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A normative foundation for equity-sensitive health evaluation: the role of relative comparisons of health gains

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Department of Economics

A normative foundation for equity-sensitive health evaluation: the role of relative comparisons of health gains*

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Abstract

We explore in this paper the relationship between equity-sensitive population health evaluation measures and normative concerns for relative comparisons of health gains. Such a relationship allows us to characterize focal equity-sensitive models for the evaluation of population health. Instances are the so-called multiplicative Quality Adjusted Life Years (QALYs) and multiplicative Healthy Years Equivalent (HYEs), as well as generalizations of the two. Our axiomatic approach assumes social preferences over distributions of individual health states experienced in a given period of time. It conveys informational simplicity, as it does not require information about individual preferences on health.

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1 Introduction

It is frequently argued that the benefit a patient derives from a particular health care intervention is defined according to two dimensions: quality of life and quantity of life (e.g., Pliskin, Shepard and Weinstein, 1980). The so-called Quality Adjusted Life Years (in short, QALYs) constitute the standard currency to deal with both health dimensions in the methodology of cost-utility analyses, probably the most widely accepted methodology in the economic evaluation of health care nowadays (e.g., Drummond et al., 2005). Nevertheless, addition of QALYs is usually criticized on equity grounds (e.g., Harris, 1987; Smith, 1987) and the importance of considering alternative (equity-sensitive) measures of population health in cost-utility analyses is widely accepted (e.g., Wagstaff, 1991; Williams, 1997; Nord, 1999; Anand, 2003).[1]

The purpose of this paper is to explore the relationship between equity-sensitive population health evaluation measures and normative concerns for relative comparisons of health gains. To do so, we follow the new axiomatic approach to the evaluation of population health, recently introduced by Hougaard, Moreno-Ternero and Østerdal (2013a). In such an approach, the health of an individual in the population is defined according to the two dimensions mentioned above (quality of life and quantity of life). Quantity of life is given by a number of life years, while no assumptions are made on how quality of life is described. The approach is informationally simple, as it does not make assumptions about individual preferences over length and quality of life, which might not be available information, either for practical or ethical reasons.[2] This is in contrast with the more standard approach to population health evaluation, where cardinal individual health utilities are assumed to be available as the basic input of the model (e.g., Bleichrodt, 1997; Dolan, 1998), or where structured individual preferences for quality and quantity of life, implying QALY-like individual utility functions, are assumed to exist (e.g., Østerdal, 2005; Harvey and Østerdal, 2010). We here, instead, address population health evaluation grounding directly on normative concerns over social preferences (on health gains).

One of the equity-sensitive population health evaluation functions for which we provide normative foundations is the so-called multiplicative QALYs function, which evaluates the health of a population by the product of the QALYs each individual in the population is

[1]For discussions on the related issue of the conceptual foundations of measuring (in)equality in health and health care, the reader is referred to Wagstaff and van Doorslaer (2000), Williams and Cookson (2000) and, more recently, Fleurbaey and Schokkaert (2012) and Hougaard, Moreno-Ternero and Østerdal (2013b).

[2]See, for instance, Dolan (2000) and Dolan and Kahneman (2008) for a discussion of the numerous conceptual and empirical challenges concerning the estimation and interpretation of individual health utility.

endowed with. Multiplicative forms of the QALY model have been frequently endorsed in the literature (e.g., Bleichrodt, 1997; Dolan, 1998). A multiplicative form induces an obvious concern for equity, as it penalizes uneven distributions of QALYs, whereas an additive form is not sensitive to such uneven distributions.[3]

QALYs can be seen as a specific computation of the so-called Healthy Years Equivalent (in short, HYEs), which refer to the socially equivalent population health distribution to a given one, in which the health outcome of one (and only one) agent is replaced by that of full health, for some quantity of time.[4] The additive HYE model evaluates population health by means of the unweighted sum of HYEs. As such, it is subject to the same criticism, on equity grounds, of its counterpart additive QALY model. We also derive in this paper normative foundations for the multiplicative HYE model in which the health of a population is evaluated by the product of the HYEs each individual in the population is enjoying.

One might argue that, for large populations, a multiplicative evaluation function might be *too equity sensitive*. For that reason, we also derive normative foundations for two families of population health evaluation functions, each generalizing the multiplicative QALY and HYE models, respectively. In such families, individual QALYs (respectively, HYEs) are submitted to an arbitrary (but increasing) function before being added. When such a function is logarithmic, we recover, precisely, the multiplicative QALY (respectively, HYE) model.

Another focal contribution within the health economics literature to develop equity-sensitive forms of evaluating a distribution of health is the so-called *fair innings* notion (e.g., Williams, 1997). Essentially, the notion reflects the feeling that everyone is entitled to some *normal* span of health. In some sense, one could consider that the multiplicative QALY and HYE models characterized in this paper are implementing a variant of the *fair innings* notion: they both aim to give a *fair* number (actually, the *average*) of quality-adjusted life years, or healthy years equivalent, to each person. Nevertheless, one might also argue that the fair innings notion is captured by Williams (1997) upon endorsing a *Bergsonian* functional form to evaluate the health distribution of a population. We shall also derive normative foundations for such functional forms in this paper.

Our model differs from the one used in Hougaard, Moreno-Ternero and Østerdal (2013a) in

[3]This is arguably the main reason why the UNDP unveiled a new methodology for the calculation of the so-called Human Development Index (e.g., Zambrano, 2013).

[4]This notion can be traced back to Mehrez and Gafni (1989) who propose it as a plausible way to reflect patient's preferences over health.

two important aspects.

First, we assume here that the quantity-of-life dimension is always strictly positive, whereas there it was only assumed to be non-negative. This seemingly innocuous aspect turns out to make a difference in both analyses. In Hougaard, Moreno-Ternero and Østerdal (2013a), the so-called social ZERO condition, which says that if an agent gets zero lifetime then her health state does not influence the social desirability of the health distribution, played an important role in simplifying the analysis. Such a condition, which is reminiscent of a widely used condition for individual utility functions on health (e.g., Bleichrodt, Wakker and Johannesson, 1997; Miyamoto et al., 1998; Østerdal 2005) is controversial, as the concept of health, in real life, is not properly understood with zero lifetime. In the analysis of this paper, we replace this condition by another saying that, when quantity of life is sufficiently small, quality of life becomes almost insignificant.

Second, we consider three core axioms, independent from those in Hougaard, Moreno-Ternero and Østerdal (2013a), which become crucial for our characterizations. These axioms convey different concerns for relative changes in lifetime, hence aiming to indeed provide the population health evaluation functions with an equity-sensitive orientation.

Our analysis is also reminiscent of some contributions within the literature on social choice and welfare. For instance, Tsui and Weymark (1997) characterized *multiplicative* social welfare orderings akin to the population health evaluation functions we derive here. More recently, Mariotti and Veneziani (2014) have introduced a stylized model to evaluate opportunities in society as "chances of success". To do so, they conceptualize *boxes of life*, in which each entry refers to the probability of success of each agent in society. In mathematical terms, their problem is equivalent to evaluating QALY distributions. It turns out that they characterize a multiplicative criterion to evaluate boxes of life, which could thus be seen as a counterpart to our characterization of the multiplicative QALY population health evaluation function considered here. The primitives of our model, however, are not profiles of QALYs (boxes of life) but rather health matrices made of duplets (referring to quantity and quality of life) each characterizing one individual. Furthermore, as mentioned above, our model excludes the possibility of zero lifetimes in the domain, as opposed to theirs. Thus, and contrary to their approach but aligned with Tsui and Weymark (1997), we rely on functional analysis for our proofs.

The rest of the paper is organized as follows. In Section 2, we introduce the model and the axioms we consider, as well as a preliminary characterization result. In Section 3, we introduce and characterize the main population health evaluation functions described above. In Section

4, we characterize other functions connecting to the notion of fair innings. We discuss the results in Section 5. For ease of exposition, we collect all proofs in an appendix.

2 The preliminaries

Let us conceptualize a policy maker with preferences defined over distributions of health for a cohort of individuals (in brief, "population") that we identify with the set $N = \{1, ..., n\}$, where $n \geq 3$.[5] The health of each individual in the population will be described by a duplet indicating the level achieved in two parameters: quality of life and quantity of life (i.e., lifetime).[6] Assume that there exists a set of possible health states, A, defined generally enough to encompass all possible health states for everybody in the population. We emphasize that A is an abstract set without any particular mathematical structure.[7] Quantity of life will simply be described by the set of strictly positive real numbers, $T = (0, +\infty)$.[8] Formally, let $h_i = (a_i, t_i) \in A \times T$ denote the health duplet of individual i. A population health distribution (or, simply, a health profile) $h = [h_1, \ldots, h_n] = [(a_1, t_1), ..., (a_n, t_n)]$ specifies the health duplet of each individual in society. We denote the set of all possible health profiles by H.[9] Even though we do not impose a specific mathematical structure on the set A, we assume that it contains a specific element, a_*, which we refer to as *perfect health* and which is univocally identified, as a "superior" state, by the policy maker.[10]

The policy maker's preferences (or social preferences) over health profiles are expressed by a preference relation $\succsim$, to be read as "at least as preferred as". As usual, $\succ$ denotes strict preference and $\sim$ denotes indifference. We assume that the relation $\succsim$ is a weak order.

A *population health evaluation function* is a real-valued function $P : H \to \mathbb{R}$. We say that

[5]Our running interpretation would be a policy maker aiming to evaluate alternative vaccinations for any of the stages in the immunization schedule of infants, or alternative screening procedures for the early detection of some cancers, which, in all cases, target a cohort of individuals of the same age.

[6]An "individual" could also be understood as the representative agent for a certain group.

[7]A could for instance refer to the resulting multidimensional health states after combining the levels of each dimension of a categorical measure, such as the so-called EQ-5D, in all possible ways.

[8]The model introduced in Hougaard, Moreno-Ternero and Østerdal (2013a), differs from this one in allowing zero lifetimes.

[9]For ease of exposition, we establish the notational convention that $h_S \equiv (h_i)_{i \in S}$, for each $S \subset N$.

[10]There are two interpretations for our model. One is to assume that individuals only experience chronic health states. Another is to consider that the health variable expresses some sort of average quality of life along the lifespan.

P represents $\succsim$ if

$$P(h) \geq P(h') \Leftrightarrow h \succsim h',$$

for each $h, h' \in H$. Note that if P represents $\succsim$ then any strictly increasing transformation of P would also do so.[11]

2.1 Structural axioms

We now list several structural axioms for social preferences that we endorse for population health evaluation functions. The first five were introduced in Hougaard, Moreno-Ternero and Østerdal (2013a), and, therefore, the reader is referred to that paper for further details about them.

First, the axiom saying that the evaluation of the population health should depend only on the list of quality-quantity duplets, not on who holds them. Formally, let Π^N denote the class of bijections from N into itself and, for each $h \in H$, and each $\pi \in \Pi^N$, let h_π denote the resulting profile after rearranging coordinates of h, according to π. Then,

Anonymity: $h \sim h_\pi$ for each $h \in H$, and each $\pi \in \Pi^N$.

The second axiom says that if the distribution of health in a population changes only for a subgroup of agents in the population, the relative evaluation of the two distributions should only depend on that subgroup. Formally,

Separability: $\left[h_S, h_{N\setminus S}\right] \succsim \left[h'_S, h_{N\setminus S}\right] \Leftrightarrow \left[h_S, h'_{N\setminus S}\right] \succsim \left[h'_S, h'_{N\setminus S}\right]$, for each $S \subseteq N$, and $h, h' \in H$.

Third is a standard technical condition, which says that, for fixed distributions of health states, small changes in lifetimes should not lead to large changes in the evaluation of the population health distribution. Formally,

Continuity: Let $h, h' \in H$, and $h^{(k)}$ be a sequence in H such that, for each $i \in N$, $h_i^{(k)} = (a_i, t_i^{(k)}) \to (a_i, t_i) = h_i$. If $h^{(k)} \succsim h'$ for each k then $h \succsim h'$, and if $h' \succsim h^{(k)}$ for each k then $h' \succsim h$.

[11] It is worth mentioning that our analysis does not deal with uncertainty. Following Broome (1993), we consider a formulation of the population health evaluation problem which contains no explicit element of risk, and in which we obtain characterizations of population health evaluation functions without assumptions on the policy maker's (or individuals') risk attitudes.

The next axiom says that replacing the health status of an agent by that of perfect health, ceteris paribus, cannot worsen the evaluation of the population health. Formally,

Perfect health superiority: $[(a_*, t_i), h_{N\setminus\{i\}}] \succsim h$, for each $h = [h_1, \ldots, h_n] \in H$ and $i \in N$.

The next one says that if each agent is at perfect health, increasing the time dimension is strictly better for the policy maker. Formally,

Time monotonicity at perfect health: If $t_i \geq t'_i$, for each $i \in N$, with at least one strict inequality, then $[(a_*, t_1), \ldots, (a_*, t_n)] \succ [(a_*, t'_1), \ldots, (a_*, t'_n)]$.

The last structural axiom we consider says that quality of life improvements become almost insignificant when lifetimes are negligible. More precisely, it says that any health profile will be strictly preferred to the resulting profile in which one agent changes to enjoy perfect health, during a sufficiently small lifetime, whereas the other duplets remain the same. Formally,

Negligible lifetimes condition: For each $h \in H$, and each $i \in N$, there exists $\varepsilon > 0$ such that $h \succ [(a_*, s), h_{N\setminus\{i\}}]$, for each $0 < s < \varepsilon$.

The previous axiom replaces the pair of axioms in Hougaard, Moreno-Ternero and Østerdal (2013a), made of the so-called *social zero condition* (described at the introduction), and the notion of *positive lifetime desirability* (society improves if any agent moves from zero lifetime to positive lifetime, for a given health state), none of which can be formalized in the current model, which does not allow for zero lifetimes.

In what follows, we refer to the set of axioms introduced above as our *structural axioms*. Our first result confirms the implications of the structural axioms.[12] They characterize the so-called *generalized* Healthy Years Equivalent (HYEs) population health evaluation function, in which HYEs are submitted to an arbitrary (but increasing) function before being added.

Generalized HYEs:

$$P^{gh}[h_1, \ldots, h_n] = P^{gh}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g(f(a_i, t_i)), \tag{1}$$

where $g : \mathbb{R}_{++} \to \mathbb{R}$ is a strictly increasing and continuous function, and $f : A \times T \to T$ is a function indicating the HYEs for each individual, i.e.,

[12] Theorem 1 is the counterpart of Theorem 1 in Hougaard, Moreno-Ternero and Østerdal (2013a), which characterizes in the model allowing for zero lifetimes the population health evaluation functions satisfying the first five structural axioms considered here, as well as the social zero condition and the axiom of positive life desirability.

- f is continuous with respect to its second variable,

- $0 < f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, and

- For each $h = [h_1, \ldots, h_n] = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

Theorem 1 *The policy maker's preferences satisfy the structural axioms, if and only if they can be represented by a generalized HYEs population health evaluation function.*

2.2 Core axioms

We now introduce three alternative axioms that will be combined, independently, to the list of structural axioms presented above. The three axioms convey a specific concern for relative comparisons of lifetimes, but each of them formalizes such a concern in a different way.

More precisely, the first one says that a proportional change in life years to individual i is socially seen as just a good as a change of life years to individual j in the same proportion, regardless of health states.[13] Formally,

Relative lifetime comparisons: For each $h \in H$, $c > 0$, and $i, j \in N$,

$$\left[(a_i, ct_i), h_{N \setminus \{i\}}\right] \sim \left[(a_j, ct_j), h_{N \setminus \{j\}}\right].$$

Now, we could restrict the scope of the previous axiom only to the case in which the two involved agents enjoy perfect health, giving rise to the following axiom. Formally,

Relative lifetime comparisons at perfect health: For each $h \in H$, $c > 0$, and $i, j \in N$,

$$\left[(a_*, ct_i), (a_*, t_j), h_{N \setminus \{i,j\}}\right] \sim \left[(a_*, t_i), (a_*, ct_j), h_{N \setminus \{i,j\}}\right].$$

Finally, we consider the axiom stating that if we have two health profiles, where each profile consists of individuals with common health duplets, then the preference between them is independent of a scaling of the life year component.[14] Formally,

Common duplets time scale invariance: For each $h = [(a,t)_{i \in N}], h' = [(a',t')_{i \in N}] \in H$, such that $h \succsim h'$, and $c > 0$, $[(a, ct)_{i \in N}] \succsim [(a', ct')_{i \in N}]$.

[13] This axiom was first formalized in a health context by Østerdal (2005).

[14] Note that this is a weak axiom as it only compares profiles in which all agents are endowed with the same duplet. In the next section, we shall explore the stronger axiom in which profiles might have different duplets.

3 The main results

We show in this section that some specific equity-oriented population health evaluation functions, defined next, can be characterized by some combinations of the axioms described in the previous section.

First, we introduce the population health evaluation function in which individual Quality Adjusted Life Years (QALYs) are multiplied to evaluate the health distribution of the population. More precisely,

Multiplicative QALYs:

$$P^{mq}[h_1,\ldots,h_n] = P^{mq}[(a_1,t_1),\ldots,(a_n,t_n)] = \prod_{i=1}^{n}\left(q(a_i)t_i\right), \tag{2}$$

where $q : A \to [0,1]$ is an arbitrary function satisfying $0 < q(a_i) \leq q(a_*) = 1$, for all $a_i \in A$.

Alternatively, we could consider the more general population health evaluation function in which HYEs, instead of QALYs, are multiplied to evaluate the health distribution of the population. Formally,

Multiplicative HYEs:

$$P^{mh}[h_1,\ldots,h_n] = P^{mh}[(a_1,t_1),\ldots,(a_n,t_n)] = \prod_{i=1}^{n} f(a_i,t_i), \tag{3}$$

where f is constructed as in (1)

It is straightforward to see that the multiplicative QALY population health evaluation function can therefore be seen as a specific instance of the multiplicative HYE population health evaluation function, in which $f(a_i,t_i) = q(a_i)t_i$, for each $(a_i,t_i) \in A \times T$.

At the risk of stressing the obvious, note that the previous two families endorse a concern for the equity of the distribution of QALYs or HYEs (more specifically, a concern for the existence of agents with poor outcomes), which is absent in their counterpart families that evaluate a health distribution with the (unweighted) sum of the QALYs or HYEs in the population.

As we mentioned in Section 2, population health evaluation functions are "immune" to monotonic transformations. More precisely, if P represents $\succsim$ then any strictly increasing transformation of P would also do so. Thus, it is straightforward to see that the following are equivalent representations of families (2) and (3):

$$P^{mq}[h_1,\ldots,h_n] = P^{mq}[(a_1,t_1),\ldots,(a_n,t_n)] = \sum_{i=1}^{n}\ln\left(q(a_i)t_i\right),$$

where q is constructed as in (2).

$$P^{mh}[h_1, \ldots, h_n] = P^{ph}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} \ln\left(f(a_i, t_i)\right),$$

where f is constructed as in (1).

A natural generalization of the above families would be obtained when QALYs (or HYEs) are submitted to an arbitrary (but increasing) function before being added. Formally,

Generalized QALYs:

$$P^{gq}[h_1, \ldots, h_n] = P^{gq}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g(q(a_i)t_i), \tag{4}$$

where $g : \mathbb{R}_{++} \to \mathbb{R}$ is a strictly increasing and continuous function, and q is constructed as in (2).

We are now ready to state the main results of our paper. The first result says that the multiplicative QALY population health evaluation function is characterized when relative lifetime comparisons is added to the structural axioms.[15] Formally,

Theorem 2 *The policy maker's preferences satisfy relative lifetime comparisons, and the structural axioms, if and only if they can be represented by the multiplicative QALYs population health evaluation function.*

Theorem 3 shows that the multiplicative HYE population health evaluation function is characterized when, instead of relative lifetime comparisons, only its weakening to perfect health is added to the set of structural axioms. Formally,

Theorem 3 *The policy maker's preferences satisfy relative lifetime comparisons at perfect health, and the structural axioms, if and only if they can be represented by the multiplicative HYEs population health evaluation function.*

Similarly, Theorem 4 shows that the *generalized* QALY population health evaluation function, P^{gq}, is characterized when common duplets time scale invariance is the added axiom to the set of structural axioms. Formally,

[15] The same functional form was characterized by Østerdal (2005) in a model in which individual QALY functions (representing individual preferences) are given.

Theorem 4 *The policy maker's preferences satisfy common duplets time scale invariance, and the structural axioms, if and only if they can be represented by a generalized QALYs population health evaluation function.*

One might argue that the generalized families (1) and (4), characterized in Theorems 1 and 4, respectively, do not necessarily include a concern for the equality of the distribution (as it indeed happens for the "logarithmic members" characterized in Theorems 2 and 3). The following results exhibit the implications of adding a concern for inequality aversion to both families. More precisely, we consider the axiom stating that a health profile in which two agents at perfect health have different time spans is dominated by the subsequent profile in which those agents keep the same perfect health status, but share a time span equal to the average of the former two. Formally,

Pigou-Dalton transfer at perfect health: For each $h = [(a_*, t_k)_{k\in N}] \in H$, and $i, j \in N$, such that $t_i \neq t_j$,

$$\left[\left(a_*, \frac{t_i + t_j}{2}\right), \left(a_*, \frac{t_i + t_j}{2}\right), h_{N\setminus\{i,j\}}\right] \succ h.$$

As shown in the next two statements, the addition of this new axiom to the list of axioms characterizing the general families, imposes that QALYs (HYEs) enter into the population health evaluation function in a (strictly) concave way.

Corollary 1 *The policy maker's preferences satisfy Pigou-Dalton transfer at perfect health, common duplets time scale invariance, and the structural axioms, if and only if they can be represented by a generalized concave QALYs population health evaluation function.*

Corollary 2 *The policy maker's preferences satisfy Pigou-Dalton transfer at perfect health and the structural axioms, if and only if they can be represented by a generalized concave HYEs population health evaluation function.*

4 Further insights

As mentioned in the introduction, the so-called *fair innings* notion is usually invoked to develop equity-sensitive forms of evaluating a distribution of health. One might argue that the fair innings notion is captured by Williams (1997) upon endorsing a *Bergsonian* functional form to evaluate the health distribution of a population. In the parlance of this paper, that would amount to consider the subfamilies arising from (1) and (4) after imposing that g is, not only a strictly increasing, continuous, and concave function (as in Corollaries 1 and 2), but also a power function. We show in this section how those families could also be characterized when adding new axioms to the set of structural axioms considered above. Nevertheless, it is worth acknowledging explicitly that, whereas the notion of a *threshold* plays a crucial role in the fair innings reasoning, it does not play any role in our formalization.

Let us start introducing the following two axioms, which were also part of the analysis in Hougaard, Moreno-Ternero and Østerdal (2013a), for the model considered therein. The first axiom says that the ranking of a pair of population health distributions does not reverse when all lifetimes are multiplied by a common positive constant. The second axiom restricts the scope of the first one to the case in which agents enjoy perfect health. Formally,

Time scale independence: For each $c > 0$, and $h = [(a_i, t_i)_{i\in N}]$, $h' = [(a'_i, t'_i)_{i\in N}]$,

$$h \succsim h' \Rightarrow [(a_i, ct_i)_{i\in N}] \succsim [(a'_i, ct'_i)_{i\in N}].$$

Time scale independence at perfect health: For each $c > 0$, and $h = [(a_*, t_i)_{i\in N}]$, $h' = [(a_*, t'_i)_{i\in N}]$,

$$h \succsim h' \Rightarrow [(a_*, ct_i)_{i\in N}] \succsim [(a_*, ct'_i)_{i\in N}].$$

We now formally define the *power* versions of the QALYs and HYEs population health evaluation functions considered above:

Power QALYs:

$$P^{pq}[h_1, \ldots, h_n] = P^{pq}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} (q(a_i)t_i)^{\gamma}, \tag{5}$$

where γ is a positive scalar, and q is constructed as in (2).

Power HYEs:

$$P^{ph}[h_1, \ldots, h_n] = P^{ph}[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} f(a_i, t_i)^{\gamma}, \tag{6}$$

where γ is a positive scalar, and f is constructed as in (1).

Two other somewhat polar families are the following:

Negative power QALYs:

$$P^{npq}[h_1, \ldots, h_n] = P^{pq}[(a_1, t_1), \ldots, (a_n, t_n)] = -\sum_{i=1}^{n} (q(a_i)t_i)^{\delta}, \tag{7}$$

where δ is a negative scalar, and q is constructed as in (2).

Negative power HYEs:

$$P^{nph}[h_1, \ldots, h_n] = P^{ph}[(a_1, t_1), \ldots, (a_n, t_n)] = -\sum_{i=1}^{n} f(a_i, t_i)^{\delta}, \tag{8}$$

where δ is a negative scalar, and f is constructed as in (1).

We obtain the following results:

Theorem 5 *The policy maker's preferences satisfy time scale invariance and the structural axioms, if and only if they can be represented by one of the following population health evaluation functions:*

1. *Power QALYs.*
2. *Multiplicative QALYs.*
3. *Negative power QALYs.*

Theorem 6 *The policy maker's preferences satisfy time scale invariance at perfect health, and the structural axioms, if and only if they can be represented by one of the following population health evaluation functions:*

1. *Power HYEs.*
2. *Multiplicative HYEs.*
3. *Negative power HYEs.*

The previous two results are of a different nature to Theorems 4 and 5 in Hougaard, Moreno-Ternero and Østerdal (2013a), which characterize the Power QALYs and HYEs population health evaluation functions in the domain allowing for zero lifetimes. This is due to the fact that the other two functional forms in the above statements do not pass the test of imposing (right) continuity at zero lifetimes.

As stated in the next two corollaries, and similarly to what we did in Section 3, the addition of the *Pigou-Dalton transfer at perfect health* axiom, to the previous results, imposes that QALYs (HYEs) enter into the power families in a (strictly) concave way, i.e., $0 < \gamma < 1$.

Corollary 3 *The policy maker's preferences satisfy Pigou-Dalton transfer at perfect health, time scale invariance, and the structural axioms, if and only if they can be represented by one of the following population health evaluation functions:*

1. *Power concave QALYs (i.e., power QALYs for $0 < \gamma < 1$).*

2. *Multiplicative QALYs.*

3. *Negative power QALYs.*

Corollary 4 *The policy maker's preferences satisfy Pigou-Dalton transfer at perfect health, time scale invariance at perfect heath, and the structural axioms, if and only if they can be represented by one of the following population health evaluation functions:*

1. *Power concave HYEs (i.e., power HYEs for $0 < \gamma < 1$).*

2. *Multiplicative HYEs.*

3. *Negative power HYEs.*

Our last results will strengthen the previous ones upon adding one of the following three related axioms, each conveying a specific attitude with respect to time tradeoffs at perfect health.

Overall time biased tradeoff at perfect health: For each $t \in T \setminus \{1\}$, $h \in H$, and $i, j \in N$,

$$\left[(a_*, t), \left(a_*, \frac{1}{t}\right), h_{N\setminus\{i,j\}}\right] \succ \left[(a_*, 1), (a_*, 1), h_{N\setminus\{i,j\}}\right].$$

Neutral time tradeoff at perfect health: For each $t \in T$, $h \in H$, and $i, j \in N$,

$$\left[(a_*, t), \left(a_*, \frac{1}{t}\right), h_{N\setminus\{i,j\}}\right] \sim \left[(a_*, 1), (a_*, 1), h_{N\setminus\{i,j\}}\right].$$

Egalitarian time biased tradeoff at perfect health: For each $t \in T \setminus \{1\}$, $h \in H$, and $i, j \in N$,

$$\left[(a_*, t), \left(a_*, \frac{1}{t}\right), h_{N\setminus\{i,j\}}\right] \prec \left[(a_*, 1), (a_*, 1), h_{N\setminus\{i,j\}}\right].$$

The next result disentangles Corollary 3.

Theorem 7 *The following statements hold:*

- *The policy maker's preferences satisfy overall time biased tradeoff at perfect health, Pigou-Dalton transfer at perfect health, time scale invariance, and the structural axioms, if and only if they can be represented by a power concave QALYs population health evaluation function.*

- *The policy maker's preferences satisfy neutral time tradeoff at perfect health, time scale invariance, and the structural axioms, if and only if they can be represented by the multiplicative QALYs population health evaluation function.*

- *The policy maker's preferences satisfy egalitarian time biased tradeoff at perfect health, time scale invariance, and the structural axioms, if and only if they can be represented by a negative power QALYs population health evaluation function.*

Similarly, the next result disentangles Corollary 4.

Theorem 8 *The following statements hold:*

- *The policy maker's preferences satisfy overall time biased tradeoff at perfect health, Pigou-Dalton transfer at perfect health, time scale invariance at perfect health, and the structural axioms, if and only if they can be represented by a power concave HYEs population health evaluation function.*

- *The policy maker's preferences satisfy neutral time tradeoff at perfect health, time scale invariance at perfect health, and the structural axioms, if and only if they can be represented by the multiplicative HYEs population health evaluation function.*

- *The policy maker's preferences satisfy egalitarian time biased tradeoff at perfect health, time scale invariance at perfect health, and the structural axioms, if and only if they can be represented by a negative power HYEs population health evaluation function.*

5 Discussion

We have presented in this paper normative foundations for several equity-sensitive population health evaluation functions. All of them share the common ground given by our structural axioms. The normative appeal of those structural axioms seems to be strong, but we have not tested their positive appeal via experiments or questionnaires, and that could certainly be a plausible line for future research.[16] Beyond those structural axioms, the main population health

[16] Amiel and Cowell (1999) provide empirical evidence in which respondents of questionnaires related to income inequality measurement reject separability. Turpcu et al., (2012) show the existence of framing effects in the empirical support for the axiom in a health context. Nevertheless, it seems that separability is well established in the health economics literature, as it underlies the use of incremental analysis in cost-effectiveness analysis, which implies that individuals for whom two treatments yield the same health should not influence the relative

evaluation functions we single out differ from each other on the specific form of *relative lifetime comparisons* they allow.[17] We have also provided normative foundations for other population health evaluation functions (including some capturing the notion of fair innings) resorting to axioms reflecting independence of the time scale.

To conclude, it is worth mentioning that our analysis is silent regarding the specific functional form one should adopt for the quality function $q : A \to [0,1]$ or the HYE function $f : A \times T \to T$. This issue is left for future research. A plausible avenue to explore it would be to extend our whole analysis in this paper to the case in which the mathematical structure of the domain of health states A is more specific, which would allow formalizing new axioms. For instance, if all the health states in the set A are assumed to be objectively ranked from worst to best, we could formalize, in the spirit of Hammond (1976), that a "health transfer" between two agents with equal lifetimes (from the one with the health state ranked higher) would be welcomed. Such a resulting axiom would translate into a further mathematical condition of the quality function $q : A \to [0,1]$ (or the HYE function $f : A \times T \to T$). It is likely to expect that the combination of new axioms of this sort would drive us towards specific functional form. In practice, these functions can be elicited directly via *person trade-offs*, which aim to derive quality weights by means of questionnaires in which respondents face hypothetical tradeoffs regarding different health profiles (e.g., Patrick, Bush and Chen, 1973).

6 Appendix. Proofs of theorems

Proof of Theorem 1. We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies the *structural axioms*. Let $h \in H$ and $i \in N$. Then, there exists $t_i^* \in T$ such that $h \sim [(a_*, t_i^*), h_{N\setminus\{i\}}]$. Assume otherwise. Then, $T = A \cup B$, where,

$$A = \{s \in T | h \succ [(a_*, s), h_{N\setminus\{i\}}]\},$$

and

$$B = \{s \in T | [(a_*, s), h_{N\setminus\{i\}}] \succ h\}.$$

By the *negligible lifetimes condition*, $A \neq \varnothing$. By *perfect health superiority* $B \neq \varnothing$. By *continuity*, A and B are open sets relative to T. As $A \cap B = \varnothing$, it would follow that T is not a connected set, a contradiction.

evaluation of these treatments (e.g., Gold et al., 1996). As for continuity, it discards *leximin* evaluations, which may have intuitive appeal for some observers.

[17] The implications of absolute lifetime comparisons is explored in Moreno-Ternero and Østerdal (2015).

Now, by *separability*, t_i^* only depends on (a_i, t_i) (and, thus, is independent of the remaining duplets of the profile). Thus, for each $i = 1, \ldots, n$, let $f_i : A \times T \to T$ be defined such that $f_i(a_i, t_i) = t_i^*$, for each $(a_i, t_i) \in A \times T$. By *anonymity*, $f_i(\cdot, \cdot) \equiv f_j(\cdot, \cdot) \equiv f(\cdot, \cdot)$, for each $i, j \in N$. By *time monotonicity at perfect health* and *perfect health superiority*, $0 < f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$ and, by *continuity*, f is a continuous function with respect to its second variable. Furthermore,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

which implicitly says that social preferences only depend on the profile of healthy years equivalent. By *separability* and *continuity*, the evaluation of the healthy years equivalents is separable and continuous. It also follows that the range of f is a connected subset of $\mathbb{R}$. Thus, by Theorem 3 in Debreu (1960), there exists a strictly increasing and continuous function $g : \mathbb{R}_+ \to \mathbb{R}$ such that

$$h \succsim h' \iff \sum_{i=1}^{n} g\left(f(a_i, t_i)\right) \geq \sum_{i=1}^{n} g\left(f(a_i', t_i')\right),$$

which concludes the proof. ■

Proof of Theorem 2 We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies the *structural axioms* and *relative lifetime comparisons*. Then, by Theorem 1, $\succsim$ can be represented by a population health evaluation function satisfying (1).

By iterated application of *relative lifetime comparisons*, and the transitivity of $\succsim$,

$$g(f(a_1, t_1)) + ... + g(f(a_n, t_n)) = g\left(f\left(a_1, \prod_1^n t_i\right)\right) + g(f(a_2, 1)) + ... + g(f(a_n, 1)).$$

For a fixed common health state $\bar{a}$, $g(f(\bar{a}, \cdot))$ therefore satisfies the following functional equation:

$$g(f(\bar{a}, t_1)) + g(f(\bar{a}, t_2)) = g(f(\bar{a}, t_1 t_2)) + g(f(\bar{a}, 1)),$$

for all $t_1, t_2 > 0$. Let $r : A \times \mathbb{R} \to T$ be the function such that $r(x, y) = g(f(x, \exp(y)))$ for each $(x, y) \in A \times \mathbb{R}$. Thus, for each fixed common health state $\bar{a} \in A$ and any $t_1, t_2 \in \mathbb{R}$, we have

$$r(\bar{a}, t_1 + t_2) + r(\bar{a}, 0) = r(\bar{a}, t_1) + r(\bar{a}, t_2), \tag{9}$$

which is precisely one of Cauchy's canonical functional equations. As r is continuous, it follows that the unique solutions to such an equation are the linear functions (e.g., Aczel, 2006; page 43). More precisely, there exist two functions $\alpha : A \to \mathbb{R}$ and $\beta : A \to \mathbb{R}$ such that

$$g(f(\bar{a}, t)) = r(\bar{a}, \ln t) = \alpha(\bar{a}) \ln t + \beta(\bar{a}),$$

for each $t \in T$.

Now, by *relative lifetime comparisons*, it follows that, for each $\bar{a}, \bar{a}' \in A$,

$$\begin{aligned} g(f(\bar{a}, t_1)) + g(f(\bar{a}', t_2)) &= g(f(\bar{a}, t_1 t_2)) + g(f(\bar{a}', 1)) \\ &= g(f(\bar{a}, 1)) + g(f(\bar{a}', t_1 t_2)). \end{aligned}$$

Thus, $\alpha(\bar{a}) = \alpha(\bar{a}') = \alpha$, and, therefore,

$$P((a_1, t_1), ..., (a_n, t_n)) = \alpha \left(\sum_{i=1}^{n} \ln(t_i) \right) + \sum_{i=1}^{n} \beta(a_i).$$

To conclude, let $q : A \to \mathbb{R}$ be such that $q(a) = \exp\left(\frac{\beta(a) - \beta(a_*)}{\alpha}\right)$, for each $a \in A$. By *perfect health superiority*, it follows that $0 < q(a) \leq q(a_*) = 1$, for all $a \in A$. Now, as the population health evaluation function is uniquely determined, up to strictly increasing transformations, we can consider the monotonic transformation of P, $P' = \exp\left(\frac{P - n\beta(a_*)}{\alpha}\right)$. Then,

$$\begin{aligned} P'((a_1, t_1), ..., (a_n, t_n)) &= \exp((P((a_1, t_1), ..., (a_n, t_n)) - n\beta(a_*))/\alpha) \\ &= \exp\left(\sum_{i=1}^{n} \ln t_i + \sum_{i=1}^{n} \left(\frac{\beta(a) - \beta(a_*)}{\alpha} \right) \right) \\ &= \prod_{i=1}^{n} q(a_i) t_i, \end{aligned}$$

as desired. ■

Proof of Theorem 3 We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies the *structural axioms* and *relative lifetime comparisons at perfect health*. Then, by Theorem 1, $\succsim$ can be represented by a population health evaluation function satisfying (1). Let $h = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \ldots, (a'_n, t'_n)] \in H$. Then, by iterated application of *relative lifetime comparisons at perfect health*, and the transitivity of $\succsim$,

$$h \succsim h' \iff [(a_*, \prod_{i \in N} f(a_i, t_i)), (a_*, 1)_{k \in N \setminus \{i\}}] \succsim [(a_*, \prod_{i \in N} f(a'_i, t'_i)), (a_*, 1)_{k \in N \setminus \{i\}}].$$

By *time monotonicity at perfect health*, and the transitivity of $\succsim$,

$$h \succsim h' \iff \prod_{i \in N} f(a_i, t_i) \geq \prod_{i \in N} f(a'_i, t'_i),$$

as desired. ■

Proof of Theorem 4 We focus on its non-trivial implication. Formally, assume $\succsim$ satisfies the *structural axioms* and *common duplets time scale invariance*. Then, by Theorem 1, $\succsim$ can be represented by a population health evaluation function satisfying (1). We now make two further claims.

Claim 1. We claim that for each $(a,t),(a',t') \in A \times T$, and $c>0$,

$$f(a,t) \geq f(a',t') \iff f(a,ct) \geq f(a',ct').$$

To prove the claim, let $h = [(a,t),\ldots,(a,t)], h' = [(a',t'),\ldots,(a',t')] \in H$ and $c>0$. Denote $h_c = [(a,ct),\ldots,(a,ct)]$ and $h'_c = [(a',ct'),\ldots,(a',ct')]$. By (1),

$$h \succsim h' \iff f(a,t) \geq f(a',t'),$$

and

$$h_c \succsim h'_c \iff f(a,ct) \geq f(a',ct').$$

By *common duplets time scale invariance*, the claim follows.

Claim 2. Let $q : A \to \mathbb{R}$ be such that $q(a) = f(a,1)$, for each $a \in A$. We claim that

$$f(a,t) \geq f(a',t') \iff q(a)t \geq q(a')t',$$

for each $(a,t),(a',t') \in A \times T$.

In order to prove the claim note that, by definition, $f(a_i,1) = f(a_*, q(a_i))$. By Claim 1,

$$f(a,t) = f(a',t') \iff f(a,ct) = f(a',ct').$$

Thus, $f(a,t) \geq f(a',t') \iff f(a_*, q(a)t) \geq f(a_*, q(a')t') \iff q(a)t \geq q(a')t'$, as desired.

By Claim 2, it follows that $f(\cdot,\cdot)$ is a monotonic transformation of the function $\tau : A \times T \to \mathbb{R}$ defined by $\tau(a,t) = q(a)t$, for each $(a,t) \in A \times T$. Then, by the above, P^{gq} represents $\succsim$, as desired. ■

Proof of Corollary 1 As before, we focus on the non-trivial implication. Formally, assume $\succsim$ satisfies the *structural axioms*, *common duplets time scale invariance*, and *Pigou-Dalton transfer at perfect health*. Then, by Theorem 4, $\succsim$ can be represented by a population health evaluation function satisfying (4).

Let $i,j \in N$ and consider the two health profiles $h = [(a_*, t_k)_{k \in N}]$, where $t_i \neq t_j$, and $h' = \left[\left(a_*, \frac{t_i+t_j}{2}\right), \left(a_*, \frac{t_i+t_j}{2}\right), h_{N \setminus \{i,j\}}\right]$. By *Pigou-Dalton transfer at perfect health*, $h' \succ h$, which, by (4), means that

$$2g\left(q(a_*)\frac{t_i+t_j}{2}\right) + \sum_{k \in N \setminus \{i,j\}} g(q(a_*)t_k) > g(q(a_*)t_i) + g(q(a_*)t_j) + \sum_{k \in N \setminus \{i,j\}} g(q(a_*)t_k).$$

Or, equivalently (as $q(a_*) = 1$),

$$g\left(\frac{t_i + t_j}{2}\right) > \frac{g(t_i)}{2} + \frac{g(t_j)}{2},$$

from where it follows that g is strictly concave, as desired. ∎

Proof of Corollary 2 As before, we focus on the non-trivial implication. Formally, assume $\succsim$ satisfies the *structural axioms* and *Pigou-Dalton transfer at perfect health*. Then, by Theorem 1, $\succsim$ can be represented by a population health evaluation function satisfying (1).

Let $i, j \in N$ and consider the two health profiles $h = [(a_*, t_k)_{k\in N}]$, where $t_i \neq t_j$, and $h' = \left[\left(a_*, \frac{t_i+t_j}{2}\right), \left(a_*, \frac{t_i+t_j}{2}\right), h_{N\setminus\{i,j\}}\right]$. By *Pigou-Dalton transfer at perfect health*, $h' \succ h$, which, by (1), means that

$$2g\left(f\left(a_*, \frac{t_i + t_j}{2}\right)\right) + \sum_{k\in N\setminus\{i,j\}} g(f(a_*, t_k)) > g(f(a_*, t_i)) + g(f(a_*, t_j)) + \sum_{k\in N\setminus\{i,j\}} g(f(a_*, t_k)).$$

Or, equivalently (as $f(a_*, t) = t$, for each $t \in T$),

$$g\left(\frac{t_i + t_j}{2}\right) > \frac{g(t_i)}{2} + \frac{g(t_j)}{2},$$

from where it follows that g is strictly concave, as desired. ∎

Proof of Theorem 5. We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies *time scale invariance* and the *structural axioms*. Then, by Theorem 1, $\succsim$ can be represented by a population health evaluation function satisfying (1).

Let $\bar{q} : A \to \mathbb{R}$ be such that $\bar{q}(a) = f(a, 1)$, for each $a \in A$. Let P denote the population health evaluation function defined by

$$P[h_1, \ldots, h_n] = P[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} g\left(\bar{q}(a_i)t_i\right).$$

By an analogous argument to that in the proof of Theorem 4 at Hougaard, Moreno-Ternero and Østerdal (2013a), we obtain that

$$\sum_{i=1}^{n} g\left(\bar{q}(a_i)t_i\right) \geq \sum_{i=1}^{n} g\left(\bar{q}(a'_i)t'_i\right) \iff \sum_{i=1}^{n} g\left(\bar{q}(a_i)ct_i\right) \geq \sum_{i=1}^{n} g\left(\bar{q}(a'_i)ct'_i\right),$$

for each $h = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$, $h' = [(a'_1, t'_1), \ldots, (a'_n, t'_n)] \in H$ and $c > 0$.

By Bergson and Samuelson (e.g., Burk, 1936; Samuelson, 1965; Moulin, 1988), there are only three possible functional forms for P:

- $P[h_1, \ldots, h_n] = P[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} \alpha_i \left(\bar{q}(a_i)t_i\right)^{\gamma}$,

- $P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = -\sum_{i=1}^{n} \alpha_i \left(\bar{q}(a_i)t_i\right)^{\delta}$,
- $P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = \sum_{i=1}^{n} \alpha_i \log\left(\bar{q}(a_i)t_i\right)$,

for some $\gamma > 0$, $\delta < 0$ and $\alpha_i > 0$ for each $i \in N$. By *anonymity*, $\alpha_i = \alpha_j$ for each $i, j \in N$, which concludes the proof. ■

Proof of Theorem 6. We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies *time scale invariance at perfect health* and the *structural axioms*. Then, by Theorem 1, $\succsim$ can be represented by a population health evaluation function satisfying (1).

By *time scale invariance at perfect health*,

$$\sum_{i=1}^{n} g\left(f(a_i,t_i)\right) \geq \sum_{i=1}^{n} g\left(f(a_i',t_i')\right) \iff \sum_{i=1}^{n} g\left(cf(a_i,t_i)\right) \geq \sum_{i=1}^{n} g\left(cf(a_i',t_i')\right),$$

for each $h = [(a_1,t_1),\ldots,(a_n,t_n)] \in H$, $h' = [(a_1',t_1'),\ldots,(a_n',t_n')] \in H$ and $c > 0$.

As in the proof of the previous theorem, by Bergson and Samuelson, and anonymity, it follows that $\succsim$ is represented by one of the following functional forms:

- $P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = \sum_{i=1}^{n} \left(f(a_i,t_i)\right)^{\gamma}$,
- $P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = -\sum_{i=1}^{n} \left(f(a_i,t_i)\right)^{\delta}$,
- $P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = \sum_{i=1}^{n} \log\left(f(a_i,t_i)\right)$,

for some $\gamma > 0$ and $\delta < 0$, for each $i \in N$, as desired. ■

Proof of Corollary 3. By Theorem 5, it only remains to show that only the concave functions of the power QALYs family satisfy Pigou-Dalton transfer at perfect health. Now, let $i, j \in N$ and consider the two health profiles $h = [(a_*, t_k)_{k\in N}]$, where $t_i \neq t_j$, and $h' = \left[\left(a_*, \frac{t_i+t_j}{2}\right), \left(a_*, \frac{t_i+t_j}{2}\right), h_{N\setminus\{i,j\}}\right]$. By *Pigou-Dalton transfer at perfect health*, $h' \succ h$, which, if preferences are represented by a member of the power QALYs family, means that

$$2q(a_*)\left(\frac{t_i+t_j}{2}\right)^{\gamma} + \sum_{k\in N\setminus\{i,j\}} q(a_*)t_k^{\gamma} > q(a_*)t_i^{\gamma} + q(a_*)t_j^{\gamma} + \sum_{k\in N\setminus\{i,j\}} q(a_*)t_k^{\gamma}.$$

Or, equivalently (as $q(a_*) = 1$),

$$\left(\frac{t_i+t_j}{2}\right)^{\gamma} > \frac{t_i^{\gamma}}{2} + \frac{t_j^{\gamma}}{2},$$

from where it follows that $0 < \gamma < 1$, as desired. ■

Proof of Corollary 4. By Theorem 6, it only remains to show that only the concave functions of the power HYEs family satisfy Pigou-Dalton transfer at perfect health. Now, let $i, j \in N$ and consider the two health profiles $h = [(a_*, t_k)_{k\in N}]$, where $t_i \neq t_j$, and $h' = \left[\left(a_*, \frac{t_i+t_j}{2}\right), \left(a_*, \frac{t_i+t_j}{2}\right), h_{N\setminus\{i,j\}}\right]$. By *Pigou-Dalton transfer at perfect health*, $h' \succ h$, which, by (6), means that

$$2f\left(a_*, \left(\frac{t_i+t_j}{2}\right)\right)^{\gamma} + \sum_{k\in N\setminus\{i,j\}} f(a_*, t_k)^{\gamma} > f(a_*, t_i)^{\gamma} + f(a_*, t_j)^{\gamma} + \sum_{k\in N\setminus\{i,j\}} f(a_*, t_k)^{\gamma}.$$

Or, equivalently (as $f(a_*, t) = t$, for each $t \in T$),

$$\left(\frac{t_i+t_j}{2}\right)^{\gamma} > \frac{t_i^{\gamma}}{2} + \frac{t_j^{\gamma}}{2},$$

from where it follows that $0 < \gamma < 1$, as desired. ■

The following technical lemma will be needed to prove the results presented next.

Lemma 1 *For each $t \in T \setminus \{1\}$, and each $\gamma \neq 0$, $t^{\gamma} + \frac{1}{t^{\gamma}} > 2$.*

Proof. Let $t \in T \setminus \{1\}$. Without loss of generality, assume $t > 1$. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f(\gamma) = t^{\gamma} + \frac{1}{t^{\gamma}}$. Thus, f is a continuous and derivable function. Furthermore, $f'(\gamma) = \left(t^{\gamma} - \frac{1}{t^{\gamma}}\right)\log t$. Thus $\gamma f'(\gamma) > 0$, which implies that $f(\cdot)$ is strictly decreasing for $\gamma < 0$ and strictly increasing for $\gamma > 0$. Thus, $f(\gamma) > f(0) = 2$, for each $\gamma \neq 0$, as desired. ■

Proposition 1 *The following statements hold:*

- *The power QALYs and HYEs population health evaluation functions satisfy overall time biased tradeoff at perfect health.*
- *The multiplicative QALYs and HYEs population health evaluation functions satisfy neutral time tradeoff at perfect health.*
- *The negative power QALYs and HYEs population health evaluation functions satisfy egalitarian time biased tradeoff at perfect health.*

Proof. The second statement is obvious, so we concentrate on the other two. Let $t \in T \setminus \{1\}$, $h \in H$, and $i, j \in N$. Then, as $q(a_*) = 1$, and $f(a_*, t) = t$, for each $t \in T$, it follows

$$P^{pq}\left[(a_*, t), \left(a_*, \frac{1}{t}\right), h_{N\setminus\{i,j\}}\right] > P^{pq}\left[(a_*, 1), (a_*, 1), h_{N\setminus\{i,j\}}\right]$$

if and only if

$$P^{ph}\left[(a_*,t),\left(a_*,\frac{1}{t}\right),h_{N\setminus\{i,j\}}\right] > P^{ph}\left[(a_*,1),(a_*,1),h_{N\setminus\{i,j\}}\right]$$

if and only if

$$t^{\gamma}+\frac{1}{t^{\gamma}}>2,$$

which follows by Lemma 1.

Similarly,

$$P^{mq}\left[(a_*,t),\left(a_*,\frac{1}{t}\right),h_{N\setminus\{i,j\}}\right] > P^{mq}\left[(a_*,1),(a_*,1),h_{N\setminus\{i,j\}}\right]$$

if and only if

$$P^{mh}\left[(a_*,t),\left(a_*,\frac{1}{t}\right),h_{N\setminus\{i,j\}}\right] > P^{mh}\left[(a_*,1),(a_*,1),h_{N\setminus\{i,j\}}\right]$$

if and only if

$$-t^{\delta}-\frac{1}{t^{\delta}}<-2,$$

which also follows by Lemma 1. ■

To conclude, Theorem 7 is a direct consequence of Corollary 3 and Proposition 1, whereas Theorem 8 is a direct consequence of Corollary 4 and Proposition 1.

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