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Indirect Elicitation of Non-Linear Multiattribute Utility Functions: A Dual Procedure Combined with DEA

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INDIRECT ELICITATION OF NON-LINEAR MULTIATTRIBUTE UTILITY FUNCTIONS. A DUAL PROCEDURE COMBINED WITH DEA

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Abstract:

In this paper, we propose an indirect method to elicit nonlinear multiattribute utility functions which is based on duality results. The idea is to obtain a utility function which is compatible with the observed behaviour of decision makers. The paper builds on a previous work by André and Riesgo (A non-interactive method to elicit non-linear multiattribute utility functions. Theory and Application to Agricultural Economics. European Journal of Operational Research. In press) but it eliminates an important shortcoming, the necessity to estimate the efficient set, by using a DEA-like approach.

Keywords: Multicriteria, Decision Making, DEA.

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1. INTRODUCTION

Multiattribute Utility Theory (MAUT), chiefly developed by Keeney and Raiffa [2], provides an analytical framework to deal with decision problems that involve conflicting criteria. In this framework, typically each alternative receives a score or monoattribute utility value for every criterion and, in turn, these scores are combined into a Multiattribute Utility Function (MAUF) to get an overall utility value[2]. Barzilai [5], [6] provided rules for multiatribute utility functions to be properly defined based on measurement theory and offered a critical view of the traditional von Neumann-Morgenstern [7] utility theory. In [8] the aggregation of individual multiattribute utility functions is studied.

This paper addresses a specific issue related to the practical implementation of MAUT: the elicitation of a non-linear MAUF using a non-interactive or indirect procedure, i.e., without the need of using direct surveys. Although a linear MAUF can be seen, in general, as more restrictive than a non-linear MAUF, most papers in the literature use linear specifications, probably because they are easier to elicit and to interpret. [9] Sumpsi *et al.* (1997) proposed a non-interactive method to elicit the weights given by DMs to each criterion, which can be understood as estimates for the parameters of a linear MAUF (see [10] or [11]). A non-interactive elicitation method can be very convenient in many real situations since it avoids some important limitations of interactive methods, basically due to the information requirements, which can be really demanding for DMs (see [4]). Therefore, the availability of an indirect elicitation method for linear MAUF's is a further argument to choose a linear structure because it enables using MAUT without the need of interactive surveys.

The only non-interactive method to elicit non-linear MAUFs we are aware of was proposed in [1]. Starting from the structure of the decision problem and the observed behavior of the DMs, the main idea is to approach the problem of finding the values of the parameters of the utility function, given the observed decision, as a dual problem of that of making the optimal decision, given the values of the parameters. Since a rational decision maker will always choose an efficient solution (see, for example, [12]), it turns out that an inefficient observed decision can not be explained as

[2] Other methods involving attributes, utility and relative measurement include the analytical hierarchy process (AHP) and simple multiattribute rating technique (SMART), which is a simple version of MAUT introduced in [3]. More recently, Brugha [4] has proposed a phased approach involving three stages: screening of alternatives, ordering and finally choosing.

the result of a rational decision making process. Following this observation, the method proposed in [1] uses a simplified version of the decision problem in which the feasible set is replaced with the efficient set. When the efficient set is not fully known, some approximation is needed and this requirement could be a relevant shortcoming in some situations. The efficient set may not be easy to approximate and any approximation error in the efficient set may be inherited by the elicited values of the parameters. In this paper, a modified version of the method in [1] is proposed. The main difference is that it is not necessary to have any approximation of the efficient set, so that the main weakness of the method by André and Riesgo is overcome.

The method proposed here has in common with the one in [1] that it rests on duality results, but it uses the original decision problem rather than a simplified version. Instead of projecting the observed decision on an approximation of the efficient set, it is projected on the true efficient set by using a procedure which is similar to DEA but it uses the constraints of the decision problem instead of a sample of observations.

Section 2 presents the problem to be solved and the method proposed to solve it. Section 3 offers an application in which two different non-linear functions are elicited and both functions turn out to be able to rationalize the observed decisions. Section 4 presents the main conclusions and some discussion.

2. METHODOLOGY

2.1 The problem

Assume that a decision maker (DM) has a vector $x \equiv (x_1, \ldots, x_k)$ of k decision variables, he is concerned about n criteria and $f_i(x)$ is the mathematical expression for the i-th criterion ($i = 1, \ldots, n$). The preferences over each criterion are represented by the mono-attribute utility functions $u_1(f_1(x)), \ldots, u_n(f_n(x))$, which we write as $u_1(x), \ldots, u_n(x)$ for simplicity. Let us postulate the existence of a multiattribute utility function $U(u_1(x), ..., u_n(x), \gamma)$, which depends on the monoattribute utilities as well as a set parameters denoted as γ. U is assumed to be increasing and concave in all the monoattribute utilities.

The decision maker is supposed to known his own preferences (including the values of the parameters γ), as well as the feasible set for his decision variables, so he solves the following problem:

$$
\begin{aligned}
&\underset{x}{Max}\, U(u_1(x),...,u_n(x),\gamma) \\
&s.t. \quad g_i(x) \le 0 \quad i = 1,\dots,m \\
&\qquad\;\; h_i(x) = 0 \quad i = 1,\dots,q
\end{aligned} \tag{1}
$$

where γ is taken as fixed, x represents the vector of decision variables, and the feasible set is determined by m inequality constraints and q equality constraints.

Assume that a researcher wants to estimate the preferences of the DM. This information may be useful, for example, to predict the reaction of the DM when some new policy is implemented, and henceforth, to forecast the ultimate effects of such a policy.

Assume also that the researcher observes the value of the decision variables actually chosen by the DM, denoted as x^{obs}. The problem of the researcher can be stated as finding a MAUF which is consistent with the observed decision, i.e., a MAUF such that x^{obs} is the optimal solution of problem (1). In order to focus on the method to elicit U, assume that the monoattribute utility functions are known. Given a specific parametric expression for U, the problem can be seen as that of estimating the value of the parameters γ.

We show below that the problem of the researcher (finding γ given x^{obs}) can be seen as a dual of the problem of the decision maker (choosing x given γ). But first, it is important to note that a rational decision maker should never choose an inefficient solution because, if the solution is inefficient, it would be possible to improve the value of some attribute(s) without worsening any other attribute and, utility being increasing in all the attributes, this would yield a higher value of utility, which contradicts the definition of optimal solution. As a consequence, an inefficient decision can not be explained as being the result of a rational decision making process. Therefore, before applying the elicitation procedure itself, we need to check if the observed solution is efficient and, if it is not, to find an efficient surrogate, by projecting the observed point on the efficient frontier.

2.2. Projecting the observed point on the efficient frontier

Let $f^{obs} \equiv \left(f_1^{obs},\dots,f_n^{obs}\right) \equiv \left(f_1\left(x^{obs}\right),\dots,f_n\left(x^{obs}\right)\right)$ denote the vector of observed values for the attributes and $u^{obs} \equiv \left(u_1^{obs},\dots,u_n^{obs}\right)$ the associated values for the monoattribute utilities. If the observed point is not efficient, it should be projected onto an efficient version which will be taken as a surrogate for the rest of the analysis. We propose to do this following a procedure similar to DEA in two steps:

The first step is to expand radially the value of the monoattribute utilities[3] as much as possible within the feasible set. This is done by solving the following problem:

$$\begin{aligned}
&\underset{x,\theta}{Max}\ \theta \\
&s.t.:\quad u_1(x) = \theta u_1^{obs}(x) \\
&\qquad\quad \ldots \\
&\qquad\quad u_n(x) = \theta u_n^{obs}(x) \\
&\qquad\quad g_i(x) \le 0 \quad i = 1,\ldots,m \\
&\qquad\quad h_i(x) = 0 \quad i = 1,\ldots,q
\end{aligned} \tag{2}$$

where θ is the expansion factor by which every attribute is multiplied. This problem is similar to DEA in the sense that the observed values of the monoattribute utilities are expanded radially (see, for example, [15]). Nevertheless, there is an important difference since in DEA the observed units are typically projected onto a linear combination of some observed efficient units in the sample, whereas in our case we do not need any sample since the efficient set is implicitly determined by the feasibility constraints (g_i and h_i). Problem (2) can be read as *finding a feasible point which expands as much as possible the monoattribute utilities*. If in (2) the optimal value of θ is larger than 1, this means that all the attributes could be improved (radially) at the same time, and the observed decision is not efficient. Denote as x' the value of the decision variables resulting from (2) and $u' \equiv (u_1^{'},\ldots,u_n^{'}) = (\theta u_1^{obs},\ldots,\theta u_n^{obs})$ the resulting values of the monoattribute utilities. If $\theta = 1$, then $u' = u^{obs}$.

Nevertheless, there is no guarantee that u' is efficient because some additional efficiency enhancement could be achieved by moving in a non-radial way. For this reason, we propose the second step which consists of displacing the new point towards an efficient point by maximizing an ad-hoc linear function such that its gradient vector is the vector connecting the so-called anti-ideal point u_* (containing the worst value of each monoattribute utility, which is typically zero) and the observed point u^{obs}, without worsening the performance of any attribute, i.e.

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[3] A similar approach (in some cases virtually identical) could be to expand the values of the attributes, but the monoattribute utilities have the advantage that they are normalized in order to represent the preferences of the decision maker.

$$
\begin{aligned}
&\underset{x}{Max}\ \Phi\left(u_1,\ldots u_n\right)=\left(u_1^{obs}-u_{1*}\right)u_1+\ldots+\left(u_n^{obs}-u_{n*}\right)u_n \\
&s.t.:\quad u_1 \geq u_1{}' \\
&\qquad\ \ldots \\
&\qquad u_n \geq u_n{}' \\
&\qquad g_i(x) \leq 0 \quad i=1,\ldots,m \\
&\qquad h_i(x)=0 \quad i=1,\ldots,q
\end{aligned}
\tag{3}
$$

The rationale behind (3) is to find an efficient point as consistent as possible with the observed behavior. If the anti-ideal point u_* is interpreted as a kind of starting point, and we observe that the decision maker has moved from u_* to u^{obs}, this can be understood as the result of moving in a utility-maximizing direction represented by the vector connecting points u_* to u_{obs}. The objective function Φ used in (3) has been selected in such a way that the vector connecting u_* and u_{obs} is the direction of steepest increase (i.e., the gradient) of function Φ. The constraints $u_1 \geq u_1{}',\cdots,u_n \geq u_n{}'$ guarantee that the new point will be located in the relevant cone. Let x^{eff} denote the vector of decision variables that solves this problem and u^{eff} the associated values of the monoattribute utilities. The observed decision is efficient if and only if $u^{eff}=u^{obs}$.

This two-step procedure is illustrated in Figure 1. Assume there are two criteria ($n=2$) and the feasible set in terms of the attributes is represented by the polygon ABCD in Panel a of the figure. The ideal point f^* and the anti-ideal point f_* are also displayed. When all the relevant points are expressed in terms of utilities we have the new representation of the feasible set displayed in Panel b, where two hypothetical observed points E^{obs} and F^{obs}, in terms of utilities, are also shown. In the first step, the observed points are projected onto E' and F' respectively. Since F' is efficient, the second step is not needed and F' is taken as the efficient surrogate for F^{obs}. On the other hand, since E' is not efficient, the second step is necessary to get the efficient surrogate for E^{obs}, E^{eff}, which coincides with point B. E^{eff} (F') is the closest point to E^{obs} (F^{obs}) which is efficient and hence can be explained as being the result of a rational decision making process. The distance between E^{obs} and E^{eff} (between F^{obs} and F') can be interpreted as some error made by the decision maker because he/she is not fully efficient.

In Figure 1, it can be seen that, in general, both steps are needed to ensure a convenient efficient projection. Step 1 is needed to increase efficiency in a radial way.

Assume, for example, that step 1 is not done and step 2 is applied directly to E^{obs} rather than E'. The shape of the efficient set is such that the result of this problem is point C, which is efficient but does not appear to be a good surrogate for E^{obs}. On the other hand, step 1 does not guarantee efficiency by itself (remember that E' is inefficient).

2.3. Eliciting the parameters of the utility function: a dual approach

As noted in [1], the problem of eliciting the values of parameters γ can be seen as a dual of the decision making problem, by using the general formulation of duality proposed in [13], [14]. The decision making problem (1) is taken as the primal problem. The Kuhn-Tucker first order conditions of this problem are the following:

$$\begin{aligned}\nabla U + \sum_{i=1}^{m}\lambda_i \nabla g_i + \sum_{i=1}^{q}\mu_i \nabla h_i = 0 \\ \lambda_i g_i = 0 \quad i = 1,\dots,m \\ \lambda_i \geq 0 \quad i = 1,\dots,q\end{aligned} \tag{4}$$

where the coefficients λ and μ are the multipliers associated to the inequality and equality constraints respectively and ∇ denotes the gradient vector with respect to the decision variables x, i.e,

$$\begin{aligned}\nabla U &\equiv \left[\frac{\partial U}{\partial x_1},\dots,\frac{\partial U}{\partial x_k}\right] \\ &= \left[\left(\frac{\partial U}{\partial u_1}\frac{\partial u_1}{\partial x_1}+\dots+\frac{\partial U}{\partial u_n}\frac{\partial u_n}{\partial x_1}\right),\dots,\left(\frac{\partial U}{\partial u_1}\frac{\partial u_1}{\partial x_k}+\dots+\frac{\partial U}{\partial u_n}\frac{\partial u_n}{\partial x_k}\right)\right]\end{aligned} \tag{5}$$

and so on. For the researcher, the decision made by the decision maker is observed (and so it is taken as given) and the problem is to find a value of γ which is consistent with the observed decision, i.e., γ are seen as unknowns and x^{obs} are taken as given data or parameters.

In order to fit the problem into Johri's setting, the primal problem (1) can be formulated as:

$$\max_{(x,\gamma)\in\Gamma} U\left(u_1(x),\dots,u_n(x),\gamma\right) \tag{6}$$

$\Gamma \equiv \{(x,\gamma) \,/\, g_i(x) \le 0 \quad i=1,\ldots,m,\ h_i(x)=0 \quad i=1,\ldots,q; \quad \gamma \, given\}$ being the feasible set for (x,γ), where γ is included as a (trivial) decision variable with a single feasible value. The *general dual problem* can be expressed as:

$$\min_{\Omega \supseteq \Gamma} \left\{ \max_{(x,\gamma)\in\Gamma} U\left(u_1(x),\ldots,u_n(x),\gamma\right) \right\} \tag{7}$$

where the minimization is carried out over all the sets Ω which include Γ. By hypothesis, x^{obs} is observed and its value could be included as an additional constraint in (7), giving rise to a so-called *restricted dual problem* where γ would be the only remaining values to be decided. If x^{obs} is not efficient, then its efficient surrogate x^{eff} should be used instead. This restricted dual problem is very similar to the one that the one we want to solve: finding a value of γ that is consistent with the observed value (or the efficient version) of the decision variables.

On the other hand, typically, the value of γ needs to satisfy some constraints for the utility function to have desirable properties (for example, the parameters being nonnegative and smaller than one, etc). Let these constraints be written as $\gamma \in \Theta$, where Θ is the feasible set for the parameters. These constraints should be also included for the resulting solution to be acceptable. Additionally, we need to guarantee that the value of γ is such that x^{eff} maximizes $U(x,\gamma)$. This can be done in an operational way by including the optimality conditions (4). Since x^{eff} is feasible by construction, the feasibility constraints $g_i(x) \le 0 \quad i=1,\ldots,m$, $h_i(x)=0 \quad i=1,\ldots,q$ do not need to be explicitly imposed. Finally, since equations (4) involve the multipliers λ and μ, which were not present in the original problem, these need to be included as decision variables in the dual problem. Summing up, the following problem needs to be solved in order to elicit the values of the parameters that are consistent with (the efficient version of) the observed values:

$$\begin{aligned} &\underset{\gamma,[\lambda_i]_{i=1,\ldots,m},[\mu_i]_{i=1,\ldots,q}}{Min} \; U\left(x^{eff},\gamma\right) \\ &s.t. \quad [4], \quad \gamma \in \Theta \end{aligned} \tag{8}$$

where the system of equations (4) is evaluated in x^{eff}.

3. Illustration

Assume a decision maker has two decision variables: $x \equiv (x_1, x_2)$ and is concerned about two criteria given by $f_1(x) = x_1$ and $f_2(x) = x_2$. The feasible set is determined by the following constraints:

$$\begin{aligned} x_1 + 3x_2 &\le 480 \\ x_1 + x_2 &\le 202 \\ 13x_1 + 5x_2 &\le 2210 \end{aligned} \tag{9}$$

and the monoattribute utility functions are given by

$$\begin{aligned} u_1(f_1(x)) &= \frac{x_1 - x_{1*}}{x_1^* - x_{1*}} \\ u_2(f_2(x)) &= \frac{x_2 - x_{2*}}{x_2^* - x_{2*}} \end{aligned} \tag{10}$$

where x_1^* (x_2^*) and x_{1*} (x_{2*}) are the best and the worst feasible values for x_1 (x_2) respectively which, after optimizing each criterion separately, turn out to be:

$$\begin{aligned} x_1^* &= 170 \quad & x_2^* &= 160 \\ x_{1*} &= 0 \quad & x_{2*} &= 0 \end{aligned} \tag{11}$$

The decision maker is observed to choose the following values of the decision variables: $x_1^{obs} = 80$, $x_2^{obs} = 120$ (implying $u_1^{obs} = 0.4706$, $u_2^{obs} = 0.7500$), which turn out to be inefficient. By solving problem (2), x^{obs} is projected onto x' so that

$$\begin{aligned} &\theta = 1.01 \\ &x_1' = 80.8, \quad u_1 = 0.4753 \\ &x_2' = 121.2, \quad u_2 = 0.7575 \end{aligned} \tag{12}$$

and, by solving (3), we conclude that $x^{eff} = x'$, so that x' is efficient. The objective is to find a multiatribute utility function that can explain the observed values as the result of a rational decision making process. Since x^{obs} is inefficient, it cannot be explained as being the result of maximizing a utility function that is increasing in both u_1 and u_2, and $x^{eff} = x'$ is taken as an efficient surrogate. There are many utility functions that can be consistent with x^{eff} and the researcher must select one of them according to some criteria or additional information. We will illustrate here the solution under two alternative specifications:

3.1. First specification: a power function

Assume first that the researcher thinks that the preferences of the decision maker are represented by the classical power function

$$U(u_1, u_2, \alpha) = (u_1)^{\alpha} (u_2)^{1-\alpha} \tag{13}$$

which, in order to be elicited, only requires parameter α to be estimated. Given the value of x^{eff}, it is immediate to check that the first and the third constraints in (9) are not binding, which implies $\lambda_1 = \lambda_3 = 0$ in the decision making problem. We can include this information for computational convenience, so that the estimation problem (8) takes the form[4]

$$\begin{aligned}
&\underset{\alpha,\lambda_2}{Min}\ (0.4753)^{\alpha} (0.7575)^{1-\alpha} \\
&s.t. \\
&\quad \frac{\alpha}{80.8}(0.4753)^{\alpha} (0.75.75)^{1-\alpha} + \lambda_2 = 0 \\
&\quad \frac{1-\alpha}{160}(0.4753)^{\alpha} (0.75.75)^{-\alpha} + \lambda_2 = 0 \\
&\quad 0 \leq \alpha \leq 1 \\
&\quad \lambda_2 \leq 0
\end{aligned} \tag{14}$$

which gives $\alpha = 0.4$ as a solution and, according to this estimation, the problem of the decision maker can be written as

$$\begin{aligned}
&\max_{x_1, x_2} \left(\frac{x_1}{170}\right)^{0.4} \left(\frac{x_2}{160}\right)^{0.6} \\
&s.t. \\
&\quad x_1 + 3x_2 \leq 480 \\
&\quad x_1 + x_2 \leq 202 \\
&\quad 13x_1 + 5x_2 \leq 2210
\end{aligned} \tag{15}$$

since (14) and (15), are dual problems, the solution of (15) is $x = (80.8, 121.2)$. This means that the MAUF $U = \left(\frac{x_1}{170}\right)^{\alpha} \left(\frac{x_2}{160}\right)^{1-\alpha}$ can be taken as a representation of the preferences of the decision maker. In other words, it allows rationalizing the value of

[4] As a matter of fact, the set of constraints of (14) are such that there is a single feasible value of α, so that the minimization in (14) is, in this case, a trivial problem.

x^{eff} as a rational decision and the distance between x^{obs} and x^{eff} can be understood as an error due to inefficiency.

3.2. Second specification: additive-multiplicative function [16]

Assume now that the researcher has the hypothesis that the preferences of the decision maker are represented by the additive-multiplicative function proposed in [16]:

$$U(u_1, u_2, k_1, k_2, k_3) = k_1 u_1 + k_2 u_2 + k_3 u_1 u_2 \tag{16}$$

where three parameters (k_1, k_2, k_3) need to be estimated. These parameters are assumed to be positive and, for the multiattribute function to be normalized between zero and one, the normalizing constraint $k_1 + k_2 + k_3 = 1$ must also hold. This specification has the advantage that it includes the linear version as a particular case (if $k_3 = 0$), so that the model does not preclude the possibility that the MAUF is linear or non-linear.

Assume also that the researcher has some additional information related to parameter k_1, so that he imposes the additional constraint $k_1 \geq 0.2$. In this case, the estimation problem is

$$\begin{aligned}
&\underset{k_1,k_2,k_3,\lambda_2}{Min}\; k_1\left(0.4753\right) + k_2\left(0.7575\right) + k_3 0.4753 \cdot 0.7575 \\
&s.t. \\
&\quad \frac{k_1}{170} + \frac{k_3 \cdot 121.2}{160 \cdot 170} + \lambda_2 = 0 \\
&\quad \frac{k_2}{160} + \frac{k_3 \cdot 80.8}{160 \cdot 170} + \lambda_2 = 0 \\
&\quad k_1 \geq 2 \\
&\quad k_2, k_3 \geq 0 \\
&\quad k_1 + k_2 + k_3 = 1 \\
&\quad \lambda_2 \leq 0
\end{aligned} \tag{17}$$

and the solution is $k_1 = 0.2000$, $k_2 = 0.3057$, $k_3 = 0.4943$. According to this estimation, the problem of the decision maker is

$$
\begin{aligned}
&\max_{x_1,x_2} 0.2\left(\frac{x_1}{170}\right)+0.3057\left(\frac{x_2}{160}\right)+0.4943\frac{x_1}{170}\cdot\frac{x_2}{160}\\
&s.t.\\
&\quad x_1+3x_2\le 480\\
&\quad x_1+x_2\le 202\\
&\quad 13x_1+5x_2\le 2210
\end{aligned}
\tag{18}
$$

Since (17) and (18), are dual problems, the solution of (18) is $x=(80.8,121.2)$, so that the MAUF $U=0.2\left(\frac{x_1}{170}\right)+0.3057\left(\frac{x_2}{160}\right)+0.4943\frac{x_1}{170}\cdot\frac{x_2}{160}$ also allows to rationalize the (efficient version of the) observed decision

4. Conclusions

The contribution of this paper is a non-interactive procedure to elicit non-linear multiattribute utility functions. The proposed method has to steps: first to check if the observed decision is efficient, and if it is not, to project it onto an efficient surrogate following a DEA-like procedure. Second, to solve an elicitation problem that is a (restricted) dual of the decision making problem. This method builds on the procedure suggested in [1] but it overcomes the need to estimate the efficient set and in this way, it eliminates a possible source of estimation errors.

This method can be applied in a framework in which the researcher has information about the problem of a decision maker and the decision actually made by this decision maker, but the parameters of the utility function are not known and need to be estimated. By construction, the duality results allow to build a utility function which is consistent with the observed decision in the sense that this decision turns out to be optimal given the elicited utility function.

We show in an application that, in general, an (efficient) decision can be rationalized by more than one utility function. The selection of any of them depends on the case study and the information that the researcher has about the problem under analysis. One of the advantages of the elicitation procedure is that it is not restricted to any given specification of the MAUF but it is, in principle, compatible with any specification satisfying the usual constraints.

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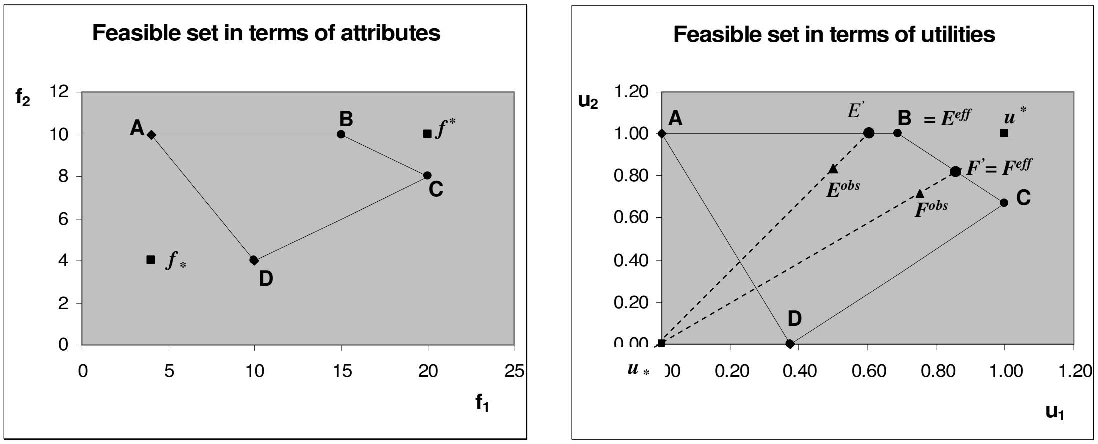
FIGURES



Panel *a* Panel *b*

FIGURE 1. Illustration of a feasible set in terms of the attributes (Panel *a*) and monoattribute utilities (Panel *b*) and projection of inefficient units onto the efficient frontier.