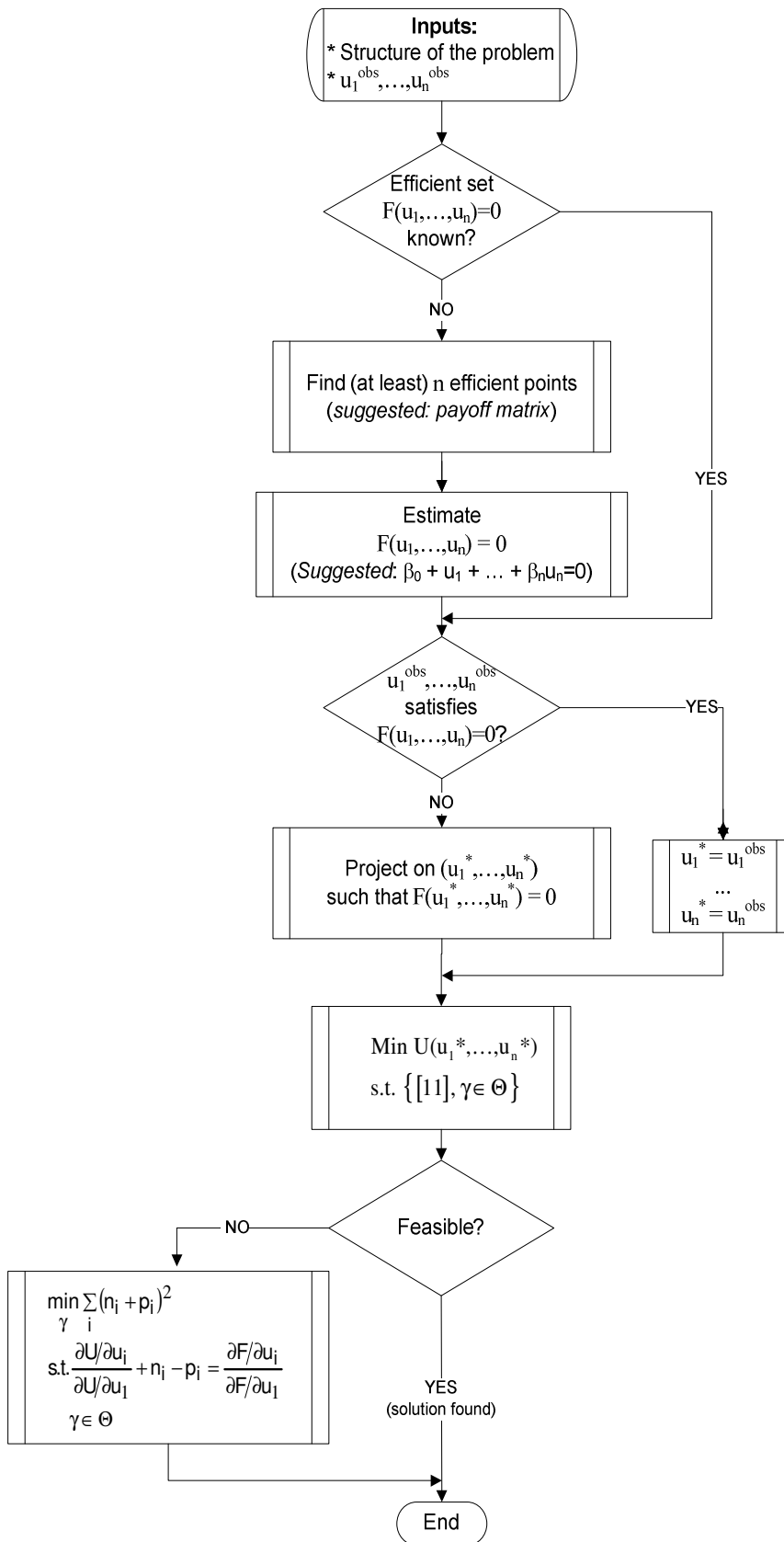


FIGURE 3

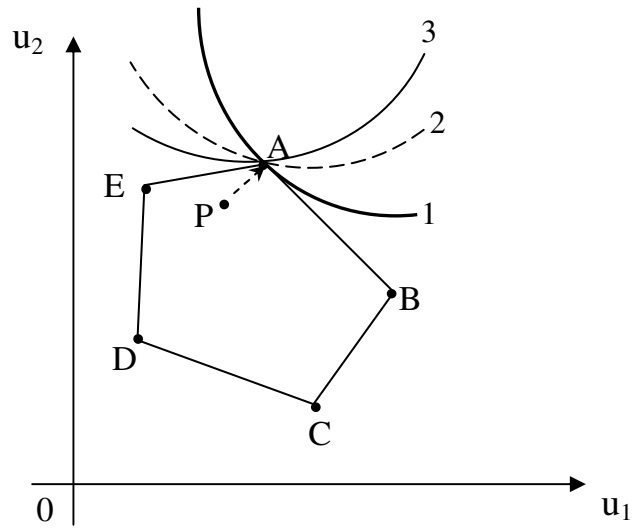


FIGURE 4

<i>Value obtained</i>	<i>Objective to be optimised</i>			<i>Observed Value</i>
	<i>u(TGM)</i>	<i>u(VAR)</i>	<i>u(K)</i>	
<i>u(TGM)</i>	<i>1</i>	<i>0.101</i>	<i>0.000</i>	<i>0.459</i>
<i>u(VAR)</i>	<i>0.000</i>	<i>1</i>	<i>0.942</i>	<i>0.755</i>
<i>u(K)</i>	<i>0.000</i>	<i>0.843</i>	<i>1</i>	<i>0.562</i>

**TABLE 1. PAYOFF MATRIX FOR THE FARMER**

<i>Crops</i>	<i>Multiplicative utility function</i>			<i>Additive utility function</i>		
	<i>Values (ha)</i>		<i>Deviation (ha)</i>	<i>Values (ha)</i>		<i>Deviation (%)</i>
	<i>Observed</i>	<i>Simulated</i>		<i>Observed</i>	<i>Simulated</i>	
Winter cereals	31.29	26.75	4.54	31.29	14.27	17.02
Maize	6.33	16.96	-10.63	6.33	13.83	-7.50
Sugar-beet	16.71	13.2	3.51	16.71	13.11	3.59
Sunflowers	2.58	0.00	2.58	2.58	15.69	-13.12
Alfalfa	0.00	0.00	0.00	0.00	0.00	0.00
Beans	0.00	0.00	0.00	0.00	0.00	0.00
Potatoes	0.00	0.00	0.00	0.00	0.00	0.00
Set-aside	4.38	4.38	0.00	4.38	4.38	0.00
<b>TOTAL</b>	61.29	61.29	<b>21.27 (34.70%)</b>	61.29	61.29	<b>41.24 (67.28%)</b>

**TABLE 2. COMPARISON BETWEEN BOTH METHODOLOGIES FOR THE FARMER'S DECISION-MAKING**