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Allocating extra revenues from broadcasting sports leagues

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Keywords: resource allocation, broadcasting, sport leagues, axioms, extra revenues.

JEL Classification: D63, C71, Z20.

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Allocating extra revenues from broadcasting sports leagues*

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Abstract

We consider the problem of sharing the revenues from broadcasting sports leagues among participating teams. We introduce axioms formalizing alternative ways of allocating the extra revenue obtained from additional viewers. We show that, combined with some other standard axioms, they provide axiomatic characterizations of three focal rules for this problem: the *uniform* rule, the *equal-split* rule and *concede-and-divide*.

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1 Introduction

In the era of streaming, sports has become the cornerstone to television programming. The popularity of televised sports events keeps increasing and, for sports organizations, the sale of broadcasting and media rights is currently their biggest source of revenue. This sale is often collective, which generates an interesting problem of resource allocation, akin to well-known problems already analyzed in the game-theory literature. Instances are airport problems (e.g., Littlechild and Owen, 1973; Hu et al., 2012), bankruptcy problems (e.g., O'Neill, 1982; Thomson, 2019), telecommunications problems (e.g., van den Nouweland et al., 1996), museum pass problems (e.g., Ginsburgh and Zang, 2003; Bergantiños and Moreno-Ternero, 2015), cost sharing in minimum cost spanning tree problems (e.g., Bergantiños and Vidal-Puga, 2007; Trudeau, 2012), or labelled network games (e.g., Algaba et al., 2019a, 2019b).

In a recent paper (Bergantiños and Moreno-Ternero, 2020), we introduced a formal model to analyze the problem of sharing the revenues from broadcasting sports leagues among participating teams. Two main rules were highlighted therein. On the one hand, the so-called *equal-split* rule, which splits the revenue generated from each game equally among the participating players (teams). On the other hand, the so-called *concede-and-divide*, which concedes each player (team) the revenues generated from its fan base (properly estimated) and divides equally the residual. Among other things, we showed that both rules are similarly characterized by just three properties. Two properties are common in both characterizations. One (*equal treatment of equals*) states that two teams with the same audiences should receive the same amount; another (*additivity*) that revenues should be additive on the audience table. The third property in each characterization comes from a pair of polar properties modeling the effect of *null* or *essential* teams. The *null team* property states that if each game played by a team has no audience, then such a team (called null) receives nothing. The *essential team* property states that if only the games played by one team have positive audience, then such a team (called essential) receives all its audience. In a follow-up paper (Bergantiños and Moreno-Ternero, 2019) we show that a third axiom (*maximum aspirations*) stating that each team receives at most the revenue generated by its overall audience, together with *equal treatment of equals* and *additivity*, characterizes the family of all rules generated by convex combinations of the *equal-split* rule and *concede-and-divide*.

A natural third rule (outside from the previous family) can also be considered for this model.

It is the rule that divides the overall revenues generated in the tournament equally among all participating teams. We refer to it as the *uniform rule*. This rule is used quite often in practice. For instance, the football competitions of England, Italy and Spain divide around one half of the revenues generated by TV broadcasting equally among all teams.

In this paper, we further explore the axiomatic approach to this problem and derive new interesting results that uncover the structure of this stylized model further. To do so, we consider new axioms that formalize alternative ways of allocating the extra revenue obtained from additional viewers.

On the one hand, we consider a group of axioms stating different ways in which a rule should react when additional viewers of some specific team appear. More precisely, assume that a given tournament has more viewers than another tournament just because the games involving a specific team (i) have more viewers. How should a rule allocate those extra viewers? Our axioms consider three possible answers. The first axiom just ignores the fact that all viewers come from games involving team i. Then, all teams should equally share the extra benefits. We show that this axiom, together with *equal treatment of equals*, characterizes the *uniform rule*. The second axiom considers that team i and the rest of the teams are in a symmetric position because the audience of team i has increased the same amount as the audience of the rest of the teams (combined). Then, the extra benefits of team i should be equal to the sum of the extra benefits of the remaining teams. We show that this second axiom together with *equal treatment of equals* characterizes the *equal-split rule*. The third axiom says that team i is fully credited for such an improvement. We show that this third axiom, together with *equal treatment of equals*, characterizes *concede-and-divide*.

On the other hand, we consider an axiom referring to the incremental effect of adding additional viewers to a game. The axiom (*equal benefits from additional viewers*) states that the involved teams in the game should be affected in the same amount. The same should happen for the non-involved teams. Our last three results show that the combination of this axiom with some other basic axioms also characterize the three rules mentioned above. More precisely, *equal benefits from additional viewers*, together with *aggregate monotonicity* (more aggregate revenues cannot hurt any team) and *non negativity*, characterize the *uniform* rule. If, instead, we add to *equal benefits from additional viewers* the *null team* axiom (mentioned above), we characterize the *equal-split* rule, whereas if we add the *essential team* axiom (also mentioned above), we characterize *concede-and-divide*.

The rest of the paper is organized as follows. We introduce the model, axioms and rules in Section 2. In Section 3, we provide the characterization results. First, those involving *equal treatment of equals*. Then, those involving *equal benefits from additional viewers*. We conclude in Section 4.

2 The model

We consider the model introduced by Bergantiños and Moreno-Ternero (2020). Let N describe a finite set of teams. Its cardinality is denoted by n. Without loss of generality, we usually take $N = \{1, 2, \ldots, n\}$. We assume $n \geq 3$.

For each pair of teams $i, j \in N$, we denote by a_{ij} the broadcasting audience (number of viewers) for the game played by i and j at i's stadium. We use the notational convention that $a_{ii} = 0$, for each $i \in N$. Let $A \in \mathcal{A}_{n\times n}$ denote the resulting matrix of broadcasting audiences generated in the whole tournament involving the teams within N.[1]

Let $\alpha_i(A)$ denote the total audience achieved by team i, i.e.,

$$\alpha_i(A) = \sum_{j\in N}(a_{ij} + a_{ji}).$$

Without loss of generality, we normalize the revenue generated from each viewer to 1 (to be interpreted as the "pay per view" fee). Thus, we sometimes refer to $\alpha_i(A)$ as the *claim* of team i. When no confusion arises, we write α_i instead of $\alpha_i(A)$.

For each $A \in \mathcal{A}_{n\times n}$, let $||A||$ denote the total audience of the tournament. Namely,

$$||A|| = \sum_{i,j\in N} a_{ij} = \frac{1}{2}\sum_{i\in N}\alpha_i.$$

A (broadcasting) **problem** is a matrix $A \in \mathcal{A}_{n\times n}$ defined as above. The family of all the problems is denoted by $\mathcal{P}$.

[1]We are therefore assuming a tournament in which each team plays each other team twice: once home, another away. Our model could be extended to tournaments in which some teams play other teams a different number of times. In such a case, a_{ij} would denote the broadcasting audience in all games played by i and j at i's stadium.

2.1 Rules

A (sharing) **rule** R is a mapping that associates with each problem an allocation indicating the amount each team gets from the total revenue generated by broadcasting games. As we have normalized the revenue generated from each viewer to 1, $R : \mathcal{P} \to \mathbb{R}^N$ is such that, for each $A \in \mathcal{P}$,

$$\sum_{i \in N} R_i(A) = ||A||.$$

We consider three focal rules. First, the one that divides the total audience equally among the teams. Formally,

Uniform, U: for each $A \in \mathcal{P}$, and each $i \in N$,

$$U_i(A) = \frac{||A||}{n}.$$

The *uniform* rule is applied in many practical situations. For instance, the football competitions of England, Italy and Spain divide an important part of the revenues generated by TV broadcasting (50%, 40% and 50%, respectively), following the *uniform* rule.

Another focal rule for this problem is the so-called *equal-split rule*, which splits equally the audience of each game. Formally,

Equal-split, ES: for each $A \in \mathcal{P}$, and each $i \in N$,

$$ES_i(A) = \frac{\alpha_i}{2}.$$

The *equal-split* rule has game-theoretical foundations as, among other things, it coincides with the Shapley value of a suitably associated TU-game to broadcasting problems (e.g., Bergantiños and Moreno-Ternero, 2020).

The third focal rule is *concede-and-divide*, which compares the audience of a team with the average audience of the other teams. More precisely, it subtracts from the total audience of a team an amount associated to each of the remaining $n-1$ teams: the average audience per game that the remaining teams yield.[2] Formally,

[2]Consequently, the rule might yield negative awards for teams with very poor audiences (with respect to the remaining teams). The interpretation for the negative awards is that those teams are free riding on the prestige of the tournament and, thus, should be taxed for it.

Concede-and-divide, CD: for each $A \in \mathcal{P}$, and each $i \in N$,

$$CD_i\left(A\right) = \alpha_i - (n-1)\frac{\sum\limits_{j,k \in N \setminus \{i\}} \left(a_{jk} + a_{kj}\right)}{(n-2)(n-1)} = \frac{(n-1)\,\alpha_i - ||A||}{n-2}.$$

This rule can be rationalized by an intuitive statistical approach (e.g., Bergantiños and Moreno-Ternero, 2020). More precisely, for each pair of teams $i, j \in N$, with $i \neq j$, we can write $a_{ij} = b_0 + b_i + b_j + \varepsilon_{ij}$, where b_0 denotes the number of generic sport fans, who watch all games in the tournament, b_k denotes the number of fans of team $k = i, j$, who watch all games of team k in the tournament, and ε_{ij} denotes the residual number of viewers. Let $\hat{b}_0$ and $\left\{\hat{b}_i\right\}_{i \in N \setminus \{k\}}$ denote the solutions to the minimization problem of $\sum_{ij} \varepsilon_{ij}^2$, after (hypothetically) removing team k from the tournament. If we then impose a *concede-and-divide* procedure to allocate a_{ij}, in which $\hat{b}_0$ is divided equally among all teams, $\hat{b}_l$ is assigned to team l, for each $l \in N \setminus \{k\}$ and $\hat{\varepsilon}_{ij}$ is divided equally among teams i and j, for each pair $i, j \in N$, with $i \neq j$, then we obtain precisely the above expression for *concede-and-divide*.

2.2 Axioms

We now consider several axioms of rules. First, the most basic form of impartiality, which is formalized by the following axiom. It says that if two teams have the same audiences, then they should receive the same amount.

Equal treatment of equals: For each $A \in \mathcal{P}$, and each pair $i, j \in N$ such that $a_{ik} = a_{jk}$, and $a_{ki} = a_{kj}$, for each $k \in N \setminus \{i, j\}$,

$$R_i(A) = R_j(A).$$

The next axiom, which is inspired by the notion of solidarity, refers to the incremental effect of adding additional viewers to a game. It states that the involved teams should be affected in the same amount. The same should happen for the non-involved teams. Formally,

Equal benefits from additional viewers: For each pair A, $A' \in \mathcal{P}$ such that $a_{ij} = a'_{ij}$, for each pair $(i, j) \neq (i_0, j_0)$, and $a_{i_0,j_0} < a'_{i_0,j_0}$, we have

$$R_{i_0}(A') - R_{i_0}(A) = R_{j_0}(A') - R_{j_0}(A),$$

and

$$R_i(A') - R_i(A) = R_j(A') - R_j(A),$$

when $\{i, j\} \subset N \setminus \{i_0, j_0\}$.

We also consider a group of axioms that are closely related, as they state how a rule should react when additional viewers (of some specific team) appear. More precisely, let A, $A' \in \mathcal{P}$ and $i \in N$ such that $a_{ij} \leq a'_{ij}$ and $a_{ji} \leq a'_{ji}$ for each $j \in N \setminus \{i\}$ and $a_{jk} = a'_{jk}$ when $i \notin \{j, k\}$. Note that tournament A' has more viewers than tournament A just because the games involving team i have more viewers. How should a rule allocate those extra viewers? Our axioms consider three possible answers.[3]

First, we just ignore the fact that all viewers come from games involving team i and assume that all teams should equally share those additional viewers. Formally,

Equal sharing of additional team viewers: For each pair A, $A' \in \mathcal{P}$, and each $i \in N$ such that $a_{ij} \leq a'_{ij}$ and $a_{ji} \leq a'_{ji}$ for each $j \in N \setminus \{i\}$ and $a_{jk} = a'_{jk}$ when $i \notin \{j, k\}$, then there exists $c \in \mathbb{R}$ such that

$$R_l(A') - R_l(A) = c,$$

for each $l \in N$.

Second, we consider that team i and the rest of the teams are in a symmetric position because the audience of team i has increased the same amount than the audience of the rest of the teams (combined). Namely, $\alpha_i(A') - \alpha_i(A) = \sum_{j \in N \setminus \{i\}} (\alpha_j(A') - \alpha_j(A))$. Thus, team i should increase as much as the rest of the teams combined. Formally,

Half sharing of additional team viewers: For each pair A, $A' \in \mathcal{P}$, and each $i \in N$ such that $a_{ij} \leq a'_{ij}$ and $a_{ji} \leq a'_{ji}$ for each $j \in N \setminus \{i\}$ and $a_{jk} = a'_{jk}$ when $i \notin \{j, k\}$, then

$$R_i(A') - R_i(A) = \sum_{l \in N \setminus \{i\}} (R_l(A') - R_l(A)).$$

Third, we assume that team i is fully credited for such an improvement. Formally,

No sharing of additional team viewers: For each pair A, $A' \in \mathcal{P}$, and each $i \in N$ such that $a_{ij} \leq a'_{ij}$ and $a_{ji} \leq a'_{ji}$ for each $j \in N \setminus \{i\}$ and $a_{jk} = a'_{jk}$ when $i \notin \{j, k\}$, then

$$R_i(A') - R_i(A) = ||A'|| - ||A||.$$

[3]Note that the three axioms are mutually exclusive.

Finally, we also introduce four additional axioms.[4]

The first one says that if a team has a null audience, then such a team gets no revenue. Formally,

Null team: For each $A \in \mathcal{P}$, and each $i \in N$, such that for each $j \in N$, $a_{ij} = 0 = a_{ji}$,

$$R_i(A) = 0.$$

The second one is sort of dual to the first one as it says that if only the games played by one team have positive audience, then such an *essential* team should receive all its claim. Formally,

Essential team: For each $A \in \mathcal{P}$, and each $i \in N$ such that $a_{jk} = 0$ for each pair $\{j,k\} \subseteq N \backslash \{i\}$,

$$R_i(A) = \alpha_i.$$

The next axiom says that if the overall audience in a tournament is higher than in another, then no team can lose from it. Formally,

Aggregate Monotonicity: For each A, $A' \in \mathcal{P}$ such that $||A|| \leq ||A'||$, we have that

$$R_i(A) \leq R_i(A'),$$

for each $i \in N$.

The last axiom simply states that no team can receive a negative amount.

Non negativity: For each $A \in \mathcal{P}$ and each $i \in N$,

$$R_i(A) \geq 0.$$

Many of the above axioms have a flavor of existing axioms in the theories of cooperative games and fair division. Equal treatment of equals is a quite standard axiom formalizing the principle of impartiality in fair division (and, also, closely related to the symmetry axiom in cooperative games). The null team axiom is inspired by the null player axiom for cooperative games. The essential team axiom parallels the notion of necessary/veto player for cooperative games. Aggregate monotonicity is a special form of the standard axiom of resource monotonicity used in fair division. Finally, the axioms of marginalism in cooperative games state how the allocation to an agent should change when his/her marginal contribution to the problem changes. In our setting, equal benefits from additional viewers, equal sharing of additional team viewers, half sharing of additional team viewers, and no sharing of additional team viewers could be considered as axioms of marginalism.

[4]The first two were introduced in Bergantiños and Moreno-Ternero (2020).

3 Characterizations

We divide this section in two parts. In the first part, we show that the combination of *equal treatment of equals* with each of the three axioms modeling the allocation of the extra revenues generated from a specific team, leads to a characterization of each of the three focal rules defined above. In the second part, we consider *equal benefits from additional viewers*, instead of *equal treatment of equals*, and show that we also characterize the same three rules with different combinations of the axioms presented above.

3.1 With equal treatment of equals

We first show that the axioms of *equal treatment of equals* and *equal sharing of additional team viewers* characterize the *uniform* rule.

Theorem 1 *A rule satisfies equal treatment of equals and equal sharing of additional team viewers if and only if it is the uniform rule.*

Proof. It is straightforward to show that the *uniform* rule satisfies the two axioms in the statement. Conversely, let R be a rule satisfying *equal treatment of equals* and *equal sharing of additional team viewers*. Let $A \in \mathcal{P}$. For each $i = 1, ..., n-1$ we define the matrix A^i obtained from A by considering only the audiences of the teams $\{1, ..., i\}$. Namely,

$$a_{jk}^i = \begin{cases} a_{jk} & \text{if } \min\{j,k\} \leq i \\ 0 & \text{otherwise.} \end{cases}$$

Notice that $A^{n-1} = A$. Let A^0 be the matrix where all entries are 0. As R satisfies *equal treatment of equals*, $R_j(A^0) = 0$ for each $j \in N$.

Let $i \in N \backslash \{n\}$. As A^{i-1} and A^i are under the hypothesis of *equal sharing of additional team viewers*, we deduce that, for each $j \in N$,

$$R_j\left(A^i\right) - R_j\left(A^{i-1}\right) = c = \frac{||A^i|| - ||A^{i-1}||}{n}.$$

Thus, for each $j \in N$,

$$\begin{aligned} R_j(A) &= \sum_{i=0}^{n-1} \left(R_j\left(A^i\right) - R_j\left(A^{i-1}\right)\right) \\ &= \sum_{i=0}^{n-1} \frac{||A^i|| - ||A^{i-1}||}{n} \\ &= \frac{||A^{n-1}||}{n} = \frac{||A||}{n} = U_j(A). \end{aligned}$$

■

Remark 1 *The axioms of Theorem 1 are independent.*

Let $\delta \equiv \{\delta_i\}_{i\in N}$ be such that $\sum_{i\in N} \delta_i = 0$ and $\delta_i > 0$ for some $i \in N$. For each A and $i \in N$, we define the rule $R^{U,\delta}$ as follows:

$$R_i^{U,\delta}(A) = \delta_i + U_i(A).$$

Then, $R^{U,\delta}$ satisfies equal sharing of additional team viewers, but violates equal treatment of equals.

The equal-split rule satisfies equal treatment of equals, but violates equal sharing of additional team viewers.

The next result characterizes the *equal-split* rule as a result of replacing *equal sharing of additional team viewers* by *half sharing of additional team viewers* in Theorem 1.

Theorem 2 *A rule satisfies equal treatment of equals and half sharing of additional team viewers if and only if it is the equal-split rule.*

Proof. It is straightforward to show that the *equal-split* rule satisfies *equal treatment of equals.* We now prove that it also satisfies *half sharing of additional team viewers.* Let A, A' and i as in the definition of the axiom. Then,

$$\begin{aligned} \sum_{j\in N\setminus\{i\}} \left(ES_j(A') - ES_j(A)\right) &= \frac{1}{2} \sum_{j\in N\setminus\{i\}} \left(\alpha_j(A') - \alpha_j(A)\right) \\ &= \frac{1}{2} \sum_{j\in N\setminus\{i\}} \left(a'_{ij} + a'_{ji} - (a_{ij} + a_{ji})\right) \\ &= \frac{1}{2}\left[\alpha_i(A') - \alpha_i(A)\right] \\ &= ES_i(A') - ES_i(A). \end{aligned}$$

Conversely, let R be a rule satisfying *equal treatment of equals* and *half sharing of additional team viewers*. We proceed by induction on the number of pairs of teams with positive audience. Formally, let

$$s = \left|\{(i,j) \in N \times N \text{ such that } a_{ij} > 0\}\right|.$$

If $s = 0$ then $A = 0$ and, by *equal treatment of equals*, $R_i(0) = 0$ for each $i \in N$.

Assume now that $s \geq 1$. Let $i \in N$ be such that there exists $i' \in N$ such that $a_{ii'} + a_{i'i} > 0$. We consider the problem $A^{ii'}$ defined as follows:

$$a_{jk}^{ii'} = \begin{cases} a_{jk} & \text{if } \{j,k\} \neq \{i,i'\} \\ 0 & \text{otherwise.} \end{cases}$$

By *half sharing of additional team viewers*,

$$R_i(A) - R_i\left(A^{ii'}\right) = \sum_{j \in N \setminus \{i\}} \left(R_j(A) - R_j\left(A^{ii'}\right)\right)$$

As

$$\sum_{j \in N} \left(R_j(A) - R_j\left(A^{ii'}\right)\right) = a_{ii'} + a_{i'i},$$

we have that

$$R_i(A) - R_i\left(A^{ii'}\right) = \sum_{j \in N \setminus \{i\}} \left(R_j(A) - R_j\left(A^{ii'}\right)\right) = \frac{a_{ii'} + a_{i'i}}{2}.$$

Thus,

$$R_i(A) = R_i\left(A^{ii'}\right) + \frac{a_{ii'} + a_{i'i}}{2}.$$

By the induction hypothesis, $R_i\left(A^{ii'}\right) = ES_i\left(A^{ii'}\right)$. Then,

$$R_i(A) = ES_i\left(A^{ii'}\right) + \frac{a_{ii'} + a_{i'i}}{2} = ES_i(A).$$

We now consider the partition of N between null teams and non-null teams. Formally, let

$$\begin{aligned} M &= \{i \in N : a_{ii'} + a_{i'i} > 0 \text{ for some } i' \in N\}, \text{ and} \\ M^c &= \{i \in N : a_{ii'} = a_{i'i} = 0 \text{ for each } i' \in N\}. \end{aligned}$$

If $M^c = \varnothing$ then the above proves that $R(A) = ES(A)$. Suppose now that $M^c \neq \varnothing$. Let $i \in M$.[5] Then, all agents in M^c have the same audiences (actually, 0) in A and $A^{ii'}$. Then, by

[5] As $s \geq 1$ we have that $M \neq \varnothing$.

equal treatment of equals, $R_j(A) = R_k(A)$ and $R_j\left(A^{ii'}\right) = R_k\left(A^{ii'}\right)$, for each $j, k \in M^c$. Thus, we can define $x = R_j(A) - R_j\left(A^{ii'}\right)$ for each $j \in M^c$.

Now,

$$\begin{aligned} a_{ii'} + a_{i'i} &= ||A|| - \left|\left|A^{ii'}\right|\right| = \sum_{j\in N} R_j(A) - \sum_{j\in N} R_j\left(A^{ii'}\right) \\ &= \sum_{j\in M}\left(R_j(A) - R_j\left(A^{ii'}\right)\right) + \sum_{j\in M^c}\left(R_j(A) - R_j\left(A^{ii'}\right)\right) \\ &= \sum_{j\in M}\left(R_j(A) - R_j\left(A^{ii'}\right)\right) + |M^c|\,x. \qquad (1) \end{aligned}$$

We have proven above that, for each $j \in M$, $R_j(A) = ES_j(A)$. By the induction hypothesis, $R_j\left(A^{ii'}\right) = ES_j\left(A^{ii'}\right)$, for each $j \in M$. As $ES_j(A) = ES_j\left(A^{ii'}\right)$ for each $j \in N \backslash \{i, i'\}$ and $\{i, i'\} \subset M$ we have that

$$\begin{aligned} \sum_{j\in M}\left(R_j(A) - R_j\left(A^{ii'}\right)\right) &= \sum_{j\in\{i,i'\}}\left(R_j(A) - R_j\left(A^{ii'}\right)\right) \\ &= a_{ii'} + a_{i'i}. \end{aligned}$$

Then, $0 = |M^c|\,x$, which implies that $x = 0$. Thus, for each $j \in M^c$, $R_j(A) = R_j\left(A^{ii'}\right)$.

As, by induction, $R_j\left(A^{ii'}\right) = ES_j\left(A^{ii'}\right)$ for each $j \in M^c$ and $ES_j\left(A^{ii'}\right) = ES_j(A)$ for each $j \in M^c$, we deduce that $R_j(A) = ES_j(A)$ for each $j \in M^c$. ■

Remark 2 *The axioms of Theorem 2 are independent.*

Let $\delta \equiv \{\delta_i\}_{i\in N}$ be such that $\sum_{i\in N} \delta_i = 0$ and $\delta_i > 0$ for some $i \in N$. For each A and $i \in N$, we define the rule $R^{ES,\delta}$ as follows.

$$R_i^{ES,\delta}(A) = \delta_i + ES_i(A).$$

Then, $R^{ES,\delta}$ satisfies half sharing of additional team viewers, but violates equal treatment of equals.

The uniform rule satisfies equal treatment of equals, but violates half sharing of additional team viewers.

The next corollary shows that we can replace *equal treatment of equals* by *null team* in the statement of Theorem 2.

Corollary 1 *A rule satisfies null team and half sharing of additional team viewers if and only if it is the equal-split rule.*

Proof. It is straightforward to show that the *equal-split* rule satisfies null team. Conversely, one just has to notice that, in the proof of Theorem 2, *equal treatment of equals* is used twice. First, to obtain that $R_i(0) = 0$ for each $i \in N$. The same conclusion could be obtained with *null team*. Second, to prove that $R_j(A) = ES_j(A)$ for each $j \in M^c$. With *null team*, such a proof is obvious because each $j \in M^c$ is a *null team* in A and, thus, $R_j(A) = 0 = ES_j(A)$ for each $j \in M^c$. ■

Remark 3 *The axioms of Corollary 1 are independent.*

$R^{ES,\delta}$ *satisfies half sharing of additional team viewers, but violates null team.*

Consider the rule that divides $||A||$ *equally among the non-null teams. Such a rule satisfies null team, but violates half sharing of additional team viewers.*

Finally, the next theorem gives a characterization of *concede-and-divide* resorting to *no sharing of additional team viewers.*

Theorem 3 *A rule satisfies equal treatment of equals and no sharing of additional team viewers if and only if it is concede-and-divide.*

Proof. It is straightforward to show that *concede-and-divide* satisfies *equal treatment of equals.* We now prove that it also satisfies no sharing of additional team viewers. Let A, A' and i as in the definition of the axiom. Then,

$$\begin{aligned} CD_i(A') - CD_i(A) &= \alpha_i(A') - \frac{\sum\limits_{j,k \in N\setminus\{i\}} \left(a'_{jk} + a'_{kj}\right)}{n-2} - \alpha_i(A) + \frac{\sum\limits_{j,k \in N\setminus\{i\}} \left(a_{jk} + a_{kj}\right)}{n-2} \\ &= \alpha_i(A') - \alpha_i(A) \\ &= \sum_{j \in N\setminus\{i\}} \left[a'_{ij} + a'_{ji} - (a_{ij} + a_{ji})\right] \\ &= ||A'|| - ||A||. \end{aligned}$$

Conversely, let R be a rule satisfying the two axioms. We proceed by induction on the number of pairs of teams with positive audience. Formally, let

$$s = \left|\{(i,j) \in N \times N \text{ such that } a_{ij} > 0\}\right|.$$

If $s = 0$ then $A = 0$ and, by *equal treatment of equals*, $R_i(0) = 0 = CD_i(0)$ for each $i \in N$.

Assume now that $s \geq 1$. Let $i \in N$ be such that there exists $i' \in N$ such that $a_{ii'} + a_{i'i} > 0$. We consider the problem $A^{ii'}$ defined as in the proof of Theorem 2.

By *no sharing of additional team viewers*,

$$R_i(A) - R_i\left(A^{ii'}\right) = ||A|| - \left|\left|A^{ii'}\right|\right| = a_{ii'} + a_{i'i}.$$

Equivalently,

$$R_i(A) = R_i\left(A^{ii'}\right) + a_{ii'} + a_{i'i}.$$

By the induction hypothesis, $R_i\left(A^{ii'}\right) = CD_i\left(A^{ii'}\right)$. Then,

$$R_i(A) = CD_i\left(A^{ii'}\right) + a_{ii'} + a_{i'i} = CD_i(A).$$

We consider the partition $\{M, M^c\}$ of N as in the proof of Theorem 2.

If $M^c = \varnothing$ then the above proves that $R(A) = CD(A)$. Suppose now that $M^c \neq \varnothing$. Let x be defined as in the proof of Theorem 2. We can prove that equation (1) also holds in this case.

We have proven above that, for each $j \in M$, $R_j(A) = CD_j(A)$. By the induction hypothesis, $R_j\left(A^{ii'}\right) = CD_j\left(A^{ii'}\right)$, for each $j \in M$.

We now consider two cases:

1. $j \in \{i, i'\}$. Then,

$$\begin{aligned} R_j(A) - R_j\left(A^{ii'}\right) &= \frac{(n-1)\,\alpha_j(A) - ||A||}{n-2} - \frac{(n-1)\,\alpha_j\left(A^{ii'}\right) - \left|\left|A^{ii'}\right|\right|}{n-2} \\ &= \frac{(n-1)\left(\alpha_j(A) - \alpha_j\left(A^{ii'}\right)\right)}{n-2} - \frac{||A|| - \left|\left|A^{ii'}\right|\right|}{n-2} \\ &= \frac{(n-1)\left(a_{ii'} + a_{i'i}\right)}{n-2} - \frac{a_{ii'} + a_{i'i}}{n-2} \\ &= a_{ii'} + a_{i'i}. \end{aligned}$$

2. $j \in M \setminus \{i, i'\}$. Then,

$$\begin{aligned} R_j(A) - R_j\left(A^{ii'}\right) &= \frac{(n-1)\left(\alpha_j(A) - \alpha_j\left(A^{ii'}\right)\right)}{n-2} - \frac{||A|| - \left|\left|A^{ii'}\right|\right|}{n-2} \\ &= -\frac{a_{ii'} + a_{i'i}}{n-2}. \end{aligned}$$

Then,

$$a_{ii'} + a_{i'i} = 2\left(a_{ii'} + a_{i'i}\right) - \left|M \backslash \{i, i'\}\right| \frac{a_{ii'} + a_{i'i}}{n-2} + \left|M^c\right| x.$$

As $\left|M^c\right| = n - 2 - \left|M \backslash \{i, i'\}\right|$ we have that

$$x = -\frac{a_{ii'} + a_{i'i}}{n-2}.$$

Let $j \in M^c$. Then,

$$R_j(A) = R_j\left(A^{ii'}\right) - \frac{a_{ii'} + a_{i'i}}{n-2}.$$

By induction, $R_j\left(A^{ii'}\right) = CD_j\left(A^{ii'}\right)$. Then,

$$R_j(A) = CD_j\left(A^{ii'}\right) - \frac{a_{ii'} + a_{i'i}}{n-2} = CD_j(A).$$

■

Remark 4 *The axioms of Theorem 3 are independent.*

Let $\delta = \{\delta_i\}_{i \in N}$ be such that $\sum_{i \in N} \delta_i = 0$ and $\delta_i > 0$ for some $i \in N$. For each A and $i \in N$, we define the rule $R^{CD,\delta}$ as follows.

$$R_i^{CD,\delta}(A) = \delta_i + CD_i(A).$$

Then, $R^{CD,\delta}$ satisfies no sharing of additional team viewers, but violates equal treatment of equals.

The uniform rule satisfies equal treatment of equals, but violates no sharing of additional team viewers.

Recently, Casajus (2018) shows that equal treatment of equals can be weakened to axiomatize Shapley's value in Young's axiomatics (e.g., Young, 1985). Casajus (2018) introduces the so called *sign symmetry* axiom that says that the sign of the allocation received by two symmetric agents should be the same. This axiom could be defined in the same way in our setting. It is not difficult to see that Theorems 1 and 2 also hold with the (weaker) *sign axiom* instead of equal treatment of equals. For Theorem 3, we know that the proof cannot be adapted. It remains an open question to know whether Theorem 3 holds with the (weaker) *sign axiom* instead of equal treatment of equals.

3.2 With equal benefits from additional viewers

We now provide a second set of characterization results, replacing *equal treatment of equals* by *equal benefits from additional viewers* in the results from the previous section, and resorting to some other axioms.

The first result in this set characterizes the *uniform* rule.

Theorem 4 *A rule satisfies equal benefits from additional viewers, aggregate monotonicity and non negativity if and only if it is the uniform rule.*

Proof. It is straightforward to show that the *uniform* rule satisfies *equal benefits from additional viewers*, *aggregate monotonicity* and *non negativity*. Conversely, let R be a rule satisfying the three axioms. We proceed by induction on the number of pairs of teams with positive audience. Formally, let

$$s = \left|\{(i,j) \in N \times N \text{ such that } a_{ij} > 0\}\right|.$$

If $s = 0$ then $A = 0$ and, by *non negativity*, $R_i(0) = 0 = U_i(0)$ for each $i \in N$.

Let $s \geq 1$. Let (i^1, i^2) such that $a_{i^1 i^2} > 0$. And let i^3 be such that $i^3 \notin \{i^1, i^2\}$. We consider the problems A^*, A^1, and A^2 defined as follows.

$$a^*_{kk'} = \begin{cases} ||A|| - a_{i^1 i^2} & \text{if } (k,k') = (i^1, i^3) \\ 0 & \text{otherwise.} \end{cases}$$

$$a^1_{kk'} = \begin{cases} a_{i^1 i^2} & \text{if } (k,k') = (i^1, i^2) \\ ||A|| - a_{i^1 i^2} & \text{if } (k,k') = (i^1, i^3) \\ 0 & \text{otherwise.} \end{cases}$$

$$a^2_{kk'} = \begin{cases} a_{i^1 i^2} & \text{if } (k,k') = (i^2, i^3) \\ ||A|| - a_{i^1 i^2} & \text{if } (k,k') = (i^1, i^3) \\ 0 & \text{otherwise.} \end{cases}$$

By *equal benefits from additional viewers*,

$$R_k\left(A^1\right) - R_k\left(A^*\right) = \begin{cases} x^1 & k \in \{i^1, i^2\} \\ y^1 & \text{otherwise,} \end{cases} \quad \text{and}$$

$$R_k\left(A^2\right) - R_k\left(A^*\right) = \begin{cases} x^2 & k \in \{i^2, i^3\} \\ y^2 & \text{otherwise.} \end{cases}$$

As we can apply the induction hypothesis to A^*,

$$\begin{aligned} R_{i^1}\left(A^1\right) &= R_{i^1}\left(A^*\right) + R_{i^1}\left(A^1\right) - R_{i^1}\left(A^*\right) = U_{i^1}\left(A^*\right) + x^1, \qquad \text{and} \\ R_{i^1}\left(A^2\right) &= R_{i^1}\left(A^*\right) + R_{i^1}\left(A^2\right) - R_{i^1}\left(A^*\right) = U_{i^1}\left(A^*\right) + y^2. \end{aligned}$$

By *aggregate monotonicity*, $R\left(A^1\right) = R\left(A^2\right)$. Thus, $x^1 = y^2$. If we proceed with i^3 instead of i^1 we can obtain that $x^2 = y^1$. If we proceed with i^2 instead of i^1 we can obtain that $x^1 = x^2$. Then, $x^1 = x^2 = y^1 = y^2$.

Now,

$$a_{i^1 i^2} = \sum_{k \in N} \left(R_k\left(A^1\right) - R_k\left(A^*\right)\right) = 2x^1 + (n-2)\, y^1 = nx^1,$$

which implies that

$$x^1 = \frac{a_{i^1 i^2}}{n}.$$

Let $i \in N$. By *aggregate monotonicity*, $R_i(A) = R_i\left(A^1\right)$. Then,

$$R_i(A) = R_i\left(A^1\right) = R_i\left(A^*\right) + R_i\left(A^1\right) - R_i\left(A^*\right) = U_i\left(A^*\right) + \frac{a_{i^1 i^2}}{n} = U_i(A).$$

■

Remark 5 *The axioms of Theorem 4 are independent.*

$R^{U,\delta}$ satisfies equal benefits from additional viewers and aggregate monotonicity but violates non negativity.

The equal-split rule satisfies equal benefits from additional viewers and non negativity, but violates aggregate monotonicity.

Let $\beta \in \Delta \backslash \left\{\left(\frac{1}{n}, ..., \frac{1}{n}\right)\right\}$ where Δ is the unit simplex. Then, the weighted version of the uniform rule according to β, ($U_i^\beta(A) = \beta_i \left\|A\right\|$) satisfies aggregate monotonicity and non negativity but violates equal benefits from additional viewers.

The next result characterizes the *equal-split* rule.

Theorem 5 *A rule satisfies equal benefits from additional viewers and null team if and only if it is the equal-split rule.*

Proof. It is straightforward to show that the *equal-split* rule satisfies *equal benefits from additional viewers*, and *null team.* Conversely, let R be a rule satisfying the two axioms. We proceed by induction on the number of pairs of teams with positive audience. Formally, let

$$s = \left|\left\{(i,j) \in N \times N \text{ such that } a_{ij} > 0\right\}\right|.$$

If $s=0$ then $A=0$ and, by *null team*, $R_i(0)=0=ES_i(0)$ for each $i\in N$.

If $s=1$, there exists (i^1,j^1) such that $a_{i^1j^1}>0$ and $a_{ij}=0$ otherwise. By *null team*, $R_i(A)=0$ for each $i\in N\backslash\{i^1,j^1\}$.

By *equal benefits from additional viewers*,

$$R_{i^1}(A)-R_{i^1}(0)=R_{j^1}(A)-R_{j^1}(0).$$

As $R_{i^1}(0)=R_{j^1}(0)=0$ we have that $R_{i^1}(A)=R_{j^1}(A)$. Thus, $R_{i^1}(A)=R_{j^1}(A)=\frac{a_{i^1j^1}}{2}$ and hence $R(A)=ES(A)$.

Let $s\geq 2$. Let (i^1,j^1) and (i^2,j^2) such that $a_{i^1j^1}>0$ and $a_{i^2j^2}>0$. Two cases are possible.

First, $(i^1,j^1)=(j^2,i^2)$. Let A' be obtained from A by making $a_{i^2j^2}=0$. By the induction hypothesis, $R(A')=ES(A')$. Using similar arguments as in the case $s=1$ (with A' instead of 0) we can deduce that $R(A)=ES(A)$.

Second, $(i^1,j^1)\neq(j^2,i^2)$. Then, there exist $i,j\in N$ such that $i\in\{i^1,j^1\}\setminus\{i^2,j^2\}$, $j\in\{i^2,j^2\}\setminus\{i^1,j^1\}$ and $i\neq j$. We consider the problems A^{-1},A^{-2}, and A^{-12} defined as follows:

$$\begin{aligned}
a^{-1}_{kk'} &= \begin{cases} 0 & (k,k')=(i^1,j^1) \\ a_{kk'} & \text{otherwise} \end{cases} \\
a^{-2}_{kk'} &= \begin{cases} 0 & (k,k')=(i^2,j^2) \\ a_{kk'} & \text{otherwise} \end{cases} \\
a^{-12}_{kk'} &= \begin{cases} 0 & (k,k')\in\{(i^1,j^1),(i^2,j^2)\} \\ a_{kk'} & \text{otherwise} \end{cases}
\end{aligned}$$

By *equal benefits from additional viewers*,

$$\begin{aligned}
R_k(A)-R_k\left(A^{-1}\right) &= \begin{cases} x^1 & k\in\{i^1,j^1\} \\ y^1 & \text{otherwise} \end{cases} \quad \text{and} \\
R_k(A)-R_k\left(A^{-2}\right) &= \begin{cases} x^2 & k\in\{i^2,j^2\} \\ y^2 & \text{otherwise} \end{cases}
\end{aligned}$$

By *equal benefits from additional viewers*, and the induction hypothesis

$$\begin{aligned}
R_i(A)-R_i\left(A^{-12}\right) &= R_i(A)-R_i\left(A^{-1}\right)+R_i\left(A^{-1}\right)-R_i\left(A^{-12}\right)=x^1 \\
R_i(A)-R_i\left(A^{-12}\right) &= R_i(A)-R_i\left(A^{-2}\right)+R_i\left(A^{-2}\right)-R_i\left(A^{-12}\right)=y^2+\frac{a_{i^1j^1}}{2} \\
R_j(A)-R_j\left(A^{-12}\right) &= R_j(A)-R_j\left(A^{-1}\right)+R_j\left(A^{-1}\right)-R_j\left(A^{-12}\right)=y^1+\frac{a_{i^2j^2}}{2} \\
R_j(A)-R_j\left(A^{-12}\right) &= R_j(A)-R_j\left(A^{-2}\right)+R_j\left(A^{-2}\right)-R_j\left(A^{-12}\right)=x^2.
\end{aligned}$$

Thus, we have the following equations:

$$x^1 - y^2 = \frac{a_{i^1 j^1}}{2}, \tag{2}$$

$$x^2 - y^1 = \frac{a_{i^2 j^2}}{2}. \tag{3}$$

As

$$\begin{aligned} \sum_{k \in N} \left(R_k(A) - R_k\left(A^{-1}\right) \right) &= a_{i^1 j^1}, \qquad \text{and} \\ \sum_{k \in N} \left(R_k(A) - R_k\left(A^{-2}\right) \right) &= a_{i^2 j^2}, \end{aligned}$$

we have the following equations too:

$$2x^1 + (n-2)\, y^1 = a_{i^1 j^1}, \tag{4}$$

$$2x^2 + (n-2)\, y^2 = a_{i^2 j^2}. \tag{5}$$

Straightforward algebraic computations allow us to show that the system of the four equations listed above has an unique solution, which is given by

$$x^1 = \frac{a_{i^1 j^1}}{2}, x^2 = \frac{a_{i^2 j^2}}{2}, \text{ and } y^1 = y^2 = 0.$$

By the induction hypothesis, $R\left(A^{-1}\right) = ES\left(A^{-1}\right)$. Given $k \in \{i^1, j^1\}$,

$$\begin{aligned} R_k(A) &= R_k\left(A^{-1}\right) + \left[R_k(A) - R_k\left(A^{-1}\right)\right] \\ &= ES\left(A^{-1}\right) + \frac{a_{i^1 j^1}}{2} \\ &= ES_k(A). \end{aligned}$$

Similarly, we can prove that, given $k \in N \setminus \{i^1, j^1\}$, $R_k(A) = ES_k(A)$. ■

Remark 6 *The axioms of Theorem 5 are independent.*

The uniform rule satisfies equal sharing of additional team viewers, but not null team.

Let R^{lowest} be the rule in which, for each game $(i, j) \in N \times N$ the revenue goes to the team with the lowest number of the two. Namely, for each problem $A \in \mathcal{P}$, and each $i \in N$,

$$R_i^{lowest}(A) = \sum_{j \in N : j > i} (a_{ij} + a_{ji}).$$

R^{lowest} satisfies null team, but violates equal sharing of additional team viewers.

Our final result is a counterpart characterization for *concede-and-divide*.

Theorem 6 *A rule satisfies equal benefits from additional viewers and essential team if and only if it is concede-and-divide.*

Proof. It is straightforward to show that *concede-and-divide* satisfies *equal benefits from additional viewers* and *essential team*. Conversely, let R be a rule satisfying the two axioms. We proceed by induction on the number of pairs of teams with positive audience. Formally, let

$$s = \left|\{(i,j) \in N \times N \text{ such that } a_{ij} > 0\}\right|.$$

If $s = 0$ then $A = 0$ and, by *essential team*, $R_i(0) = 0 = ES_i(0)$ for each $i \in N$.

If $s = 1$, there exists (i^1, j^1) such that $a_{i^1j^1} > 0$ and $a_{ij} = 0$ otherwise. By *essential team*, $R_i(A) = a_{i^1j^1}$ for each $i \in \{i^1, j^1\}$.

By *equal benefits from additional viewers*, for each $i, j \in N \setminus \{i^1, j^1\}$,

$$R_i(A) - R_i(0) = R_j(A) - R_j(0).$$

As $R_i(0) = R_j(0) = 0$ we have that $R_i(A) = R_j(A)$. As $\sum_{i \in N} R_i(A) = a_{i^1j^1}$ we deduce that $R_i(A) = -\frac{a_{i^1j^1}}{n-2}$ for each $i \in N \setminus \{i^1, j^1\}$. Thus, $R(A) = CD(A)$.

Let $s \geq 2$. Let (i^1, j^1) and (i^2, j^2) such that $a_{i^1j^1} > 0$ and $a_{i^2j^2} > 0$. We consider two cases:

First, $(i^1, j^1) = (j^2, i^2)$. Let A' be obtained from A by making $a_{i^2j^2} = 0$. By the induction hypothesis, $R(A') = CD(A')$. By essential team, $R_i(A) = a_{i^1j^1} + a_{i^2j^2} = CD_i(A)$ for each $i \in \{i^1, j^1\}$. Using similar arguments as in the case $s = 1$ (with A' instead of 0) we can deduce that $R_i(A) = CD_i(A)$ for each $i \in N \setminus \{i^1, j^1\}$.

Second, $(i^1, j^1) \neq (j^2, i^2)$. Then, there exists $i, j \in N$ such that $i \in \{i^1, j^1\} \setminus \{i^2, j^2\}$, $j \in \{i^2, j^2\} \setminus \{i^1, j^1\}$ and $i \neq j$. We consider the problems A^{-1}, A^{-2}, and A^{-12}, x^1, y^1, x^2 and y^2 defined as in the proof of Theorem 5. Similarly to such a proof we can obtain the following system of equations

$$\begin{cases} x^1 - y^2 = a_{i^1j^1} - \frac{a_{i^2j^2}}{n-2}, \\ x^2 - y^1 = a_{i^2j^2} - \frac{a_{i^1j^1}}{n-2}, \\ 2x^1 + (n-2)\, y^1 = a_{i^1j^1}, \\ 2x^2 + (n-2)\, y^2 = a_{i^2j^2}. \end{cases}$$

The unique solution to this system is

$$x^1 = a_{i^1j^1}, x^2 = a_{i^2j^2}, \ y^1 = -\frac{a_{i^1j^1}}{n-2}, \text{ and } y^2 = -\frac{a_{i^2j^2}}{n-2}.$$

By the induction hypothesis, $R(A^{-1}) = CD(A^{-1})$. Given $k \in \{i^1, j^1\}$,

$$\begin{aligned} R_k(A) &= R_k\left(A^{-1}\right) + \left[R_k(A) - R_k\left(A^{-1}\right)\right] \\ &= CD\left(A^{-1}\right) + a_{i^1 j^1} \\ &= CD_k(A). \end{aligned}$$

Similarly, we can prove that, given $k \in N \setminus \{i^1, j^1\}$, $R_k(A) = CD_k(A)$. ■

Remark 7 *The axioms of Theorem 6 are independent.*

The uniform rule satisfies equal sharing of additional team viewers, but not essential team.

We consider the rule defined as $CD(A)$ when the problem A has essential teams and $ES(A)$ when there are no essential teams in A. This rule satisfies essential team but not equal sharing of additional team viewers.

3.3 Summary

The next table summarizes our findings.

Axioms / Rules	**U**	**ES**	**CD**
Equal treatment of equals	YES^{Th1}	YES^{Th2}	YES^{Th3}
Equal sharing of additional team viewers	YES^{Th1}	NO	NO
Half sharing of additional team viewers	NO	YES^{Th2,Cor1}	NO
No sharing of additional team viewers	NO	NO	YES^{Th3}
Equal benefits from additional viewers	YES^{Th4}	YES^{Th5}	YES^{Th6}
Null team	NO	YES^{Th5,Cor1}	NO
Essential team	NO	NO	YES^{Th6}
Aggregate monotonicity	YES^{Th4}	NO	NO
Non negativity	YES^{Th4}	YES	NO

Most of the statements of the table have been proven in the text. The remaining are straightforward.

4 Discussion

We have explored in this paper new axioms (mostly referring to the allocation of extra resources) for the problem of sharing the revenues from broadcasting sports leagues. These axioms provide normative support for three focal rules in this setting. Two of these rules had been characterized already in Bergantiños and Moreno-Ternero (2020). The main novelty of the results presented here, with respect to those, is to dismiss *additivity*, an axiom with long tradition in axiomatic work (e.g., Shapley, 1953), but also with strong implications. More precisely, the additivity requirement in our setting precludes the allocation of revenue a_{ij} to depend on any other information contained in the matrix A. Our results here demonstrate that this feature is also a by-product of the combination of more fundamental axioms.

We note that the rules we consider in the paper treat equally a_{ij} and a_{ji}. But we should stress that the model does not preclude different (asymmetric) treatment of those audiences. One could, for instance, easily think of asymmetric generalizations of our rules in which a_{ij} and a_{ji} are not equally treated. A canonical case would be that in which teams fully collect the revenue (audience) from their home (or, dually, away) games. This rule would be natural for the (related) problem of sharing the revenue collected from selling tickets to attend games at stadiums.

To conclude, we note that one could also be interested into approaching our problems with a (cooperative) game-theoretical approach. This is a typical course of action in some of the related problems listed in the introduction. In Bergantiños and Moreno-Ternero (2020), we associate to our problems a natural *optimistic* cooperative TU game in which, for each subset of teams, we define its worth as the total audience of the games played by the teams in that subset. The Shapley value (e.g., Shapley, 1953) of such a game yields the same solutions as the *equal-split* rule for the original problem. It is straightforward to show that the egalitarian value (e.g., van den Brink, 2007) of that game yields the same solutions as the *uniform* rule considered (and characterized) here. Casajus and Huettner (2013), van den Brink et al., (2013) and Casajus and Yokote (2019) characterize the family of values arising from the convex combination of the Shapley value and the egalitarian value. In our setting, this would correspond to the following family of rules: $\left\{EU^{\lambda}\right\}_{\lambda\in[0,1]}$ where, for each $\lambda \in [0,1]$, and each $A \in \mathcal{P}$, $EU^{\lambda}(A) = \lambda ES(A) + (1-\lambda)U(A)$. In the three papers mentioned above, the characterizations are obtained without using the axiom of additivity (as in this paper). Besides, they use some axioms specifying how payoffs vary when some input changes (as in this paper). A natural question that arises is then whether it is possible to characterize the family of rules $\left\{EU^{\lambda}\right\}_{\lambda\in[0,1]}$ by adapting some of the axioms from those papers to our setting. The answer is beyond the objective of this paper and is left for further research.

It is also left for further research to explore the logical implications of other axioms related to the principle of solidarity, with a strong tradition in the theory of justice (e.g., Moreno-Ternero and Roemer, 2006). We have used in this paper one of the axioms within this group, *aggregate monotonicity*, which is a special form of the standard axiom of *resource monotonicity* in fair allocation (e.g., Moreno-Ternero and Roemer, 2012). Other monotonicity notions, reflecting, for instance, the effect on each team when the audiences of a given team increase, would be interesting to analyze as they might provide normative foundations for new rules.

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