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Benedetto Molinari (U. Pablo Olavide)

Francesco Turino (U. Alicante y U. di Bologna)

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Department of Economics

Advertising, Labor Supply and the Aggregate Economy. A Long Run Analysis. *

Benedetto Molinari[†]
Universidad Pablo Olavide

Francesco Turino[‡]
Universitat d'Alacant
Università di Bologna

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Abstract

This paper studies the influence of persuasive advertising in a neoclassical growth model with monopolistically competitive firms. Our findings show that advertising can significantly affect the stationary equilibrium of a model economy in which the labor supply is endogenous. In this case, for empirically plausible calibrations, we find that the equilibrium level of hours worked, GDP, and consumption increase with the amount of resources invested in advertising. These findings are consistent with a new stylized fact provided in this paper: over the past decade, per-capita advertising expenditures have been positively correlated with per-capita output, consumption and hours worked across OECD countries. Because of the connection between advertising and labor supply, we show that our model improves on its neoclassical counterpart in explaining both within-country and cross-country variability of hours worked per capita.

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[†]Department of Economics, Universidad Pablo Olavide (Sevilla, Spain – EU). Contact email: benedetto.molinari@upf.edu

[‡]Fundamentos del Análisis Económico, Universitat d'Alacant (Alicante, Spain – EU). Contact email: fturino@merlin.fae.ua.es

1 Introduction

The goal of this paper is to study the influence of persuasive advertising in a neoclassical growth model with an endogenous labor supply and monopolistically competitive firms. Building on the work of Dixit and Norman (1978), we introduce advertising into the dynamic representative agent's framework by assuming that consumers' tastes are endogenously determined, in that they depend on firms' spending on advertising. This assumption generates a positive linkage between the demand of consumption goods and producers' advertising as the result of consumers' optimization behavior. Such a linkage, in turn, provides a rationale for firms' advertising spending, thereby introducing advertising into a dynamic general equilibrium set up as an endogenous firm's decision policy.

The main reason to study the influence of advertising in a general equilibrium framework is to investigate its potential effects on the aggregate economy. The literature on advertising has often speculated about the way advertising would affect macro variables.[1] The basic argument supporting this idea relies on the indirect effect that advertising may have on aggregate demand. Although advertising itself is a relatively small sector of aggregate production, by its own nature it may have a relevant effect on aggregate consumption.[2] Since consumption is a major component of aggregate demand, through this channel advertising may create important distortions in the economy. In this paper, we push this argument further by claiming that such distortions can be properly assessed only in a dynamic general equilibrium context. Suppose, for instance, that advertising stimulates aggregate consumption at the expense of saving. Then, it would contemporaneously increase consumption and crowd out investment, therefore having an unclear net effect on the aggregate demand. A partial equilibrium analysis would clearly fail to account for this trade-off effect. Moreover, by reducing investment, advertising may restrict future production capacity, thus creating a distortion between future demand and supply of goods. A static model would miss this connection. Additionally, advertising may imply a reallocation of resources across sectors, thereby indirectly creating pressures on prices in the productive factors markets and thus distorting the aggregate supply.

The model provided in this paper allows us to identify the conditions under which the presence of advertising significantly affects the aggregate economy in the long run. We show that the effect of advertising on the main economic aggregates depends crucially on the endogenous response of labor. From the standpoint of households, advertising operates in our framework as an endogenous taste shock that makes households more inclined to substitute consumption for leisure. All else being equal, this implies that an increase in aggregate advertising shifts the labor supply schedule to the right, thereby making consumers willing to work more in order to consume more. In the general equilibrium, the larger supply of labor also increases the equilibrium levels of GDP, consumption and investment. If instead we assume an exogenous labor supply, the results are the opposite: by reducing the equilibrium level of GDP and increasing the average markup, advertising essentially exacerbates the distortions caused by the monopolistically competitive structure of the goods market.

The effect of advertising on labor supply operates as a powerful mechanism that magnifies the long-run impact of advertising on the main economic aggregates. By calibrating the model to the U.S. economy, our framework predicts that the equilibrium levels of worked hours, GDP, and consumption in the U.S. are, respectively, about 9%, 8%, and 8.2% higher than in a counterfactual economy in which U.S. firms could not advertise their products.

[1] See Bagwell (2007) for an exhaustive survey on the related literature.

[2] See Molinari and Turino (2009) for a more recent analysis that supports the importance of advertising in the aggregate economy.

The relationship established in this paper between aggregate advertising and labor supply is of particular interest in light of the literature on differentials in hours worked across countries, e.g., Alesina, Glaeser and Sacerdote (2005) and Prescott (2004). Our paper contributes to this literature by showing that advertising could be one determinant of such differentials. In this perspective, we document a novel stylized fact: over the past decade, per-capita advertising expenditures have been positively correlated with per-capita output, consumption and hours worked across the OECD countries. This is another important contribution of our paper. Also, we provide two numerical exercises that contrast the models' predictions with actual data. Our results show that the model with advertising better explains both within-country and cross-country variability of labor supply than the neoclassical model. On one hand, by allowing for only cross-country heterogeneity in the size of the advertising sector, our model is able to explain between 25% and 33% of the observed differences in hours worked between some selected European countries and the US. On the other hand, by performing a business cycle accounting exercise in the spirit of Chari, Kehoe and McGrattan (2007), we show that our model predicts an increasing pattern for the US labor supply during the boom in the 1990s that resembles the observed increase. We also show that this prediction sensibly improves on the one made by the canonical RBC model, which fails to predict any upward trend in hours worked.

This paper is not the first to advocate a potential relationship between advertising and labor supply. Among the others, Brack and Cowling (1983) provided empirical evidence in favor of such a linkage for the US economy; more recently, Fraser and Paton (2003) empirically supported the same relationship for the UK economy. Our paper improves on this literature by providing a theoretical framework to rationalize such a relationship and, using this framework, showing that advertising can have an important effect not only on labor supply but also on several macro aggregates.

Another implication of our model is that, for empirically plausible calibration, the presence of advertising results in a higher equilibrium level of hours worked, output and, simultaneously, firms' markup. This feature is particularly interesting for at least two reasons. First, it provides a theoretical counterexample to standard results, showing that an increase in market power is not necessarily associated with a lower level of both hours worked and output. Second, it suggests that utility-diminishing advertising can still be welfare enhancing. By reducing the aggregate leisure in the economy, advertising mitigates the distortion associated with the monopolistically competitive structure of the goods market, thus possibly bringing the economy closer to the competitive equilibrium. In the model economy, however, this positive effect on welfare is offset by the increase in firms' markup due to advertising. To understand which effect prevails, we performed a welfare analysis that takes into account the endogenous nature of consumers' taste, as shown by Benhanib and Bisin (2002). For empirically plausible values of model parameters, we found that the consumer is worse off with advertising. Interestingly, unlike canonical results, welfare losses in our framework are driven by the "overworking" effect induced by advertising and not by the reduction in consumption due to higher prices (higher markup). Consumption, in fact, generally increases with advertising, as we will show in Section 4.2.

The rest of the paper is organized as follows. Section 2 documents some empirical evidence. Section 3 describes the model economy. Section 4 quantifies the long-run effects resulting from the presence in the economy of advertising activities by firms. Section 5 contrasts model predictions with actual data on worked hours from both within-country and cross-country perspectives. Section 6 provides the results for the welfare analysis. Section 7 concludes. All proofs are presented in the Appendix.

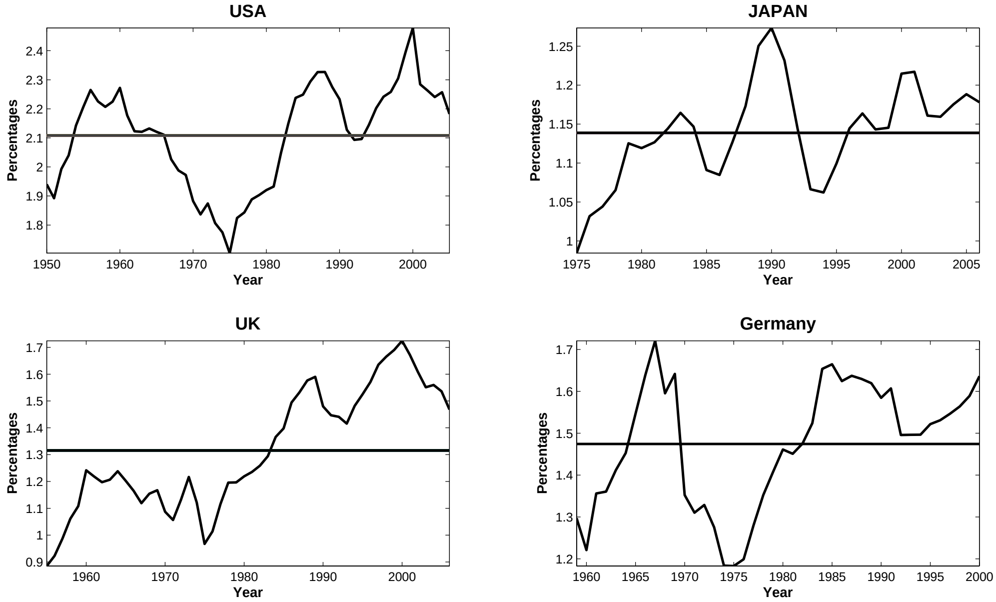

Figure 1: **Advertising share in the US, Japan, the UK and Germany**. Advertising share is calculated as the ratio of total advertising expenditures (all the media) to GDP. The horizontal line indicates the sample average mean.

2 Stylized Facts

This section provides empirical stylized facts for the advertising industry in several OECD countries. In what follows, we define *aggregate advertising* in a specific country as the total spending of domestic and foreign firms that advertise their products in that country's media. Our dataset contains annual figures of GDP, aggregate consumption, hours worked, population and advertising expenditures. Data were collected from several different sources. Details are provided in the Appendix.

We began our analysis by documenting the relative importance of the advertising sector across countries. To this end, figure 1 provides graphs of aggregate advertising expenditures as a percentage of GDP (henceforth, advertising share) in the US, Japan, Germany, and the UK. In terms of resources invested in this sector, one observes that advertising is a sizeable industry in all the countries considered, being equivalent to more than 1% of GDP. There are, however, remarkable cross-countries differences. Germany and the UK are similar, with an advertising share that on average accounts for slightly less than 1.4% of the GDP. The US has the highest value for the advertising share, more than 2%. The lowest value is in Japan, with almost 1.15%. There is not a clear trend in any of the figures we considered; rather, it seems that the advertising share has fluctuated around a constant mean, thereby indicating that, in each country, the average growth rate of advertising and GDP are approximately the same. The observed fluctuations are probably due to cyclical episodes. In fact, as shown by Molinari and Turino (2009), advertising has a well defined pattern over the course of the business cycle, being pro-cyclical and highly volatile.

Table 1 provides summary statistics for advertising expenditures, GDP, and consumption in the G7 countries. To render the figures comparable, all data are in dollars with constant PPP and prices. For each country, in addition to the average advertising share, the table also provides

Table 1: **Summary statistics for selected countries**.

Country	Time	$\frac{Adv}{GDP}\%$	$\frac{Adv}{Pop}$	$\frac{Adv}{Inv}\%$	$\frac{GDP}{Pop}$	$\frac{Cons}{GDP}\%$	$\frac{Cons}{Pop}$
USA	1984-2005	2.27	1.09	16.4	48.1	67.7	33.0
GBR		1.54	0.56	10.7	36.1	61.0	22.3
DEU		1.49	0.53	9.82	35.6	57.0	20.5
JPN		1.16	0.40	5.20	34.9	54.1	19.2
CAN	1996-2005	0.90	0.38	6.27	42.1	55.3	23.3
FRA		0.69	0.28	4.74	39.9	54.9	22.6
ITA		0.67	0.25	4.17	37.5	58.6	22.2

Note: *Inv* refers to total private fixed investment net of housing while *Pop* is the total person aged 15-64. All the variables are expressed in dollars with constant PPP and constant prices. Average means over the selected period of time.

the average mean of both per-capita advertising and advertising expenditures as a percentage of non-residential fixed investment. Per-capita advertising proxies the intensity of advertising in the economy−the number of messages for individual−while the ratio of advertising expenditures to non-residential fixed investment measures the relative importance of advertising as a firm's investment policy.[3]

Three main features of the data are worth emphasizing. First, the table shows that advertising expenditures, ranging from a minimum of 4.17% to a maximum of 16.4%, account for a relevant part of the aggregate productive investment in the G7 countries, displaying once again a remarkable degree of cross-country variability. The data for the US economy are particularly striking: in this country, the amount of resources invested in the advertising sector are equivalent to 16.4% of all productive investment, thereby suggesting that advertising is a particularly important component of the US firms' investment budget.

Second, independent of whether we use the share of GDP or per-capita expenditures as an indicator, our data show a clear positive cross-country correlation between per-capita real GDP and advertising. In each block of the table, the countries with the highest level of advertising are also characterized by the highest level of per-capita GDP. Moreover, this interesting feature holds true when we extend our sample to 18 OECD countries. As shown in figure 2 (panel A), there in fact exists a strong positive cross-country correlation between per-capita GDP and per-capita advertising. The estimated elasticity is positive (0.45) and statistically significant at the 5% level, with an R^2 coefficient of 0.42 (see table 2, column A).

Third, our data also show that consumption[4] and advertising are positively correlated across the G7 countries. This feature of the data is particularly important for the purposes of this paper, as the literature on the macroeconomic effect of advertising has often emphasized that the link between advertising and the economy, if it exists, must be established with consumption rather than with GDP.[5] As originally proposed by Galbraith (1967), the idea is that marketing activities in general, and advertising in particular, by promoting new and larger desires for material consump-

[3]In the literature, advertising is typically intended as a type of investment for firms−an intangible asset that, by affecting demand for more than one period, provides revenues that extend into the future. See Arrow and Nerlove (1962).

[4]Either as a percentage of GDP or in per-capita terms.

[5]See, for instance, Jacobson and Nicosia (1981).

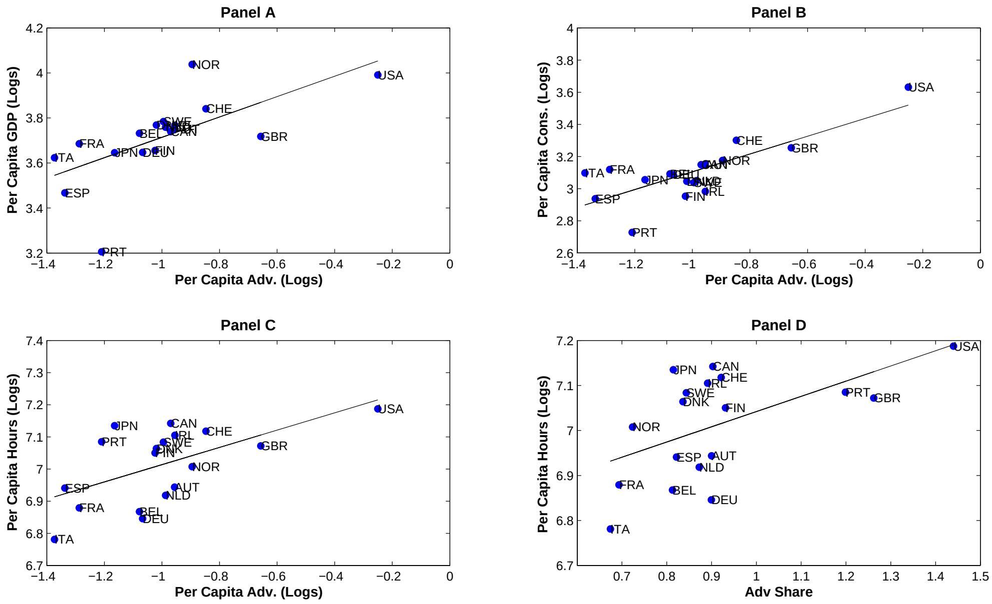

Figure 2: **Scatter plots.** Panels A, B, and C graph respectively the logs of per capita GDP, Consumption, and Hours against per capita advertising. Panel D graphs the logs of per capita hours versus advertising share (in percentage). Average means over the period of time 1996-2005. Source for aggregate advertising expenditures: WARC. See the data appendix for details.

tion may have important market-enhancing effects, thereby affecting not only the distribution of market shares across firms, as suggested by the conventional wisdom,[6], but also the market size of consumption goods and eventually aggregate consumption.[7] Since consumption is the major component of aggregate demand, advertising may possibly affect GDP and, therefore, the aggregate economy. This characteristic of advertising has been recognized by the specialized literature, and most empirical studies on the macroeconomic effects of advertising focus on its potential link with aggregate consumption. In general, there exists evidence of bi-directional causation (in the Granger sense) between advertising and consumption.[8] However, those studies focus on individual countries, while our data allow for a cross-countries analysis.

In order to explore this issue, panel B of figure 2 plots per-capita consumption expenditures versus per-capita advertising for 18 OECD countries. As we can see, there exists a positive and significant relationship between these two variables, thereby providing cross-country evidence of the connection between advertising and aggregate consumption. The estimated elasticity is positive (0.55) and statistically significant at the 5% level, with an R^2 of 0.61.

When comparing the United States with European countries, it has been often noted that Americans work more than Europeans. The literature has provided several different explanations for this fact. Prescott (2004) suggested that the observed differences in hours worked between Europe and the US can be explained by differences in marginal tax rates on labor income. Alesina

[6]See Simon (1970).

[7]Galbraith (1967) was among the first to suggest that advertising may have important market-enhancing effects. In his vision, because of advertising, the economy becomes more consumption based.

[8]See, for instance, Ashley et al. (1980) or, more recently, Jung and Seldom (1994). For a structural approach based on a fully fledged DSGE model with advertising, see Molinari and Turino (2009).

Table 2: **Simple Regressions**

Regressors	(A) $\frac{GDP}{Pop}$	(B) $\frac{Cons}{Pop}$	(C) $\frac{Hours}{Pop}$	(D) $\frac{Hours}{Pop}$	(E) $\frac{Hours}{Pop}$
Per-Capita Adv	0.452	0.553	0.269	-	-
	(0.004)	(0.000)	(0.011)	-	-
Advertising Share	-	-	-	0.338	0.717
	-	-	-	(0.015)	(0.058)
Constant	4.166	3.659	7.282	6.704	6.392
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	0.420	0.612	0.343	0.318	0.250

Columns A-D report the estimated relationship graphed in figure 2. Column E report estimates for the relationship by hours worked and advertising share by excluding the US, the UK, and Portugal (PRT) from the sample. All the variables, with the exception of advertising share, are in logs. P-values are reported in parenthesis. 18 OECD countries. Average mean over 1996-2006. Source of advertising expenditures data: WARC.

et al. (2005) indicated that the major differences between Europe and the US are largely due to the European labor market regulations advocated by politically powerful unions. Blanchard (2004) argues that Europeans enjoy leisure more than Americans do. Cowling and Poolsomnute (2007) take a different stand on the issue, arguing that *"the intensity of creation of wants through advertising and marketing might be an influence on decisions made by Americans about how much time they should devote to paid work, and how much time to leisure"*. The argument of the authors is based on the vision that advertising *"creates a continuing dissatisfaction with current levels of consumption, that may encourage people to offer a larger fraction of their time for the generation of income in order to satisfy their increased demands for material consumption"*. As a consequence, the pressure to consume provided by advertising will also affect the labor supply. Evidence for such a phenomenon is documented by Brack and Cowling (1983) for the US labor supply and, more recently, by Fraser and Paton (2003) for the UK economy.

In order to explore this issue in a cross-country dimension, in panels B and C of figure 2 we report per-capita hours worked (in logs) versus per-capita advertising and advertising share. In both cases, we find a positive cross-country correlation, with an estimated elasticity that is always positive and statistically significant at the 5% level (see table 2). However, the correlation between advertising share and hours worked (see panel D) appears to be less clear than in the other cases: the R^2 coefficient is significantly lower than for all the other estimates, and, in particular, the relationship appears to be driven by the observations for the US, the UK, and Portugal. For this reason, we repeated the regression by excluding those countries from the sample. As shown in column E of table 2, results did not change dramatically: although the estimates in this case are less precise than before, the estimated elasticity is still positive and statistically significant at the 10% level.

To summarize, our empirical analysis showed that advertising is a sizeable sector in the most industrialized countries, displaying a substantial degree of cross-country variability. In the US, advertising absorbs far more resources than it does in other countries. As a new stylized fact, we provide evidence in favor of a positive cross-country correlation between advertising and the main macro aggregates, such as GDP, consumption and hours worked.

3 The Model

In this section, we lay out a baseline model that captures the main features of advertising in a general equilibrium setup. The basic structure of the model is the standard neoclassical growth model augmented with a monopolistically competitive structure of product markets a la Dixit-Stiglitz. The main difference with the canonical framework is that, in our model, firms may promote their products by incurring advertising expenditures. Following Dixit and Norman (1978), we introduced this feature into the representative agent's framework by assuming that consumers' tastes are endogenously determined, depending on the aggregate expenditures in advertising activities by firms. This assumption implies a positive linkage between demand for consumption goods and producers' advertisements as a result of individual optimization behavior, thereby providing a rationale for firms' spending on advertising activities.

3.1 Households

The economy consists in a continuum of differentiated goods indexed by $i \in [0, 1]$, each produced by a monopolistically competitive firm. We assume that the representative consumer has preferences for consumption and hours worked described by the following utility function:

$$U(\widetilde{C}_t, H_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(\widetilde{C}_t/A_t\right)^{(1-\sigma)} - 1}{1-\sigma} - \xi \frac{H_t^{1+\phi}}{1+\phi} \right] \tag{3.1}$$

where $\widetilde{C}_t$ is the consumption aggregate, H_t is the time devoted to work, and ξ is a parameter affecting the disutility of labor. To ensure that the economy evolves along a balanced growth path, we assume that representative household derives utility from $\widetilde{C}_t$ relative to the level of technology, A_t that evolves at the constant rate $\gamma_a > 1$. As in An and Schorfheide (2007), we interpret this term as an exogenous habit stock component. The composite consumption aggregate $\widetilde{C}_t$ is defined as follows:

$$\widetilde{C}_t = \left(\int_0^1 \left(c_{i,t} + B\left(g_{i,t}, A_t\right)\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.2}$$

where $\varepsilon > 1$ is the pseudo-elasticity of substitution across varieties; $g_{i,t}$ is the goodwill stock associated with good i, which will be defined shortly; and $B(\cdot)$ is a decreasing and convex function of the goodwill stock controlling for the effect of advertising on consumer's preferences and satisfying $B(0, A_t) \geq 0 \ \forall \ A_t \in \mathbb{R}_+$.[9] As we will see in the next section, the dependence of $B(\cdot)$ on the rate of technology A_t is a necessary condition to guarantee the existence of a balanced growth path equilibrium.

Building on the work of Arrow and Nerlove (1962), we modeled the dynamic effect of advertising by assuming that current and past advertising for a good combine to create the producer's *goodwill*,

[9]The consumption aggregate (3.2) is a Stone-Geary-type non-homothetic utility function. Depending on whether the term $B(g_{i,t})$ is assumed to be positive or negative, the utility displays a saturation point or a subsistence level with respect to each variety consumed.

which, in turn, is defined as the intangible stock of advertising that affects the consumer's utility at time t, as shown in (3.2). The stock of goodwill evolves according to the law of motion:

$$g_{i,t} = \omega z_{i,t} + (1 - \delta_g)\, g_{i,t-1} \tag{3.3}$$

where $z_{i,t}$ is a firm's investment in new advertising at time t, $\delta_g \in (0, 1)$ is the depreciation rate of the goodwill and $\omega > 0$ is a parameter determining the advertising efficiency that might reflect institutional aspects, such as specific regulations, e.g. advertising bans or taxation.

Equation (3.1) is the key to making room for advertising in this model. It represents a non-homothetic version of the consumption aggregate originally proposed by Dixit and Norman (1978).[10] In this formulation, advertising is intended to be purely persuasive in the sense that it affects consumer choice by modifying his/her tastes without conveying any information about the characteristics of the good.[11] For each variety i, this effect is controlled by function $B(\cdot)$, whose properties are restricted in order to guarantee a positive linkage between a firm's advertisements and sales. This feature is made apparent by explicitly deriving the demand curve for each individual variety. The latter is the solution to the dual problem of minimizing consumption expenditures subject to the aggregation constraint (3.2), that is:

$$c_{i,t} = max \left\{ \left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \widetilde{C}_t - B(g_{i,t}, A_t); 0 \right\} \tag{3.4}$$

where

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$

is the nominal price index. Given (3.4), it is easy to verify that the monotonically decreasing behavior of the function $B(\cdot)$ implies that the demand of each variety is increasing with the firm's advertising effort. This feature hinges on the property that consumers' preferences are endogenously determined in our model. In fact, an increase in the advertising spending by a firm raises the goodwill stock of its own product and, at the same time, affects the consumers' taste by increasing the marginal utility of that particular variety. As a result, the consumer's willingness to pay for that good rises, thereby causing an upward shift in the demand schedule. Furthermore, the convexity of function $B(\cdot)$ guarantees a sort of saturation effect by inducing decreasing returns from advertising.

It is interesting to note that our framework implicitly embeds the combative nature of advertising. By differentiating equation (3.4) with respect to the goodwill stock, and assuming that a

[10] Non-homothetic preferences have recently received growing attention in macroeconomics. Ravn et al. (2006) modeled habit formation at the level of individual varieties using a non-homothetic consumption aggregator, which is essentially the same as the one we assumed here. There are indeed several similarities between our framework and the recent "deep habits" literature. See Molinari and Turino (2009) for further discussion on this point.

[11] Although the way advertising affects consumers' decisions is a rather controversial issue, we choose to focus on persuasive advertising for several reasons. First, as emphasized in Kaldor (1950), advertising, since it is pursued by an interested party, largely tries to persuade rather than to inform consumers and therefore is persuasive in nature. Second, recent studies of behavioral economists provided evidence on how consumers' tastes are distorted by advertising. Among others, Gabaix and Laibson (2006) showed that, in context of informative advertising, information revelation may break down in the presence of consumers that fail to foresee "shrouded attributes," such as hidden fees or maintenance costs. Finally, focusing on non-informative advertising allows us to get rid of complications, such as modeling informational asymmetries, that are impossible to analyze within the representative household framework.

fraction $\lambda > 0$ of firms increase their advertising levels, we get:

$$\frac{\partial c_{i,t}}{\partial g} = \int_0^{\lambda} \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} \frac{\partial B(g_{i,t}, A_t)}{\partial g} di \; < \; 0 \, \forall \, i \; \in [\lambda, 1]$$

In other words, when (enough of) the other firms increase their spending in advertising, the effect on firm i's demand is negative. Consequently, for a given level of consumption expenditures, any asymmetrical distribution in the goodwill stocks merely redistributes demand among firms, thereby causing an asymmetrical distribution in market shares.

The rest of the model is standard. We assume that the capital stock, K_t, held by the representative consumer evolves over time according to the law of motion:

$$K_{t+1} = I_t + (1 - \delta_k) K_t \tag{3.5}$$

where the investment in period t, I_t is assumed to be a composite good produced by aggregating differentiated goods via the following technology:

$$I_t = \left(\int_0^1 (i_{i,t})^{\frac{\varepsilon - 1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon - 1}} \tag{3.6}$$

As before, for any level of I_t, purchases of each variety $i \in [0, 1]$ in period t must solve the dual problem of minimizing total investment expenditures subject to the aggregation constraint (3.6), that is:

$$i_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} I_t \tag{3.7}$$

Notice that advertising does not directly affect the investment expenditures decision. This assumption implies that any link between advertising and investment is driven by general equilibrium effects. This allows us to embed in the model a possible crowd-out effect of advertising on investment, as we will see in the next section.

The consumer supplies labor services per unit of time and rents whatever capital he owns to firms. The labor and capital markets are perfectly competitive, so that each consumer takes as given the wage rate W_t paid per unit of labor services and the rental rate R_t paid per unit of capital. In addition, the consumer receives pure profit from the ownership of firms, Π_t. The flow budget constraint faced by the representative consumer is then given by the following equation:

$$\int_0^1 p_{i,t} \left(c_{i,t} + i_{i,t}\right) di \leq W_t H_t + R_t K_t + \Pi_t \tag{3.8}$$

The intertemporal maximization problem for the representative consumer can be stated as consisting of choosing processes $\widetilde{C}_t$, H_t so as to maximize the utility function (3.1) subject to (3.5) and (3.8).[12] The first-order necessary conditions for an interior maximum of U are

$$A_t^{(\sigma - 1)} \widetilde{C}_t^{-\sigma} = \lambda_t \tag{3.9}$$

[12]Notice that in the derivation of the first-order necessary conditions for this maximization problem, we rewrite the budget constraint (3.8) by using the following property:

$$\int_0^1 P_{i,t}(c_{i,t} + i_{i,t}) di = P_t \tilde{C}_t + P_t I_t - \int_0^1 P_{i,t} B(g_{i,t}, A_t) di$$

$$\lambda_t = \beta \left\{ \lambda_{t+1} \left[R_t + (1 - \delta_k) \right] \right\} \tag{3.10}$$

$$\xi H_t^{\phi} = W_t \lambda_t \tag{3.11}$$

where λ_t is the Lagrange multiplier for the budget constraint (3.8). Equation (3.10) is the familiar Euler equation that gives the intertemporal optimality condition. Equation (3.11), under the assumption of a perfect competitive labor market, instead describes the supply of hours.

The optimality conditions (3.9), (3.10), and (3.11) mimic those of the standard neoclassical growth model, but with the remarkable difference that the definition of the shadow price λ_t depends not only on aggregate consumption but also on aggregate goodwill. Consequently, consumers' decisions about labor and investment are affected by the level of aggregate advertising.[13]

This mechanism plays a pivotal role in determining the general equilibrium results that we will explore in the next section. A partial equilibrium analysis is useful for understanding how advertising affects the consumers' behavior. Suppose, for instance, that advertising expenditures increase exogenously for a sufficiently large fraction of firms. Given our assumptions, $\int B\left(g_{i,t}, A_t\right) di$ decreases, and, as a consequence, the consumer's shadow price λ_t increases. Consider now the labor supply schedule (3.11). An increase in λ_t implies that the agent values consumption more than leisure, since the marginal rate of substitution increases for any given wage. Hence, the labor supply schedule shifts to the right; i.e., the agent is willing to work more in order to consume more.

An increase in λ_t also affects the consumer's saving decisions by changing the intertemporal elasticity of substitution in the Euler equation (3.10). However, since (3.10) is a function of the ratio of current λ_t over future λ_{t+1} marginal utility, the sign of the effect of higher advertising depends on the relative response of current over future goodwill. In this simple example, the eventual effect is easily predictable. Goodwill is an $AR(1)$ process, and we assume a one-time increase in advertising; thus, current consumption will increase. In general, an increase in advertising due to an exogenous shock, while unambiguously shifting the labor supply schedule to the right, has an effect on the saving function that is determined by the dynamic response of expected future goodwill to a shock, which itself depends on several different general equilibrium effects that combine together. In particular, however, whenever the expected growth rate of goodwill is positive, the consumer finds it more convenient to postpone his consumption, since he foresees that his marginal utility will be higher in the future. Conversely, when the growth rate of goodwill is negative, the consumer experiences an *urge to consume* and increases his demand for current consumption.

3.2 Firms

In this model, firms make decisions on pricing policy, production plans, and budgets for advertising activities. We assume that each firm uses two types of input: labor and capital. To produce goods, firms have access to a common technology of the following form:

$$y_{i,t} = k_{i,t}^{1-\alpha} \left(h_{p,t}(i) A_t \right)^{\alpha} - A_t F \tag{3.12}$$

[13]In particular, insofar as $\widetilde{C}_t$ has a negative first derivative with respect to the aggregate goodwill, then advertising will increase both the marginal utility of aggregate consumption and the opportunity cost of leisure.

where $y_{i,t}$, $k_{i,t}$, and $h_{p,t}(i)$ respectively denote firm i's output, capital stock, and the amount of production-related labor. A_t measures the (labor augmented) technological progress evolving at a positive constant rate γ_a; $\alpha \in (0,1)$ and F is a fixed cost.

Each firm may promote its products by incurring advertising expenditures. As in Grossmann (2008), we assume that firms produce advertising in house by using a common technology that requires only labor, that is:

$$z_{i,t} = A_t \left(h_{a,t}(i)\right)^{\alpha} \tag{3.13}$$

where $z_{i,t}$ and $h_{a,t}(i)$ denote firm i's advertising effort and the amount of marketing-related labor, respectively. By getting rid of complications related to the specification of an advertising sector, this assumption greatly simplifies the analysis without affecting our main conclusions.

Using equations (3.4) and (3.7), the demand schedule faced by each firm can be written as:

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon}\left(\widetilde{C}_t + I_t\right) - B(g_{i,t}, A_t) \tag{3.14}$$

The non-homotheticity of the consumption aggregate (3.2) implies that the demand schedule features a non-constant elasticity of demand. Using (3.14), it is in fact easy to verify the following:

$$\xi\left(y_{i,t}, g_{i,t}\right) = \left|\frac{\partial y_{i,t}}{\partial P_{i,t}}\frac{P_{i,t}}{y_{i,t}}\right| = \varepsilon\left(1 + \frac{B(g_{i,t}, A_t)}{y_{i,t}}\right) \tag{3.15}$$

which shows that the price elasticity of demand depends, over time, on the ratio $B(g_{i,t}, A_t)/y_{i,t}$. This feature hinges on the property that total demand (3.14) is composed of two terms: one is perfectly inelastic with respect to the price, while the other has a constant elasticity. As a consequence, the resulting price elasticity is a combination of the elasticities of these two components, and the value of price elasticity depends, over time, on the relative importance of the inelastic term over total demand.

Most importantly, the elasticity of demand of each variety decreases with the producer's investment in advertising. This feature has a natural interpretation in terms of the degree of substitutability among goods. Intuitively, we can think that advertising activities by a firm, which attach peculiar attributes to the product, increase the consumers' perceived differentiation with respect to rival products. In our framework, when a firm increases its spending in advertising, it directly affects consumers' tastes, thus making that firm's product more valuable in terms of utility. Thus, the cost for the consumer of switching from that good to another increases. Or, equivalently, the degree of substitutability between goods decreases. Because of this perception of product differentiation, the consumer is now willing to pay a higher price for the advertised good, and the producer's market power increases.

We next incorporate the demand (3.14) into the profit-maximizing problem of firms. The demand for production-related inputs is the solution of the dual problem of minimizing total cost, given by $W_t n_{p,t}(i) + R_t k_{i,t}$, subject to the production constraint (3.12). The resulting optimal ratio of factors is of the form:

$$\frac{k_{i,t}}{h_{p,t}(i)} = \left(\frac{1-\alpha}{\alpha}\right)\frac{W_t}{R_t} \tag{3.16}$$

The corresponding total cost function, $CT\left(y_{i,t}\right)$, and the associated marginal cost, $\varphi_{i,t}$, are given by the following equations, respectively:

$$CT(y_{i,t}) = \frac{D}{A_t^{\alpha}} W_t^{\alpha} R_t^{1-\alpha}\left(y_{i,t} + A_t F\right) \tag{3.17}$$

$$\varphi_{i,t} = \frac{D}{A_t^{\alpha}} W_t^{\alpha} R_t^{1-\alpha} \tag{3.18}$$

where $D = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} \frac{1}{\alpha}$ is a positive constant.

Firm i's intertemporal problem can be stated as choosing a sequence of prices $P_{i,t}$ and an amount of advertising-related labor $h_{a,t}(i)$ in order to maximize the discounted value of all future profit flows. Formally, firm i solves the following problem:

$$\max_{h_{a,t}(i),P_{i,t}} \sum_{t=0}^{\infty} r_{0,t} \left(\frac{\pi_{i,t}}{P_t}\right)$$

subject to

$$\pi_{i,t} = P_{i,t} y_{i,t} - CT(y_{i,t}) - W_t h_{a,t}(i)$$

$$g_{i,t} = \omega z_{i,t} + (1 - \delta_g)\, g_{i,t-1}$$

$$z_{i,t} = A_t \left(h_{a,t}(i)\right)^{\alpha}$$

$$y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} \left(\widetilde{C}_t + I_t\right) - B(g_{i,t}, A_t)$$

where $r_{0,t}$ is the firm's discount factor,[14] and $CT(y_{i,t})$, is defined as in (3.17). The first-order conditions for an interior maximum are the following:

$$P_{i,t} = \frac{\varepsilon\left(1 + \frac{B(g_{i,t},A_t)}{y_{i,t}}\right)}{\varepsilon\left(1 + \frac{B(g_{i,t},A_t)}{y_{i,t}}\right) - 1} \varphi_{i,t} \equiv \mu_{i,t}\varphi_{i,t} \tag{3.19}$$

$$\phi_{i,t} = \frac{W_t}{\alpha\omega A_t} (h_{a,t}(i))^{1-\alpha} \tag{3.20}$$

$$\phi_{i,t} = -\frac{\partial B(g_{i,t}, A_t)}{\partial g_{i,t}} (P_{i,t} - \varphi_{i,t}) + (1 - \delta_g)\left(\phi_{i,t+1} r_{t,t+1}\right) \tag{3.21}$$

Equation (3.19) describes the familiar firm's pricing policy. The firm exploits its monopolistic power by charging a positive markup ($\mu_{i,t}$) over the marginal cost. Equation (3.20) defines the shadow price $\phi_{i,t}$ as the marginal cost of producing advertising. Equation (3.21) is the optimal advertising policy, stating that a firm chooses the optimal level of goodwill by equating its marginal benefit with its marginal costs. Substituting equation (3.20) into (3.21), solving the resulting equation forward and using the advertising production function (3.13) yields:

$$z_{i,t}^{\frac{1-\alpha}{\alpha}} = \alpha\omega \frac{A_t^{\frac{1}{\alpha}}}{W_t} \sum_{j=0}^{\infty} (1 - \delta_g)^j\, r_{t,t+j} \left[-\frac{\partial B(g_{t+j,i}, A_{t+j})}{\partial g_{t+j,i}} (p_{i,t+j} - \varphi_{t+j})\right] \tag{3.22}$$

[14] Under the assumptions of a perfect financial market and households holding the ownership of the firms, the stochastic discount factor is defined as:

$$r_{0,t} = \beta^t \left(\frac{\lambda_t}{\lambda_0}\right)$$

where $\beta \in (0, 1)$ is the consumer's subjective discount factor, and λ_t the consumer's shadow price, as defined in equation (3.9).

This expression shows that the optimal amount of advertising chosen by firms is sensitive to both cost and demand conditions. On one hand, exogenous reductions of the wage rate, or exogenous increases in technology, push the marginal cost of advertising downward, thereby raising the firm's incentive to advertise. On the other hand, since the marginal benefit of advertising depends positively on net revenues, which in turn are determined by the demand,[15] then exogenous variations of demand affect the firm's spending in advertising in the same direction. Interestingly, the marginal benefit of advertising increases in the stochastic discount factor $r_{t,t+j}$, implying that any change in the discount factor, such as variations in the interest rate due to monetary authority or variations due to exogenous shocks driving economic fluctuations, affect advertising expenditures.

Finally, it is interesting to note that firms' pricing and advertising policy are directly related. According to equation (3.22), a firm will find it convenient to increase its advertising budget in response to an increase in the unit net revenue from sales caused by a higher relative price. Using the terminology of Iwasaki *et al.* (2008), this means that, in our framework, firms play a supermodular game, since their pricing and advertising policies are complementary strategies.[16]

3.3 The Symmetric Equilibrium

The equilibrium for the model economy is derived by imposing a clearing condition in all markets. Let Y_t denote total output obtained by integrating (3.12) over the firms' index. The clearing condition on the goods market requires:

$$Y_t = C_t + I_t \tag{3.23}$$

while equilibrium in the market for labor factor requires:

$$H_t = \int_0^1 \left(H_{a,t}(i) + H_{p,t}(i) \right) di \tag{3.24}$$

Notice that conditions (3.23) and (3.24), together with the equations describing the optimal choices of all agents in the economy, imply that the capital market also clears.

Given the symmetry embedded in our model, there exists an equilibrium in which all firms set the same price, produce the same quantities, and invest the same amount of resources in advertising. Thus, we restrict our analysis by focusing on such symmetric equilibrium. In addition, we will normalize the price of consumption goods, $p_{i,t}$ to 1, so that all remaining prices are defined in terms of contemporaneous consumption.

The next proposition summarizes sufficient conditions to guarantee that a balanced growth path exists, that is, an equilibrium in which all the variables grow at a positive constant rate, with the exception of the interest rate, labor and the aggregate markup, which instead stay constant.

Proposition 1. *Consider an economy in which monopolistically competitive firms may promote their products by incurring advertising expenditures. Suppose furthermore that consumer preferences are defined as in equation (3.1) and that the technology for producing goods and advertising*

[15]See equation (3.19).

[16]Supermodular games are a general class of noncooperative games where n players simultaneously choose a set of strategies. Iwasaki *el al.* (2008) discuss the general property of advertising that unequivocally leads to a supermodular game in the context of an oligopolistic market in which firms simultaneously choose advertising budgets and pricing policies.

are given as in equations (3.12) and (3.13), respectively. Then, a sufficient condition for a balanced growth path equilibrium to exist is that, whenever

$$\frac{G_t}{A_t} = \frac{G_{t+1}}{A_{t+1}}$$

implies both:

$$\frac{B(G_{t+1}, A_{t+1})}{B(G_t, A_t)} = \gamma_a$$

and

$$B_g(G_{t+1}, A_{t+1}) = B_g(G_t, A_t)$$

where $B_g(G_t, A_t)$ denotes the partial derivative of function $B(\cdot)$ with respect to the first argument and evaluated at the aggregate goodwill stock, G_t.

Assuming that function $B(\cdot)$ satisfies the requirements of proposition 1, it is convenient to express the model economy in terms of detrended variables, for which there exists a deterministic steady state. Denoting with $\widehat{S}_t = S_t/A_t$ the original variable S_t detrended by the rate of technology A_t and letting $X_t = (\widehat{G}_t,\ \mu_t,\ \widehat{Z}_t,\ H_t,\ H_{a,t},\ H_{p,t},\ \widehat{C}_t,\ \widehat{K}_t,\ \widehat{I}_t,\ \widehat{Y}_t,\ R_t,\ \widehat{W}_t,\ \widehat{\widehat{C}}_t)$ be the vector of the detrended endogenous variables, a symmetric equilibrium for the model economy can be defined as a pair of initial conditions $(K_0, G_0) \in \mathbb{R}^2_+$ and a process $\{X_t\}_{t=0}^{\infty}$ that satisfies the following system of equations:

$$\widehat{W}_t = \alpha\mu_t^{-1}\left(\frac{\widehat{K}_t}{H_{p,t}}\right)^{1-\alpha} \tag{3.25}$$

$$R_t = (1-\alpha)\,\mu_t^{-1}\left(\frac{H_{p,t}}{\widehat{K}_t}\right)^{\alpha} \tag{3.26}$$

$$\widehat{\widehat{C}}_t^{-\sigma} = \frac{\beta}{\gamma_a}\left\{\widehat{\widehat{C}}_{t+1}^{-\sigma}\left[R_{t+1} + (1-\delta_k)\right]\right\} \tag{3.27}$$

$$\widehat{\widehat{C}}_t = \widehat{C}_t + \widehat{B}(\widehat{G}_t, A_t) \tag{3.28}$$

$$H_t = H_{a,t} + H_{p,t} \tag{3.29}$$

$$\widehat{Y}_t = H_{p,t}^{\alpha}\widehat{K}_t^{1-\alpha} - F \tag{3.30}$$

$$\widehat{Z}_t = H_{a,t}^{\alpha} \tag{3.31}$$

$$\widehat{G}_t = \frac{(1-\delta_g)}{\gamma_a}\widehat{G}_{t-1} + \omega\widehat{Z}_t \tag{3.32}$$

$$\gamma_a\widehat{K}_{t+1} = (1-\delta_k)\,\widehat{K}_t + \widehat{I}_t \tag{3.33}$$

$$\widehat{Y}_t = \widehat{C}_t + \widehat{I}_t \tag{3.34}$$

$$\xi H_t^{\phi} = \widehat{W}_t\widehat{\widehat{C}}_t^{-\sigma} \tag{3.35}$$

$$\widehat{W}_t H_{a,t}^{1-\alpha} = -\alpha\omega\, B_g(\widehat{G}_t, A_t)\,(1-\mu_t^{-1}) + \frac{\beta(1-\delta_g)}{\gamma_a}\left(\frac{\widehat{\widehat{C}}_{t+1}}{\widehat{\widehat{C}}_t}\right)^{-\sigma}\widehat{W}_{t+1}H_{a,t+1}^{1-\alpha} \tag{3.36}$$

$$\mu_t = \frac{\varepsilon\left(1 + \frac{\widehat{B}(\widehat{G}_t, A_t)}{\widehat{Y}_t}\right)}{\varepsilon\left(1 + \frac{\widehat{B}(\widehat{G}_t, A_t)}{\widehat{Y}_t}\right) - 1} \tag{3.37}$$

3.4 The Steady State

The steady-state equilibrium is derived from the above system of equations by assuming that the vector X_t is constant over time. As shown in the Appendix, we can conveniently summarize the equilibrium relationships (3.25)-(3.34) by introducing a map $V : \mathbb{R}^3_+ \rightarrow \mathbb{R}^{10}_+$ such that $V_t = V(J_t)$, where $V_t = (\, H_{p,t},\ R_t, \widehat{K}_t,\ \widehat{W}_t,\ \widehat{Y}_t,\ \widehat{I}_t,\ \widehat{\widehat{C}}_t,\ \widehat{G}_t,\ \widehat{Z}_t,\ \widehat{\widehat{C}_t})$, and $J_t = (H_t, \mu_t, H_{a,t})$. The steady state equilibrium of the model is then characterized in next proposition.

Proposition 2. *A stationary perfect foresight equilibrium is as a sequence $\{V_t, J_t\}_{t=0}^{\infty}$ such that $V_t = V(H, \mu, H_a)\ \forall\ t$ and where the vector $J_t = (H, \mu, H_a) \in \mathbb{R}^3_+$ satisfies*

$$-\omega B_g(\widehat{G}, A_t) H_a^{\alpha-1} = \left(\frac{\upsilon_1}{\mu - 1}\right) \mu^{\frac{\alpha-1}{\alpha}}$$

$$H = (1-\alpha)\, H_a + \varepsilon \widehat{B}(\widehat{G}, A_t) \left[\frac{\mu^{\frac{1}{\alpha}}\,(\mu - 1)}{\mu\,(1-\varepsilon) + \varepsilon}\right] \left(\frac{R}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}}$$

$$\xi H^{\phi} = \alpha \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \mu^{-\frac{1}{\alpha}} \left(\widehat{C} + \widehat{B}(\widehat{G}, A_t)\right)^{-\sigma}$$

where $\upsilon_1 = \left(\frac{\gamma_a - \beta(1-\delta_g)}{\gamma_a}\right)\left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}$ and where R, $\widehat{C}$ and $\widehat{G}$ and defined as in equations (C.2), (C.7) and (C.9) in the appendix.

By analyzing proposition (2), we identify three channels through which advertising affects the steady-state equilibrium. First, given equation (3.37), the long-run markup depends upon aggregate advertising. This is a price channel through which advertising ends up affecting all other endogenous variables. Second, by virtue of equation (3.29), the production of advertising activities absorbs labor, thereby reducing the total amount of resources available for producing goods. Unless the steady state results in a greater amount of hours worked, this channel implies that the equilibrium level of GDP is lower than it would have been without advertising. Finally, the aggregate goodwill stock affects the marginal evaluation of consumption. As discussed in section 3.1, this mechanism modifies the representative consumer's decisions about labor and savings, thereby modifying the aggregate supply of productive factors. We will see that the pressure provided by this mechanism on the supply of hours turns out to be crucial in determining the macroeconomic effects of advertising.

The next proposition summarizes the long-run effects that unequivocally result from the presence of firms' advertising expenditures in the economy.

Proposition 3. *In the steady state equilibrium, the ratio of consumption to GDP and the labor income share increase with advertising.*

Basically, advertising affects the steady-state equilibrium by generating a redistribution of resources from capital to labor. As stated in the proposition, the share of GDP that remunerates labor services is higher than it would be without advertising. This feature hinges on the property that, at the steady state, the interest rate is independent of advertising (see equation (3.27)), while the marginal rate of substitution between leisure and consumption does depend on advertising. Therefore, all adjustments between the productive factors occur through changes in the wage rate. This affects the factor markets asymmetrically, resulting in a general equilibrium in which the labor share increases. In addition, the resource constraint (3.34) implies that the ratio of investment to GDP unequivocally decreases with advertising, thereby making the economy more consumption based, as originally conjectured by Galbraith (1967).

4 Quantitative Properties

The three channels discussed in the previous section, by inducing countervailing effects on the endogenous variables, make the actual impact of advertising in aggregate not easily predictable. Consider, for instance, the supply of hours. Although we know that advertising shifts the labor supply schedule to the right, the general equilibrium effect is ambiguous. Given equation (3.35), movements in consumption may offset the impact of advertising on marginal utility so that the labor supply schedule may shift upward or downward or may remain unchanged. This depends on the relative strength of the different mechanisms at play and, eventually, on model parametrization. Furthermore, because the system of equations representing the steady-state equilibrium is non-linear, an explicit solution for the vector of endogenous variables cannot be found. Therefore, in what follows, we perform several numerical experiments in order to evaluate the aggregate effects of advertising.

4.1 Calibration

In order to explore the quantitative properties of our model economy, we need to assign specific numerical values to the structural parameters. To this end, we first parameterize the function $B(\cdot)$ by considering the following family of functions:

$$B(g_{i,t}, A_t) = \frac{A_t}{1 + \theta \frac{g_{i,t}}{A_t}} \tag{4.1}$$

where $\theta > 0$ is a parameter that controls for the effect of advertising on the consumer's preferences. It is easy to verify that (4.1) satisfies all the assumptions made so far about $B(\cdot)$. It is increasing and convex in the goodwill stock, $g_{i,t}$, and it satisfies the requirements of proposition 1. Also, notice that this formulation implies that the marginal utility of consumption is bounded. Because of this bound, the demand schedule (3.4) features a maximum price above which the demand is zero: when the price is too high the marginal benefit of consuming that good is smaller than its cost, and the consumer drops it from his basket of purchases. In this fashion, firms have an incentive to advertise their products to reduce the bound. In the absence of advertising, the bound is constantly equal to 1, while with advertising, the bound depends on the level of goodwill, whose effect is larger with larger θ. Hence, this parameter is interpreted as a measure of the effectiveness of advertising in affecting consumers' tastes.

Second, we calibrate model parameters, $\{\beta, \sigma, \phi, \xi, \varepsilon, \theta, \alpha, \delta_g, \delta_k, \gamma_a, \omega, \theta\}$, as follows. The standard parameters of real business cycle models are calibrated according to the values commonly used in the literature, while the others are chosen such that the steady-state values of model variables match selected long-run moments in the US postwar data. In particular, the discount parameter β is set to $(1.04)^{-.25}$, implying a yearly nominal interest rate of about 4%. The growth rate of technology γ_a is set to 1.005 so that the annual growth rate of GDP is 2%. The depreciation rate of capital δ_k is equal to 3% per quarter, and the gross elasticity of substitution across varieties is equal to 6. The preference parameter ξ is chosen to ensure that the consumer devotes in equilibrium 1/4 of his time to labor activities. Following Ravn, Schmitt-Grohe and Uribe (2006), we set the intertemporal elasticity of substitution to 0.5, the labor elasticity of output α to 0.75, and the Frisch elasticity of labor supply to 1.3. These restrictions imply that the preference parameters σ and ϕ are 2 and 0.77, respectively.

The values of advertising related parameters have been assigned using the following strategy. The goodwill depreciation rate has been fixed at 0.3, implying that the half life of goodwill stock is about two quarters. This value is consistent with the empirical evidence provided by Clarke

Table 3: **Calibration**

Parameter	Value	Description
β	.9952	Subjective discount factor
ε	6	Elasticity of substitution across varieties
δ_k	0.03	Capital depreciation rate
ξ	2.351	Preference Parameter
δ_g	0.3	Goodwill depreciation rate
ϕ	0.77	Inverse of Frisch Elasticity of Labor Supply
θ	2.62	Intensity of advertising in the utility function
α	0.75	Labor elasticity of output
σ	2	Inverse of Intertemporal Elasticity of Substitution
ω	0.949	Advertising Efficiency
γ_a	1.005	Growth Rate of Technology

(1976): the effect of advertising on the firm's demand basically vanishes after one year. The intensity of advertising in the utility function θ is set to 2.62, a value consistent with the empirical evidence reported in Molinari and Turino (2009), while ω is fixed such that, conditional on all other parameters, the steady-state value of advertising over GDP is equal to 2.27%, consistently with US average over the 1948-2005 period.[17] The time period in the model is one quarter. Table 3 summarizes the set of calibrated parameters.

4.2 Steady State Effects

We now quantitatively characterize the steady-state equilibrium of our model economy. In order to disentangle the effects resulting from the presence of advertising expenditures by firms, we will compare our model with a benchmark framework in which firms cannot advertise their products. To this end, we will first perform a static comparative exercise by analyzing the effect on the steady-state of alternative values for the advertising efficiency parameter, ω. All else being equal, given equation (3.36), this parameter controls for the optimal level of goodwill chosen by firms, thereby determining the aggregate amount of resources absorbed by advertising. In particular, when $\omega = 0$ firms have no incentive to promote their products through marketing activities, and the optimal level of advertising is zero. Thus, we take this as our benchmark model. Results are reported in figure 3 and in the first panel of table 4. Figure 3 displays the steady-state values of selected endogenous variables as a function of the advertising efficiency parameter ω. Figures are expressed as percentage deviations from their corresponding values in the benchmark model without advertising. The first panel of table 4 summarizes the quantitative effects on several endogenous variables by calibrating all parameters to the US economy. As a robustness check, the table also provides the results obtained by setting the elasticity ε to alternative values.

Several remarks are in order. First, the advertising share monotonically increases with ω. For larger values of this parameter, the rate of transformation of advertising to goodwill increases, so that one unit of advertising becomes more effective in enhancing demand. This raises the firms'

[17]This number refers to the ratio of advertising expenditures to net GDP, where exports are subtracted from GDP, because exported goods are not sold based on domestic advertising.

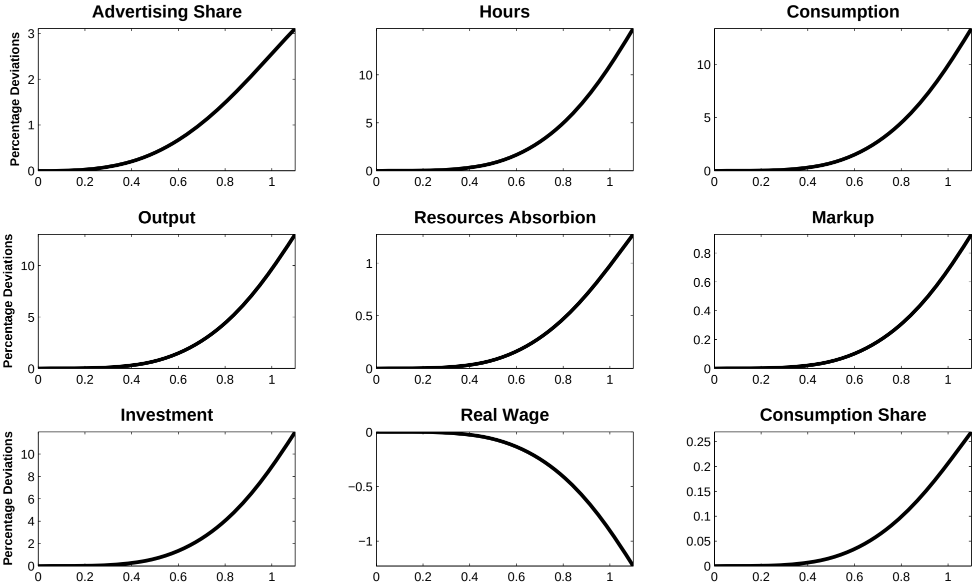

Figure 3: **Steady state allocations as a function of advertising productivity ω. Endogenous labor supply**. All the variables are expressed as percentage deviation with respect to the benchmark model without advertising (i.e. $\omega = 0$). Resource absorption refers to the ratio of advertising-related hours to total hours worked.

incentive to promote their products through marketing activities, thereby increasing the share of their revenues devoted to advertising expenditures. In the aggregate, this implies that the amount of resources absorbed by advertising increases with ω.

Second, the presence of advertising results in a steady-state equilibrium characterized by a higher level of hours worked, output and its components. Moreover, all these variables monotonically increase with ω. In other words, if we compare two economies that are identical in all structural parameters except the value of ω, our model predicts that in the economy with the larger advertising share we should also observe a higher level of hours worked, output and consumption. This is clearly consistent with the positive cross-country correlations documented in section 2. Additionally, the size of the effects triggered by the presence of advertising in the steady-state equilibrium turns out to be quantitatively important. For instance, our model predicts (see table 4) that, in the US economy, advertising induces an increase in hours worked in the range 7.31%-10.9%, depending on the value chosen for the gross elasticity of substitution among varieties ε. Similar magnitudes characterize variations in output, consumption and investment. However, the ratio of advertising-related labor to total hours worked $-$which we called resource absorption$-$ also increases with ω, thereby reducing the amount of labor available for producing consumption goods. As a result, the increase in the equilibrium level of GDP, ranging from a minimum of 6.31% to a maximum of 9.90%, is lower than the increase in total hours worked. In addition, the redistribution effect discussed in section 3.4 implies a substantial difference in the variation of consumption and investment, even though the increase in the consumption share is instead quantitatively more contained.

The mechanism providing upward pressure on the supply of hours through movements in the aggregate goodwill stock is crucial in generating the effects discussed so far. This is apparent by noticing that the real wage monotonically decreases with ω, thereby indicating that the increase in total hours worked is necessarily driven by the excess of supply that occurs in the labor market. Then, the larger availability of working hours ends up increasing the equilibrium level of GDP and its components. To further explore this feature, it is useful to compare the model derived in section 3 with two alternative models where we assume: (i) that labor supply is exogenous and (ii) that the aggregate goodwill has no effect on the marginal utility of aggregate consumption. In both cases, advertising does not directly affect the supply of labor, and therefore the general equilibrium effects are driven by price movements and resource absorption induced by advertising. Specifically, the first alternative model is based on the same setup used in section 3; the only difference is that the consumer inelastically supplies a fixed amount of working hours.[18] In the second model, we follow Molinari and Turino (2009) by assuming that the function $B(\cdot)$ is given by:

$$\frac{B(g_{i,t}, A_t)}{A_t} = S(g_{i,t}, A_t) + \int_0^1 \left(1 - S\left(g_{i,t}, A_t\right)\right) di \tag{4.2}$$

where

$$S(g_{i,t}, A_t) = \frac{1}{1 + \theta \frac{g_{i,t}}{A_t}}$$

Under this specification of $B(\cdot)$, the effectiveness of firm i's advertising on its own demand depends not only on its goodwill stock, $g_{i,t}$, but also on the level of goodwill of its competitors. In the symmetric equilibrium, function $B(\cdot)$ is equal to 1 for any value of $\widehat{G}_t$ and the marginal utility of consumption thus becomes independent of the aggregate stock of goodwill. Consequently, the effects of advertising on labor supply and aggregate consumption are indirect. Among other things, this formulation implies that advertising is a zero-sum game for firms, since it just redistributes demand across firms without affecting the market size. We will refer to this case as purely combative advertising. The results are summarized in panels 2 and 3 of table 4.

Compared with the baseline, the alternative models deliver two main results that highlight the importance of the labor supply channel for the macroeconomic consequences of advertising. First, the assumption that advertising does not directly affect labor supply implies largely different effects on the GDP and its components. Either with purely combative advertising or with an exogenous labor supply, the presence of advertising results in lower levels of output, consumption, and investment. Moreover, without the labor supply channel, any increase in the equilibrium level of hours worked, which is driven by an excess of demand for labor, is completely absorbed by the production of advertising activities. This is apparent by noticing that with purely combative advertising the equilibrium level of GDP and its components decrease even though the equilibrium level of hours worked increases (see panel 3). Second, the effect of advertising on the main aggregates is substantially smaller in both of these alternative models than in the baseline. For example, the variation of GDP with respect to the benchmark model (in absolute value) is about 14 times lower when the labor supply is assumed to be exogenous and when the elasticity of substitution among goods is set to 6. This means that the labor supply channel operates as a powerful amplification mechanism, which indirectly magnifies the impact of advertising to GDP.[19]

Another interesting implication of our model is that the presence of advertising results in a higher level of both hours worked and output even though the steady-state equilibrium is char-

[18] More precisely, in the model's simulations, we set $H = 1/4$.

[19] For example, notice that in the baseline calibration, our model predicts that a 2% amount of aggregate resources absorbed by the advertising sector implies an increase in GDP of 8%.

Table 4: **Results and Models Comparisons**

ε	ΔY	ΔC	ΔI	ΔH	$\Delta\mu$	$\Delta(C/Y)$
			Baseline Model			
5	6.31	6.50	5.66	7.31	0.55	0.17
6	8.09	8.27	7.42	9.09	0.54	0.17
7	9.90	10.1	9.23	10.9	0.52	0.17
			Exogenous Labor Supply			
5	-0.73	-0.50	-1.53	0	0.34	0.23
6	-0.54	-0.37	-1.14	0	0.26	0.17
7	-0.56	-0.39	-1.17	0	0.27	0.17
			Purely Combative Advertising			
5	-0.17	-0.01	-0.74	0.01	-0.01	0.16
6	-0.17	-0.01	-0.75	0.01	-0.01	0.16
7	-0.18	-0.01	-0.76	0.01	-0.01	0.17

Note: Δx refers to the percentage deviation of the original variable x with respect to its benchmark value without advertising ($\gamma = 0$). Panel 1 displays the results for the baseline model. Panel 2 displays the results for the model with an exogenous labor supply. Panel 3 provides the results for the model in which function $B(\cdot)$ is specified as in equation (4.2).

acterized by a larger aggregate markup (see figure 3). Unlike the standard results,[20] this feature provides a theoretical counterexample, showing that an increase in the market power is not necessarily associated with a lower level of hours worked and output. To understand this result intuitively, it is worth comparing our model with the canonical framework. In the standard Dixit-Stiglitz model with monopolistic competition in the goods market, firms' market power results in a wedge between the marginal rate of substitution from leisure to consumption and the wage rate that makes the consumer less willing to work. This effect results in a suboptimal equilibrium level of both hours worked and output. On the contrary, in our framework, advertising decreases the elasticity of substitution among goods but, at the same time, increases the marginal utility of aggregate consumption. As a result, the consumer feels disaffection with the current level of consumption that leads him to supply more working hours, while firms' gain market power that they exploit by charging a higher markup over the marginal cost. The negative effect related to the increase in firms' market power is then offset by the stronger substitution effect induced by advertising, and eventually the equilibrium level of hours worked increases. This mechanism provides us with a case where utility-diminishing advertising can be welfare enhancing. We will return to this point in section 6.

In the last part of this section, we propose a sensitive analysis to evaluate the extent to which the model's predictions are sensitive to alternative calibrations of the structural parameters. In particular, given the crucial importance of the labor supply channel to our results, we restrict our attention to the parameters that affect the labor supply schedule, that is, the inverse of intertemporal elasticity, σ, and the inverse of the Frisch elasticity of labor supply, ϕ. Figure 4

[20]See, for instance, Blanchard and Kyotaky (1987).

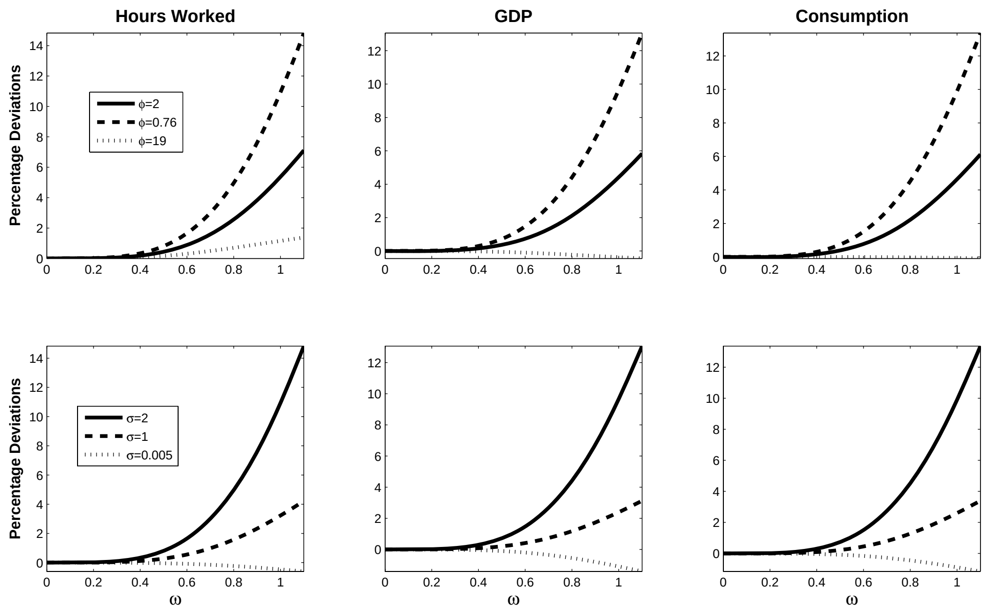

Figure 4: **Steady State Hours Worked, Output, and Consumption as function of advertising productivity for various value of ϕ, and σ.** All the variables are expressed as percentages deviation from the benchmark value in the model without advertising expenditures ($\omega = 0$)

illustrates the results, displaying, for alternative values of ϕ and σ, the graphs of hours worked, output and consumption as functions of ω. All the variables are expressed as percentage deviations from their benchmark values.

As shown in the first panel of figure 4, the size of the steady-state effects declines with larger ϕ. Intuitively, increasing this parameter implies a lower value of the Frisch elasticity and therefore a smaller reaction of labor supply to variations in wage and marginal utility of consumption. Hence, the downward pressures provided by the increase in the aggregate markup and the resource absorption due to advertising activities offsets the effect generated by the increase in the supply of hours, thereby reducing the size of the steady-state effect on output and consumption for any given ω. As matter of fact, for large enough values of ϕ, the equilibrium levels of these two aggregates decrease even though the equilibrium level of hours worked increases.

Regarding the intertemporal elasticity of substitution, notice that the effect of advertising on the main aggregates declines with lower σ. In this case, the substitution effect between consumption and leisure becomes smaller, and therefore the consumer's willingness to work declines. As such, the labor supply reacts less to equal movements in the aggregate goodwill, and consequently total hours worked, output, and consumption all increase less. Moreover, when σ is set to a sufficiently low value, the main results are reverted. As shown in figure 4 (see the dotted line), hours worked and the main aggregates monotonically decrease with ω. In such a circumstance, although the aggregate markup is still affected, the labor supply schedule becomes almost independent of the aggregate goodwill stock, and thus advertising ends up exacerbating the distortions associated with the monopolistically competitive structure of the goods market. As a result, hours worked, output and consumption all decrease with larger ω.

To summarize, in this section we have shown that the steady-state equilibrium levels of hours worked, outputs, and its components are larger with advertising than they would be otherwise. Moreover, as long as the representative consumer's labor supply is sufficiently elastic to movements in the marginal utility of consumption, the sizes of these effects are quantitatively important. However, the assumptions of endogenous labor supply and advertising with market-enhancing effects are both crucial for these results. On the contrary, the presence of advertising ends up exacerbating the distortions associated with the monopolistically competitive structure of the goods market, providing quantitatively small effects.

5 Advertising and Labor Supply

Beyond the macroeconomic consequences of advertising, the connection between the aggregate goodwill stock and the representative consumer's decisions about working activities is interesting per se. The connection suggests that aggregate advertising might potentially be an important determinant of labor supply. In section 2, we showed that the amount of resources absorbed by the advertising sector features a substantial within-country variation and, at the same time, shows remarkable differences across countries. Thus, advertising, by providing pressures on labor supply that vary over time and across countries, may potentially contribute to the observed within-country variations of labor supply, as well as cross-country differences in hours worked per capita. This feature is particularly important in light of the literature that studies the determinants of the labor supply decisions−an issue that has spurred a large amount of research in all branches of economic theory. In this perspective, this paper identifies an alternative mechanism that affects labor supply decisions that has so far been overlooked by economic theory. In the following sections, we study this connection by testing the ability of our model to explain both within-country and cross-country variations in labor supply.

5.1 The US boom in the 1990s

In the 1990s, the US economy experienced a decade of sustained economic growth. As we can see in figure 5, during this period, per-capita hours worked steadily increased, dropping back to the level of the early 1990s only with the recession in 2001. As emphasized by McGrattan and Prescott (2007), the basic neoclassical growth model, while accounting well for the post-war US economy prior to the 1990s, fails to reproduce the last economic boom. The reason for this failure is that, by incorporating variations in total factor productivity (TFP) and in marginal taxes on labor income, the neoclassical growth model predicts an after-tax real wage below its secular trend that, in turn, implies a decline in the predicted hours worked when, in fact, there was an increase. Other factors must have played a role in determining the behavior of the US during that period of time. McGrattan and Prescott suggested that the huge increase in intangible investments, such as those in building organizations, advertising, or expenditures in research and development, might be the missing piece of the puzzle to explain the economic boom in the 1990s.

McGrattan and Prescott, however, used their intuition to draw from an opportunely modified neoclassical growth model, an implicit aggregate series of intangible assets during the 1990s. With respect to their paper, then, we completed a reverse engineering exercise. We possess the actual series of advertising, and we used it to check whether our model can replicate the behavior of actual labor series during the 1990s. We suspect that advertising goes in the right direction to explain labor dynamics because, during the 1990s, advertising expenditures in the US experienced sustained growth. This is apparent in figure 1, which shows that advertising, relative to GDP, grew consistently over the 1990s, reaching its maximum postwar peak in 1999. According to

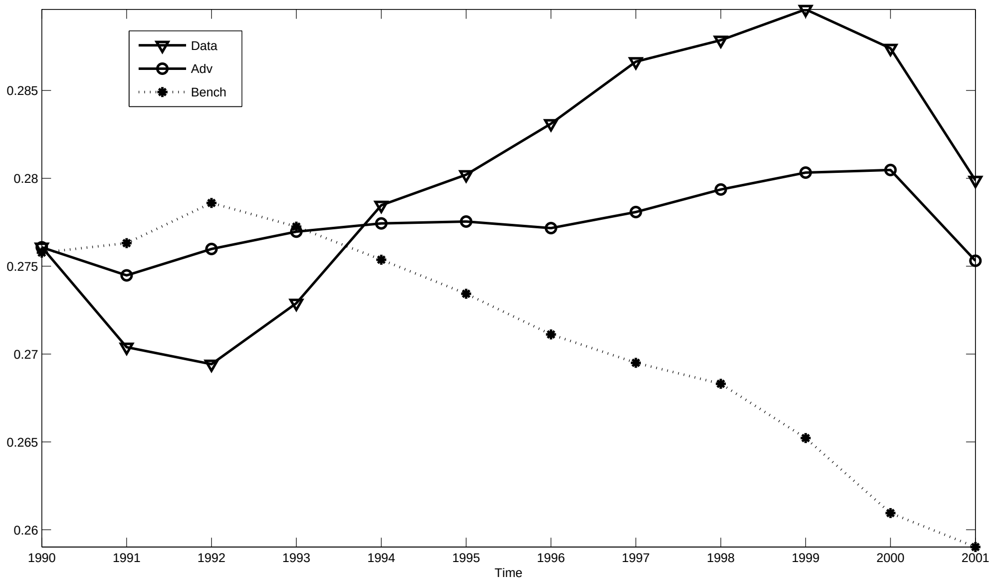

Figure 5: **Hours worked during the US boom in the 1990s**. Model's predictions vs actual data. All the data are taken from McGrattan and Prescott (2007). Bench refers to the model without advertising ($\omega = 0$).

our model, this should provide an upward pressure on the labor supply, thus possibly explaining the observed *puzzling* dynamics in hours worked. In order to address this issue, we performed a business cycle accounting (BCA) exercise like that developed by Chari, Kehoe, and McGrattan (2007). Namely, we made use of data on investment, GDP, hours worked, advertising expenditures, and taxes in order to recover from our model's equilibrium conditions three exogenous wedges that allow for a perfect match between the data and model predictions.[21] More specifically, we used the intratemporal condition (3.9) to recover the labor wedge, the Euler equation (3.10) to recover the investment wedge, and the optimal advertising policy (3.21) to recover the advertising wedge. Furthermore, in order to recover a sequence for TFP, we modified both production functions (3.12) and (3.13) by introducing a multiplicative term that captures purely transitory variations in productivity. Also, we introduced in our model taxes on labor income, τ_t, so that the representative consumer's intratemporal condition becomes:

$$\xi H_t^{\phi} = (1 - \tau_t)\widehat{W}_t \left(\widehat{C}_t + \widehat{B}(\widehat{G}_t, A_t)\right)^{-\sigma} \tag{5.1}$$

Then, we compared actual data for hours worked with the model's predictions obtained by shutting down the labor wedge and instead allowing for taxes, TFP, and other wedges to vary. Since the labor wedge has been proven to be the main determinant of labor dynamics within the basic neoclassical model,[22] our exercise was able to disentangle the effect of advertising on the labor supply during the 1990s.

Figure 5 compares the model's predictions for hours worked with the US actual hours worked per capita. As a benchmark case, we reported predicted hours worked per capita in the model

[21] Details on the procedure are provided in the Appendix.

[22] See, for instance, Shimer (2009).

without advertising ($\omega = 0$). As can be seen in this figure, the canonical neoclassical growth model predicts a constant decline in hours worked, when they were in fact increasing. As in McGrattan and Prescott's study, the model with a constant labor wedge fails to predict the boom in the 1990s. On the contrary, the model with advertising predicts the dynamics of labor remarkably better, showing a constant increase in hours worked per capita. To understand this result intuitively, recall that our framework naturally provides a theory for the household's disutility of work to be time varying. This is apparent from equation (5.1), which shows that for a given level of consumption the elasticity of the labor supply with respect to the after-tax real wage depends on the aggregate level of goodwill over time. In particular, for a given wage rate, the larger the aggregate goodwill stock, the stronger the consumers' willingness to work. Given the sustained growth observed in advertising share, this effect appears to have been particularly strong during the 1990s. In fact, while we did not find any significant difference in the recovered TFP, the labor wedge in the model with advertising is remarkably lower than the one we obtained in the benchmark.[23] The effect of variations on the stock of goodwill on the marginal utility of consumption overcompensates for the effect caused by the dynamics of the after-taxes real wage, thereby implying an increasing pattern in per-capita hours worked, as observed in the data. Although the fit of our model is far from perfect and clearly implies that other factors may have played a role in determining the dynamics of labor, the experiment shows that our framework substantially improves on the neoclassical model, confirming that the connection between advertising and labor supply operated as an important mechanism in determining the aggregate level of hours worked, at least in the US economy.

5.2 Cross-country comparison

In this section, we test the ability of our model to explain the observed cross-country differences in hours worked. To this end, we compare actual data of per-capita hours worked in the US and selected European countries with the model's predictions for H_t.[24] Given the availability of international data on advertising expenditures, we restricted the analysis to the period 1996-2006. As evident from table 5, our sample is characterized by large differences in average hours worked and advertising shares between the US and Europe. In both cases, the US is the leading country. Therefore, this is an interesting case study to evaluate the contribution of advertising in explaining the huge differences in hours worked between the US and Europe.

The analysis starts by assuming that the only source of cross-country heterogeneity is given by the size of the advertising sector. The first panel of table 5 refers to this comparison. Figures are obtained by setting the structural parameters to their baseline values, with the exception of the advertising efficiency parameter, ω, which is chosen by targeting the country-specific advertising share, and the preference parameter, ξ, which instead is calibrated to match the average hours worked in the US over the years 1996-2006. Following this calibration, any difference in per-capita hours worked predicted by the model is imputed to the different advertising share by country. In the second panel of table 5, we repeat the experiment by introducing other sources of cross-country heterogeneity, namely TFP and marginal taxes on labor income. Predictions here are obtained by calibrating the structural parameters as in panel (I), with the exception of taxes parameter τ, and TFP parameter, Υ.[25] In particular, τ is chosen by targeting the country-specific average marginal tax over the considered period of time, and Υ is chosen to match the average TFP by

[23] See figure 7 in the Appendix.

[24] Predictions are obtained by computing the steady state equilibrium of the model economy.

[25] Υ_t is the multiplicative term we add to the production functions for both goods and advertising. This factor captures purely transitory variations in total factor productivity. See the Appendix for further details.

Table 5: **Cross-Country comparison, baseline model predictions.**

	Labor Supply (US=1)			**Prediction Factors**		
Country	Actual	Predicted	Diff	Adv/Gdp	Tax Rate τ	TFP (US=1)
(I) Heterogeneity in Advertising						
Germany	0.71	0.93	0.25	1.28	0	1
France	0.73	0.91	0.32	0.97	0	1
Italy	0.66	0.91	0.26	0.89	0	1
United Kingdom	0.89	0.96	0.33	2.11	0	1
USA	1	1	-	2.76	0	1
(II) Heterogeneity in Advertising, TFP and Taxes						
Germany	0.71	0.71	0.99	1.28	0.59	0.89
France	0.73	0.70	1.11	0.97	0.59	0.92
Italy	0.66	0.65	1.03	0.89	0.64	0.98
United Kingdom	0.89	0.91	0.78	2.11	0.44	1.06
USA	1	1	-	2.76	0.44	1
(III) Heterogeneity in Taxes and TFP (Benchmark Model)						
Germany	0.71	0.80	0.69	0	0.59	0.89
France	0.73	0.80	0.75	0	0.59	0.92
Italy	0.66	0.74	0.76	0	0.64	0.98
United Kingdom	0.89	0.96	0.36	0	0.44	1.06
USA	1	1	-	0	0.44	1

Note: Labor Supply and Total Factor Productivity are both relative to the corresponding US values. $Diff = (1 - H^{actual})\ /\ (1 - H^{predicted})$ indicates the fraction of actual difference in hours worked between a country x and the US economy that is explained by the model. Panel (I) provides the results for the baseline model by assuming cross-country heterogeneity in the advertising sector. Panel (II) same as panel (I) but letting both taxes on labor income and TFP to vary. Panel (III) displays the results for the benchmark model ($\gamma = 0$) by assuming cross-country heterogeneity in both marginal tax on labor income and TFP. Marginal taxes on labor income, τ, are taken from Prescott (2004). Predictions are obtained by computing the steady state equilibrium of the model. Further details are provided in appendix.

country.[26] The third panel of table 5 replicates the same analysis in panel (II), except that it sets $\omega = 0$. By doing so, we drop advertising expenditures from the model, and therefore all the predicted cross-country differences in hours worked are imputed to the cross-country variability in both TFP and taxes on labor income. We report results for the US, Germany, France, Italy, and the UK.

Several remarks are worth emphasizing. First, according to our model's predictions, differences in the size of the advertising industry explain an important part of the cross-country variability in hours worked: as we can see from the first panel of table 5, allowing for this source of heterogeneity

[26] More specifically, TFP is recovered from data using the model equilibrium conditions. Marginal taxes on labor income are taken from Prescott (2004). Note that, in all experiments, we adjusted the data in order to be consistent with the model. See the Appendix for further details.

alone explains between 1/4 and 1/3 of the actual difference between the US and selected European countries. Second, our experiment confirms to a large extent the results provided in Prescott (2004): over the decade 1996-2006, as in the early 1990s, cross-country heterogeneity in both taxes on labor income and TFP appears to be crucial in explaining the US-Europe difference in average hours worked. In fact, allowing for these sources of heterogeneity notably improves the predictions of our framework with respect to the case of heterogeneity in the advertising share alone. Moreover, as we can see from the second panel of table 5, the results in this case are remarkable. The model fit is almost perfect. It is worth emphasizing, however, that heterogeneity in the advertising sector still plays an important role in explaining the variability in the labor supply across countries, even when we allow for variance in taxes and TFP. To see this, note that the model's fit worsens sensibly once we drop advertising heterogeneity from the model (see panel 3 of table 5). Therefore, although taxes and TFP are surely important, our experiments show that, during the decade 1996-2006, other factors contributed to the differences in hours worked between the US and Europe, and we identify advertising as one of these factors.

As a final remark, it is worth noticing that the mechanism behind our results is consistent with the idea, originally suggested by Blanchard (2004), that cross-country differences in hours worked might simply reflect cultural differences in the evaluation of leisure. According to Blanchard's vision, if Europeans enjoy leisure more than Americans, then any equal increase of wage would asymmetrically affect their labor supplies because the income effect would be stronger in Europe than in the US. In our model, the mechanism is exactly the same but with the key difference that cross-country variability on the marginal evaluation of leisure is endogenously determined by the effect of advertising on the consumer's preferences. In this perspective, our model predicts that even if Americans and Europeans shared the same *ex ante* preferences toward consumption and leisure, Americans would nonetheless have worked more because of the stronger desire for material consumption due to the larger advertising spending in the US market.

6 Welfare Analysis

In section 4.2, we showed that the presence of advertising results in a steady-state equilibrium characterized by a larger level of both consumption and worked hours, in spite of the fact that the economy experiences a larger aggregate markup at the same time. This feature of the model suggests that utility-diminishing advertising can be welfare improving, as it can potentially induce a lower level of aggregate leisure, thus mitigating the distortion of monopolistic competition. In what follows, we perform a welfare analysis to test whether this is in fact the case.

The evaluation of the welfare consequences of advertising is, however, complicated by the fact that preferences are endogenously determined, as has been emphasized by Dixit and Norman (1978). If advertising affects consumers' taste, there are at least two natural yardsticks for welfare comparisons: the pre-advertising and post-advertising tastes,[27] and it is not clear *a priori* which of these yardsticks is the most appropriate. For this reason, we follow Benhabib and Bisin (2002) by introducing a welfare criterion that takes into account both pre- and post-advertising preferences. More precisely, denoting the representative consumer's allocation pair and his equilibrium utility function by $(C(\omega), H(\omega))$ and $U(C(\omega), H(\omega), \omega)$, respectively,[28] we will make use of the following criterion:

Definition 1. *The consumer's welfare increases due to advertising if and only if it increases with*

[27] That is, the utility functions used to derive, respectively, pre- and post-advertising demands.

[28] In this way, $\omega = 0$ corresponds to utility and equilibrium allocations of pre-advertising case.

respect to post-advertising preferences, so that

$$U(C(\omega), H(\omega), \omega) \geq U(C(0), H(0), \omega)$$

and it also increases with respect to pre-advertising preferences,

$$U(C(\omega), H(\omega), 0) \geq U(C(0), H(0), 0)$$

with at least one inequality holding strictly.

In other words, for a given $\omega > 0$, we will say that the consumer is better off with advertising if, independently of whether we are using pre- or post-advertising preferences as a welfare yardstick, he prefers the post-advertising allocation $(C(\omega), H(\omega))$ to the pre-advertising allocation $(C(0), H(0))$.

The top panel of figure 6 provides the graphs for the representative consumer's utility function for alternative values of parameter ω. All the remaining parameters are calibrated to their baseline values.[29] In order to facilitate comparisons, the bottom panel of the figure provides graphs for welfare gain (in terms of steady-state consumption) associated with a policy that completely bans advertising.[30] Several remarks are in order. First, while the welfare gain strictly increases with ex-ante preferences, it displays a hump-shaped pattern with ex-post preferences, becoming negative for values of ω that are larger than the cutoff point ω^* (see the graph at the right-bottom corner). According to our criterion, this implies that the consumer is unequivocally worse off with advertising for any $\omega \in [0, \omega^*]$.[31] In this case, the presence of advertising ends up exacerbating the welfare losses caused by the monopolistic competitive structure of the goods market. Unlike canonical results, however, welfare losses in our framework are driven by the "overworking" effect induced by advertising and not by the reduction in consumption due to higher prices (higher markup). Consumption, in fact, increases with ω, as shown in section 4.2.

Second, our experiment shows that persuasive advertising does not necessarily imply a welfare loss for the society. To the contrary, if we use ex-post preferences as a yardstick for welfare comparisons, for a large enough value of ω, consumers are instead better off with advertising. In this perspective, notice that in the calibration that replicates the US economy, in which $\omega = 0.949 > \omega^*$, we cannot conclude that the consumer is worse off with advertising, since he prefers the allocation with advertising when choosing according to post-advertising preferences.

Finally, our model predicts that a consumer's satisfaction and per- capita income are negatively correlated. As we have seen in section 4, conditional to the baseline calibration, total output is in fact an increasing function of advertising efficiency, while, as shown in figure 6, the utility associated with the equilibrium allocations is instead decreasing, independent of whether we focus on pre- or post-advertising preferences. This result is consistent with the fact that no clear relationship between average income and average happiness can be found across countries (see Graham (2005)). In fact, the literature on the economics of happiness suggests that other factors, such as social aspirations or relative income considerations, contribute to an individual's sense of well-being. For instance, Laynards (2005) stresses the extent to which social status affects people's happiness. Also, the increasing flow of information about the living standards of others can increase frustrations with relative income differences. From this perspective, individuals may engage in consumption not only for its intrinsic value, but also for its value in signaling their relative position in the income distribution. Heffetz (2009) provides evidence for such a phenomenon,

[29] As shown in section 4.2, under this calibration the equilibrium levels of hours worked and consumption monotonically increase with ω.

[30] More precisely, for any given ω, the welfare gain is defined as the value of λ that solves the equation: $U(\lambda C(\omega), H(\omega), \omega) = U(C(0), H(0), \omega)$

[31] In fact, a negative welfare gain indicates that the consumer's welfare is larger with advertising than otherwise.

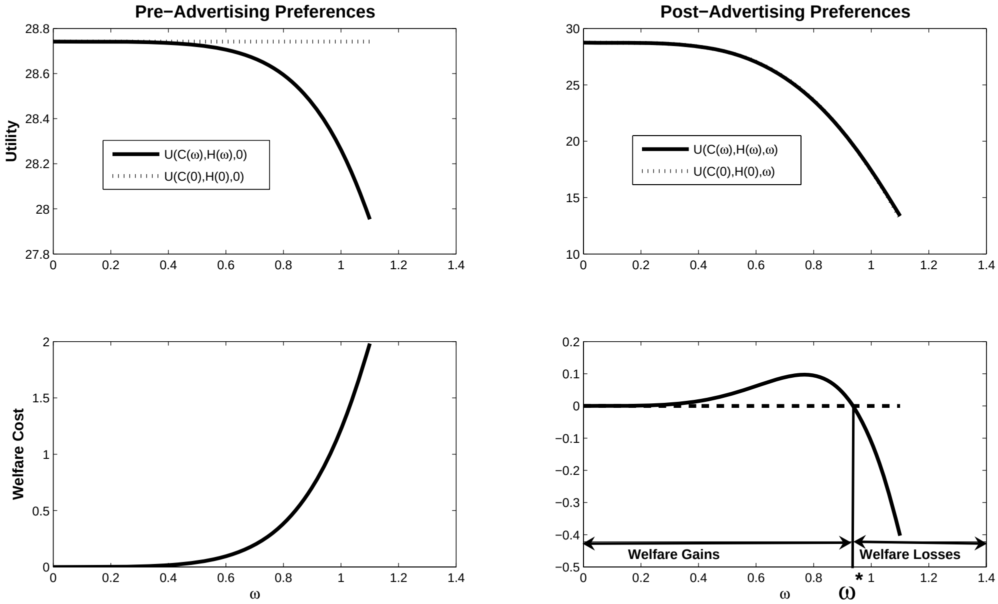

Figure 6: **Welfare Analysis**. Steady state utility functions. Left panel: pre-advertising preferences. Right panel: post-advertising preferences. The bottom panel illustrates the welfare gain, in terms of percentage of steady-state consumption, of a policy that completely bans advertisements.

showing that households' consumption decisions are affected by the so-called *socio-cultural visibility* of consumer expenditures, which he defines as *the speed with which members of society notice a household's expenditures on different commodities*. Our framework embeds many of these features, providing a mechanism that makes endogenous the welfare losses driven by the increasing aspirations for material consumption. It is interesting to notice that, although we did not assume preferences for relative consumption (or status), the core mechanism of our model is consistent with the potential connection between advertising and socio-cultural visibility of consumption goods. As shown in Krähmer (2006), if consumers use brands for image and visibility, then advertising, by informing individuals of the brand name, makes for a good potential signaling device, thereby inducing households to engage in conspicuous consumption. This mechanism creates a connection between advertising and aggregate consumption expenditures that is similar to the one we obtained in our model. Moreover, in our framework, as in models with preferences for relative consumption, e.g., Fisher and Hof (2000), if a consumer's decisions about working activities are endogenous, then advertising, by producing an overworking effect, might reduce the individual's sense of well-being even though he consumes more.

7 Conclusion

In this paper, we studied the influence of persuasive advertising in a neoclassical growth model. Our findings show that the long-run equilibrium levels of hours worked, outputs, and its components are higher with advertising than they would be otherwise. Furthermore, as long as the representative consumer's labor supply is sufficiently elastic to movements in the marginal utility of consumption, the size of these effects is quantitatively important. These results are consistent with a new stylized fact provided in this paper: advertising is positively correlated with hours worked, GDP and consumption across the OECD countries. However, the assumptions of an endogenous labor supply and advertising with market-enhancing effects are both crucial for our theoretical results. On the contrary, the presence of advertising ends up exacerbating the distortions associated with the monopolistically competitive structure of the goods market, providing quantitatively small effects. Because of the connection between advertising and labor supply, we also showed that our framework improves on the neoclassical model in explaining both within-country and cross-country variability of labor supply. Among other things, our findings suggests that cross-country variability in the size of the advertising sector contributes to the explanation for the huge observed differences in hours worked between the US and Europe. Finally, we provided a welfare analysis that allowed us to identify some portions of the model parametric space where the effect of advertising on the consumer's welfare is unambiguous. In these cases, the consumer is worse off with advertising, but the effect is fairly small.

To conclude, the main insight from our paper is that understanding the potential linkage between advertising and labor supply is crucial to properly assess the macroeconomic effects of advertising. Despite the evidence reported by Brack and Cowling (1983) and Fraser and Paton (2003), the literature is still lacking an appropriate empirical analysis that documents this relationship by involving a cross-country comparison of advertising and labor supply. Exploring this issue is therefore the next item on our research agenda.

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Appendix

A Data

Advertising expenditures data

- **Germany**: Investment in advertising including expenditures on salaries, media and the production of means of publicity for the period 1950-2000. Sources Rehme G., and Weiser S. (2007) table 7.
- **United Kingdom**: Annual advertising expenditures all the media. Data from 1950 to 1991 are provided to us by courtesy of Stuart Fraser. The data from 1991 to 2005 are taken from IPA (www.ipa.co.uk).
- **USA**: Data from 1948 to 1999 are obtained from an updated version of Robert J. Coens (McCann-Erikson, Inc.) original data published in Historical Statistics of the United States, Colonial Times to 1970. The data for 2000 to 2005 are obtained from the Newspaper Association of America (http://www.naa.org). The aggregate data include spending for advertising in newspapers, magazines, radio, broadcast television, cable television, direct mail, billboards and displays, Internet, and other forms.
- **Japan**: Data from 1975 to 2005. Source DENTSU (www.dentsu.com)
- **Others OECD countries** data from 1996 to 2006. Source World Advertising Research Center (WARC). Advertising Media and Forecasts.

Macro aggregates.

- Output, Consumption and Investment are from the OECD dataset. Investment are net of housing.
- Per capita hours worked are taken from Groningen Growth and Development Centre and the Conference Board, Total Economy Database, January 2007, http://www.ggdc.net
- Data for marginal taxes on labor income are taken taken from Prescott (2004)
- Macro aggregates for the business cycle accounting exercise are taken from McGrattan and Prescott (2007).
- Country specific capital for the cross-country analysis are taken from Kiel Institute for the world economy (http://www.ifw-kiel.de/forschung/datenbanken/netcap).

B Proof of proposition 1

A balanced growth path an equilibrium where all the endogenous variables grow at a constant rate. Given the structure of our model economy, we want to show that any function $B(\cdot)$ satisfying the requirements stated in proposition 1 implies the existence of such an equilibrium. Before proceeding, it is worth noting that the assumption of labor-augmenting Cobb-Douglas technology for the production of goods, the capital accumulation equation (3.33) and the good market clearing condition (3.34) imply that the steady-state rates of growth of output, capital, investment and consumption are all equal to the growth rate of labor augmenting technical progress, γ_a. These conditions together with the clearing condition (3.28) imply constancy of both advertising and production related labor. Given the production function of advertising activities (3.31) and the accumulation equation (3.3), this also implies that the goodwill stock and advertising have the same rate of growth γ_a. Thus, denoting one plus the growth rate of a variable X as γ_X, in the balanced growth path equilibrium it must be true that:

$$\gamma_Y = \gamma_K = \gamma_I = \gamma_C = \gamma_Z = \gamma_G = \gamma_a > 1 \tag{B.1}$$

and

$$\gamma_H = \gamma_{H_a} = \gamma_{H_p} = 1 \tag{B.2}$$

Using the terminology of King, Plosser and Rebelo (1988), equations (B.1)-(B.2) describe the technologically feasible steady state. Notice, moreover, that the same conditions also implies: (i) the marginal product of capital is constant over time; (ii) the marginal product of labor grows at the rate γ_a.

Next, we need to show can that any function $B(\cdot)$ that satisfies the restrictions stated in the proposition implies that conditions (B.1)-(B.2) are compatible with all the optimality conditions of agents in the economy. To this end, notice first that rewriting the intratemporal condition (3.35) as follows:

$$\xi H_t^{\phi} = \frac{W_t}{A_t}\left(\frac{\widetilde{C}_t}{A_t}\right)^{-\sigma}$$

indicates that for hours worked to be constant it is required that $\widetilde{C}_t$ and A_t grow at the same rate γ_a. Thus, by rewriting its rate of growth as follows:

$$\frac{\widetilde{C}_t}{\widetilde{C}_{t-1}} = \left(\frac{C_t}{C_{t-1}} - \frac{B(G_t, A_t)}{B(G_{t-1}, A_{t-1})}\right)\frac{C_{t-1}}{\widetilde{C}_{t-1}} + \frac{B(G_t, A_t)}{B(G_{t-1}, A_{t-1})}$$

we note that $\widetilde{C}_t$ grows at the rate γ_a if and only if also $B(G_t, A_t)$ does. Given that the goodwill stock grows at the steady state rate γ_a, for this requirement to be satisfied it is sufficient that:

$$\frac{B(G_{t+1}, A_{t+1})}{B(G_t, A_t)} = \gamma_a$$

whenever:

$$\frac{G_t}{A_t} = \frac{G_{t+1}}{A_{t+1}}$$

In addition, by virtue of equations (3.25), (3.26), (3.27) and (3.37), this condition also implies that: (i) both the aggregate mark-up and the interest rate stay constant; (ii) the representative consumer's Euler equation is satisfied; (iii) the real wage grows at the rate γ_a.

To conclude the proof it remains to show that equation (3.21) is satisfied along the balanced growth path. To this end, notice first that equation (3.20) implies that in the balanced growth equilibrium the advertising marginal cost stays constant, which, from equation (3.21), implies:

$$B_g(G_t, A_t) = B_g(G_{t+1}, A_{t+1})$$

Therefore, a sufficient condition for this requirement to be satisfied is that:

$$\frac{G_t}{A_t} = \frac{G_{t+1}}{A_{t+1}}$$

implies:

$$B_g(G_{t+1}, A_{t+1}) = B_g(G_t, A_t)$$

This concludes the proof.

C Proof of proposition 2

We begin the proof by showing how to construct a map $V : \mathbb{R}_+^3 \to \mathbb{R}_+^{10}$ that links a subset of endogenous variables to the vector $J_t = (H, \mu, H_a)$. To this end, let us assume that the economy is at the steady-state. From the clearing condition in the labor market we can derive en expression for production-related labor:

$$H_p = H - H_a \equiv H_p(H, \mu, H_a) \tag{C.1}$$

and for the Euler equation (3.27) an expression for the long run net interest rate:

$$R = \frac{\gamma_a - \beta(1-\delta_k)}{\beta} \equiv R(H, \mu, H_a) \tag{C.2}$$

By making use of equation (3.26), the ratio of production-relate labor to capital can be expressed as follows:

$$\frac{H_p}{\widehat{K}} = \left(\frac{R}{1-\alpha}\right)^{\frac{1}{\alpha}} \mu^{\frac{1}{\alpha}}$$

which, in turn, allows us to explicitly derive an expression for both capital stock and wage rate. That is:

$$\widehat{K} = \mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{-\frac{1}{\alpha}} (H - H_a) \equiv \widehat{K}(H, \mu, H_a) \tag{C.3}$$

$$\widehat{W} = \alpha \mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \equiv \widehat{W}(H, \mu, H_a) \tag{C.4}$$

No entry condition implies that the fixed cost is of the form:

$$F = \left(1 - \frac{1}{\mu}\right) H_p^{\alpha} K^{1-\alpha} - W H_a$$

Substituting this equation into the definition of output (3.30), and using equations (3.25) and (3.29) into the resulting expression yields:

$$\widehat{Y} = \mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} [H - (1-\alpha) H_a] \equiv \widehat{Y}(H, \mu, H_a) \tag{C.5}$$

An expression for the equilibrium level of investment can be derived using the low of motion for the capital stock. Accordingly:

$$\widehat{I} = \bar{\delta}_k \, \mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{-\frac{1}{\alpha}} (H - H_a) \equiv \widehat{I}(H, \mu, H_a) \tag{C.6}$$

where $\bar{\delta}_k = \gamma_a - (1 - \delta_k)$. Thus, using the goods market clearing condition, the equilibrium level of consumption can be rewritten as follows:

$$\widehat{C} = \mu^{-\frac{1}{\alpha}} \left(\frac{R}{1-\alpha}\right)^{-\frac{1}{\alpha}} \left[\frac{R}{(1-\alpha)} - \bar{\delta}_k\right] [H - (1-\alpha) H_a] \equiv \widehat{C}(H, \mu, H_a) \tag{C.7}$$

Finally, by virtue of equations (3.28), (3.31) and (3.32) we get:

$$\widehat{Z} = H_a^{\alpha} \equiv \widehat{Z}(H, \mu, H_a) \tag{C.8}$$

$$\widehat{G} = \left[\frac{\gamma_a \omega}{\gamma_a - (1 - \delta_g)}\right] H_a^{\alpha} \equiv \widehat{G}(H, \mu, H_a) \tag{C.9}$$

$$\widehat{\widehat{C}} = \widehat{C} + \widehat{B}(\widehat{G}, A_t) \equiv \widehat{\widehat{C}}(H, \mu, H_a) \tag{C.10}$$

Therefore, letting $V_t = (\, H_{p,t},\, R_t, \widehat{K}_t, \widehat{W}_t,\, \widehat{Y}_t,\, \widehat{I}_t,\, \widehat{\widehat{C}}_t,\, \widehat{G}_t,\, \widehat{Z}_t,\, \widehat{\widehat{C}}_t)$, we can conveniently summarize the equilibrium relationships (C.1)-(C.10) by introducing a map $V : \mathbb{R}_+^3 \to \mathbb{R}_+^{10}$ such that $V_t = V\,(H_t, \mu_t, H_{t,a})$.

To conclude the proof, we next derive a set of equilibrium conditions that allows us to identify among all the vectors $J_t = (H, \mu, H_a)$ those representing a steady-state equilibrium. To this end, notice first that the intratemporal condition (3.35) and the optimal advertising policy (3.36) can be respectively rewritten as follows:

$$\xi H^{\phi} = \alpha \left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \mu^{-\frac{1}{\alpha}} \left(\widehat{C} + \widehat{B}(\widehat{G}, A)\right)^{-\sigma} \tag{C.11}$$

$$-\omega B_g(\widehat{G}, A_t) H_a^{\alpha-1} = \left(\frac{\upsilon_1}{\mu - 1}\right) \mu^{\frac{\alpha-1}{\alpha}} \tag{C.12}$$

where $\upsilon_1 = \left(\frac{\gamma_a - \beta(1-\delta_g)}{\gamma_a}\right)\left(\frac{R}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}}$ and where R, $\widehat{C}$ and $\widehat{G}$ and defined as in equations (C.2), (C.7) and (C.9). The optimal price policy instead implies that the price elasticity takes the following form:

$$\varepsilon\left(1+\frac{\widehat{B}(\widehat{G},A_t)}{\widehat{Y}}\right)=\frac{\mu}{\mu-1}$$

which, in turn, allows us to express aggregate output as follows:

$$\widehat{Y}=\varepsilon\widehat{B}(\widehat{G},A_t)\left[\frac{\mu-1}{\mu(1-\varepsilon)+\varepsilon}\right]$$

Thus, substituting equation (C.5) into the previous one, and solving for H, yields:

$$H=(1-\alpha)\,H_a+\varepsilon\widehat{B}(\widehat{G},A_t)\left[\frac{\mu^{\frac{1}{\alpha}}\,(\mu-1)}{\mu\,(1-\varepsilon)+\varepsilon}\right]\left(\frac{R}{1-\alpha}\right)^{\frac{1-\alpha}{\alpha}} \tag{C.13}$$

Hence, at the steady-state the vector $J_t = (H, \mu, H_a)$ is the solution of the system of equations (C.11)-(C.13). Therefore, a stationary perfect foresight equilibrium for the model economy can be now defined as a sequence $\{V_t, J_t\}_{t=0}^{\infty}$ such that $V_t = V(H, \mu, H_a)\ \forall\ t$ and where the vector $J_t = (H, \mu, H_a) \in \mathbb{R}^3_+$ satisfies equations (C.11)-(C.13).

D Proof of proposition 3

Let $s_h(H, H_a)$ denotes the labor income share as a function of total hours worked, H, and advertising-related labor, H_a. By virtue of equations (C.4) and (C.5) it follows:

$$s_h(H,H_a)\equiv\frac{W\,H}{Y}=\alpha\left[\frac{H}{H-(1-\alpha)H_a}\right]>\alpha\equiv\ s_h(H,0)\,\forall\,H_a\ \in\ (0,H)$$

which shows that with advertising the labor income share is unequivocally larger than otherwise (i.e. $H_a = 0$). To prove that the consumption share increases with advertising, it is enough to note that equations (C.6) and (C.7) jointly imply:

$$\frac{I}{Y}(H,H_a)=\frac{\bar{\delta}_k(1-\alpha)}{R}\left[\frac{H-H_a}{H-(1-\alpha)H_a}\right]<\frac{\bar{\delta}_k(1-\alpha)}{R}\equiv\ \frac{I}{Y}(H,0)\,\forall\,H_a\ \in\ (0,H)$$

E Details on the BCA exercise

Simulation of the model is performed by following several steps and using yearly data over the period of time 1990-2003. Figures for macro-aggregates are taken from McGrattan and Prescott (2007). .

Step 1

To perform the BCA exercise, we begin by slightly modifying the model presented in section 3. More precisely, we introduce proportional taxes on labor income, τ_t, so that the representative consumer's intratemporal condition becomes:

$$\xi H_t^{\phi}=(1-\tau_t)\widehat{W}_t\left(\widehat{C}_t+\widehat{B}(\widehat{G}_t,A_t)\right)^{-\sigma} \tag{E.1}$$

and we modify both the production functions for goods and advertising activities in order to introduce a factor, Υ_t, capturing purely transitory variations in total factor productivity (TFP). That is:

$$\widehat{Y}_t=\Upsilon_t H_{p,t}^{\alpha}\widehat{K}_t^{1-\alpha}-F \tag{E.2}$$

$$\widehat{Z}_t=\Upsilon_t H_{a,t}^{\alpha} \tag{E.3}$$

Step 2. Calibration

As second step, we need to assign numerical values to all the model structural parameters. Some of them are set to their baseline values as reported in table (3), while the other are calibrated in order to: (i) take into account the yearly frequency of the data; (ii) match specific moments for the 1990 US (detrended) data. More specifically, we set the growth rate of technology, γ_a to 1.02, implying a yearly growth rate of GDP of 2%; the depreciation rate δ_g is fixed to 0.7, implying a half-life for the goodwill stock of about half year; the elasticity ε is set to 9, implying, as in the baseline case, an average mark-up equal to 5%. Parameters σ, θ, and ϕ are set to their baseline values. The remaining parameters $(\beta;\omega;\alpha;\delta_k;\xi;\Upsilon)$ are calibrated by using the US level of detrended variable in 1990 for GDP (normalized to 1), capital stock, hours worked, consumption, advertising expenditures, and by setting the yearly real interest rate to 1.041.

Step 3. Constructing the series of exogenous and endogenous variables

In order to simulate the model, we need first to construct sequences of all the endogenous and exogenous variables, including three exogenous wedges. To this end, we make use of data for output $\widehat{Y}_t$, investment $\widehat{I}_t$, advertising expenditures $\widehat{Z}_t$, hours worked H_t and taxes on labor income, τ_t. Exogenous wedges are introduces into the model in order to force the agents' optimality conditions so that, by computing the perfect foresight equilibrium, the model's predictions coincide with the observable data.

More specifically, we use the accumulation equation (3.33) with observations for the initial capital stock (the 1990 value) and investment to construct sequences of capital stocks. A series of aggregate consumption is obtained by using the resources constraint (3.34) with observations for detrended output and investment. Sequences of TFP is recovered by first noting that, by virtue of equations (3.29) and (E.3), the production function can be rewritten as follows:

$$\begin{aligned}
\widehat{Y}_t &= \Upsilon_t H_{p,t}^{\alpha} \widehat{K}_t^{1-\alpha} - F \\
&= \Upsilon_t \left(H_t - H_{a,p}\right)^{\alpha} \widehat{K}_t^{1-\alpha} - F \\
&= \Upsilon_t \left(H_t - \left(\widehat{Z}_t/\Upsilon_t\right)^{\frac{1}{\alpha}}\right)^{\alpha} \widehat{K}_t^{1-\alpha} - F
\end{aligned}$$

so that Υ_t can be expressed as:

$$\Upsilon_t = \left[\frac{(\widehat{Y}_t + F)^{\frac{1}{\alpha}} + \widehat{K}_t^{\frac{1-\alpha}{\alpha}} \widehat{Z}_t^{\frac{1}{\alpha}}}{\widehat{K}_t^{\frac{1-\alpha}{\alpha}} H_t}\right]^{\alpha} \tag{E.4}$$

Sequences of $H_{a,t}$ and $H_{p,t}$ are derived by using equations (3.29) and (E.3) with observations for $\widehat{Z}_t$, H_t and Υ_t. To perfectly match data for detrended advertising expenditures we modify the accumulation equation (3.32) by introducing an advertising wedge, $X_{z,t}$, as follows:

$$\widehat{G}_t = \frac{(1-\delta_g)}{\gamma_a} \widehat{G}_{t-1} + \omega X_{z,t} \widehat{Z}_t$$

This modifies the optimal advertising policy, which now is given by:

$$\frac{\widehat{W}_t H_{a,t}^{1-\alpha}}{X_{z,t}} + \alpha\omega_t \frac{\partial B(\widehat{G}_t, A_t)}{\partial G_t} (1-\mu_t^{-1}) + \frac{\beta(1-\delta_g)}{\gamma_a} \left(\frac{\widehat{\tilde{C}}_{t+1}}{\widehat{\tilde{C}}_t}\right)^{-\sigma} \frac{\widehat{W}_{t+1} H_{a,t+1}^{1-\alpha}}{X_{z,t+1}} \tag{E.5}$$

A sequence of labor wedges, $X_{h,t}$, is recovered by combining the representative consumer's intratemporal condition (3.35) with the optimal demand of production related labor (3.25) as follows:

$$X_{h,t} = \alpha \frac{(1-\tau_t)}{\xi\mu_t} \left(\frac{Y_t}{H_{p,t}}\right) \left[\frac{\tilde{C}_t^{-\sigma}}{H_t^{\phi}}\right] \tag{E.6}$$

while the investment wedge, $X_{k,t}$, is obtained from the representative consumer's intertemporal condition (3.27) as follows:

$$X_{k,t+1} = X_{k,t} \left\{\frac{\beta\left\{\tilde{C}_{t+1}^{-\sigma}\left[R_{t+1} + (1-\delta_k)\right]\right\}}{\tilde{C}_t^{-\sigma}}\right\} \tag{E.7}$$

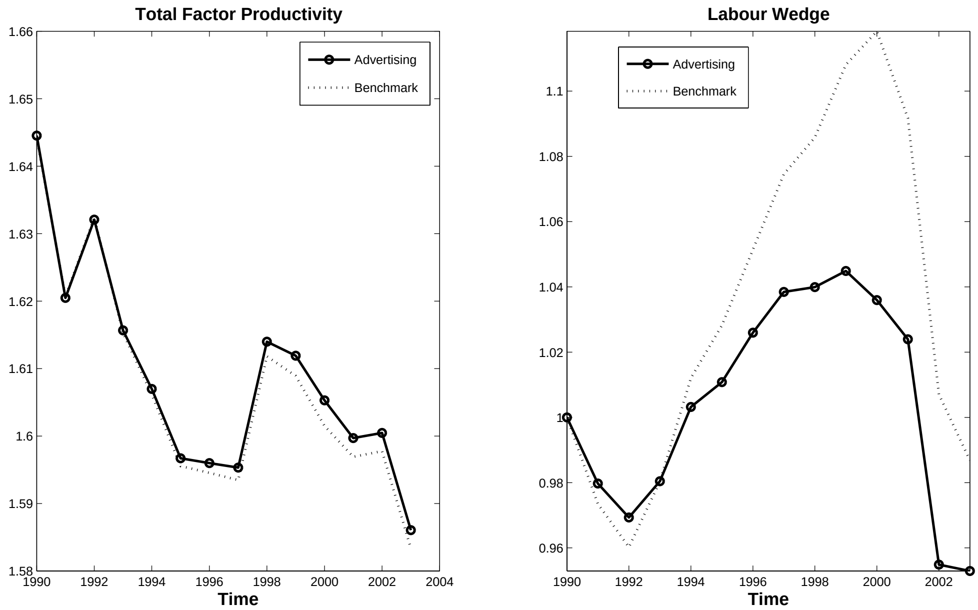

Figure 7: **Total Factor Productivity and the Labor Wedge**. *Note*: Benchmark refers to the model without advertising expenditures ($\gamma = 0$).

where the sequence $X_{k,t+1}$ is constructed by assuming $X_{k,1} = 1$ and R_{t+1} is defined as the following:

$$R_{t+1} = \frac{(1-\alpha)}{\mu_{t+1}} \frac{Y_{t+1}}{K_{t+1}} \tag{E.8}$$

Finally, using equations (E.5)-(E.7) and equations (3.25), (3.26), (3.28) and (3.37) with observations for GDP, Investment, Consumption, labor related variables, TFP and taxes on labor income we simultaneously recover sequences of $\widehat{W}_t$, R_t, μ_t, $\widehat{\widehat{C}}_t$, G_t, $X_{z,t}$, $X_{h,t}$ and $X_{k,t}$.

By construction, if we compute a perfect-foresight equilibrium path for this model, assuming households take as given time paths for TFP, tax on labor income and wedges, we get a perfect match between the model predictions and the data. Figure provides graph for the estimated series for the labor wedge and TFP in our framework, along with the corresponding sequences obtained in the benchmark case without advertising.

Step 4. Simulation

To analyze the ability of our model to explain the dynamics of hours worked during the 1990s, we compute the perfect-foresight equilibrium path by setting the labor wedge constantly equal to 1. By doing so, we are able to disentangle the effect upon the labor supply of advertising expenditures by comparing the model's predictions with data. Figure 5 provides the results.

F Details on Cross-Country comparison.

To perform the cross-country analysis, we compute the steady state equilibrium of our model economy by allowing for cross-country heterogeneity in the advertising sector, TFP and taxes on labor income. As in the case of the BCA exercise, we need first to construct a sequence for country-specific TFP. To this end, we make use of equation (E.4) with yearly data on detrended GDP and capital stock, where the latter is constructed by using the accumulation equation (3.33) with detrended data of investment. Data for the country-specific initial capital stock is taken from Kiel Institute for the world economy (http://www.ifw-kiel.de/forschung/datenbanken/netcap). In computing the country-specific steady-state, in order to introduce cross-country heterogeneity in TFP, we set the TFP parameter, Υ, to the country average over the period of time 1996-2006. Marginal taxes on labor income are taken from Prescott (2004).