

## 1. COURSE DESCRIPTION

Degree:	Finanzas y Contabilidad	
	(English teaching)	
Double Degree:	Derecho y Finanzas y Contabilidad	
	(English teaching)	
Course:	MATHEMATICS FOR BUSINESS I	
	(Matemática Empresarial I - English teaching)	
Module:	Basic Training in Business and Economic Sciences	
Department:	Economics, Quantitative Methods and Economic History	
Term:	First term	
<b>Total Credits:</b>	6	
Year:	1 <sup>st</sup>	
Type of Course:	Basic	
Course Language:	English	

Teaching model:	C1	
a. General/background:		50%
b. Theory-into-practice/developmental		50%
knowledge-building		
c. Guided Academic Activities:		



## 2. COURSE COORDINATOR

**Course coordinators** 

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**Faculty: Business Administration** 

**Department: Economics, Quantitative Methods and Economic History** 

**Academic Area: Quantitative Methods** 

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#### 3. ACADEMIC CONTEXT

## 3.1. Course Description and Objectives

#### **COURSE DESCRIPTION**

Introduction to Mathematics for Business.

#### **OBJECTIVES**

- To provide students with mathematical knowledge and techniques; these will be useful to complete their higher education and to carry out their professional life.
- To supply the student with the basic, indispensable tools needed in order to easily interpret and tackle mathematical models associated with the economic problems that can be found in other subjects and in the business world.
- To give elemental tools from Linear Algebra, Matrix Theory, and Elements of Functions, to facilitate the comprehension of economic results.
- To make the student familiar with the daily mathematical vocabulary, and to make him used to a logic reasoning to carry out the resolutions of problems.
- To introduce the student to the use of the computational software program *Mathematica*, and to facilitate the application of this computational tool for the resolution of problems posed within the course.

## 3.2. Contribution to the Training Plan

Mathematics for Business I is a 6-credit core subject, belonging to the following plans: Finance and Accounting Degree, and Double Degree in Finance and Accounting, and Law. It is taught in the first year of both Degrees, and it depends on the Academic Area of Quantitative Methods in the Department of Economics, Quantitative Methods and



Economic History (Departamento de Economía, Métodos Cuantitativos e Historia Económica).

Due to the contents of these Degrees, this course must be essentially practical, so that it can be applied to other subjects. The contents of the course have been selected considering the requirements of the others. We want to emphasize the utility of mathematical tools in other subjects, such as: Introducción a la Economía (Introduction to Economics), Economía de la Empresa (Business Administration), Microeconomía (Microeconomics), Matemática Financiera (Financial Mathematics), Matemática Empresarial II (Mathematics for Business II), Estadística para Finanzas I (Statistics for Finance I), Macroeconomía (Macroeconomics), Estadística para Finanzas II (Statistics for Finance II), Métodos Estadísticos y Econométricos en Finanzas (Statistical and Econometric Methods for Finance), Matemática Financiera Avanzada (Advanced Financial Mathematics) and Cálculo Actuarial y Seguros (Actuarial Calculus and Insurance).

The importance of mathematical results explained in this course is rooted in their capacity to open different ways of solving problems from a range of different fields. The lecturers will show the straightest applications for each introduced topic, making the learning process more dynamic, and increasing the motivation of the student towards Mathematics.

In spite of the practical nature of this course, it cannot be reduced to a simple collection of methods to solve particular problems. Its formative nature must be taken into account, which will allow the student to develop skills in logic reasoning and in comprehension of formal language. It is necessary to make the student realize the importance of studying quantitative techniques because they are useful, but it is opportune to establish a minimum level of rigor which cannot be left aside under the pretext that Mathematics implies a basic knowledge for the economist and the businessperson.

### 3.3. Recommendations or Prerequisites

Although this is not a formal prerequisite, in order to understand the course, it is required to have basic knowledge from the Bachillerato and previous years. Especially for those students who do not have a *Bachillerato de Ciencias*, it would be appropriate a first stage where contents will be leveled.



#### 4. SKILLS

## 4.1. Degree Skills Developed during this Course

#### Instrumental:

- Synthesis and abstraction skills.
- Organizing and planning skills.
- Development of written and oral communication skills.
- Elementary computer skills and analytical abilities from different sources.
- Ability to solve problems and apply theory into practice.

#### Personal:

- Ability to criticize and self-criticize.
- Ethical involvement at work.
- Ability to work under stressful conditions.

### **Systemic**:

- Self-study and research skills.
- Ability to face up new situations.
- Awareness of quality and success.

## Specific:

- Knowledge of basic mathematical techniques in Linear Algebra and Calculus; application of the presented techniques in order to resolve specific problems in business.
- Ability to connect the concepts studied in this course with other subjects.

### 4.2. Module Skills Developed during this Course

#### Instrumental:

- Synthesis and abstraction skills.
- Organizing and planning skills.
- Development of written and oral communication skills.
- Elementary computer skills and analytical abilities from different sources.
- Ability to solve problems and apply theory into practice.

#### Personal:

- Ability to criticize and self-criticize.
- Ethical involvement at work.
- Ability to work under stressful conditions.



## Systemic:

- Self-study and research skills.
- Ability to face up new situations.Awareness of quality and success.

## 4.3. Course-specific Skills

- Knowledge of basic mathematics for business techniques.



### 5. COURSE CONTENT (COURSE TOPICS)

#### LEARNING MODULES

- Linear Algebra and Matrix Theory (Unit 1)
- Differential Calculus and Integral Calculus, in one or several variables (Units 2 to 4)
- Sequences and Series (Unit 5)

#### **COURSE TOPICS**

## UNIT 1: Basic Elements on Linear Algebra and Matrix Theory

- 1.- Representation of Economic Data through Real Matrices. Types of Matrices and Matrix Operations.
- 2.- Considering Variables of Several Dimensions. Vector Operations. Linear Dependence and Linear Independence.
- 3.- Linear Models of Several Equations. Solving and Interpreting Systems of Linear Equations.
- 4.- Computer Applications.

<u>Description</u>: Many economic models can be formulated in terms of systems of linear equations, for example the static and dynamic models of equilibrium or the multisectorial models. All these formulations as well as Matrix Theory are used in the applied econometric models, simplifying them considerably, at times. From a historic point of view, Matrix Theory has been developed due to the need of studying methods to solve such systems.

The main purpose of this unit is to enable the student to solve any linear equation system that he can find. The fact that the student will be dealing with these systems later on in this subject and in others makes it absolutely essential to be able to handle them. This unit will therefore be treated in a practical way. In the setting out and solutions of the systems of equations presented here, matrices are extremely important, because they simplify the analysis of consistency and the determination of the number of solutions it has.

All concepts in this unit are neither new to the student nor extremely complex. Therefore those who work daily and follow the explanations well will feel at ease handling these tools.

### UNIT 2: Real-valued Functions of One Real Variable

- 1.- Analysis of Basic Functions in the Context of Economics, Business and Management. Domain, Continuity and Graphical Representation.
- 2.- Absolute Variation and Relative Variation.



- 3.- Computing Derivatives of Functions of One Real Variable. Higher Derivatives. Economic Interpretation.
- 4.- Computer Applications.

<u>Description</u>: In the world of economy it is fundamental to establish relations between the different economic variables which allow us to easily analyze the influence from one to other. Such relations are usually expressed through functions and the changes produced in the variables are studied with the help of the concept of derivative. In this second unit special attention is paid to the real functions of one variable as the previous step to analyze the functions of several variables, which will be seen in the third unit. So, the main aim of this unit is to provide students with a deeper understanding of the computation of derivatives as well as other questions.

The main challenge the student will face in this unit is to master the computation of derivatives. Although it is an aspect which, in general, has been looked at in Bachillerato, some students have not practiced enough and need to devote some time to it. In order to compute derivatives of one variable functions, it is absolutely essential to go ahead with the partial derivatives which will be introduced in the next unit, and these, on the other hand, are essential to solve optimization problems, faced in the future.

#### **UNIT 3: Functions of Several Variables**

- 1.- Real-valued Function. Functions in Economics. Real-valued Function of Several Variables. Graph of a Function. Some Type of Functions According to Their Economic Interpretation. Contour Lines (or Level Set).
- 2.- Continuity. Properties.
- 3.- Partial Derivatives. Gradient. Marginal Values. Elasticity. Interpretation.
- 4.- Homogeneous Functions.
- 5.- Computer Applications.

<u>Description</u>: In the world of business and economics, it is common to express the relations between different economic variables through several variable functions. As common examples, we can point out: utility function, which measures the level of satisfaction which the consumption of an amount of goods gives to an individual; production function, which indicates the level of production obtained from the amount used; costs functions, which allow to calculate the cost associated to the process of production, knowing the amount of each of the productive factors which have been employed; etc.

In this unit, some of the concepts previously introduced are widened and the main purpose here is that the student acquires the ability to deal with functions of several variables and with the computation of partial derivatives, relating them to different economic aspects. We also explain homogeneous functions since they are fundamental in the Economic Theory. Economic Analysis supplies many examples which naturally hold homogeneous functions: productive processes with increasing or decreasing scale



outputs, some demand functions, production functions, etc.

Taking into account the problems which this unit involves, the most significant one is the lack of handling in the computation of derivatives of one variable functions which obviously limit a proper computation of partial derivatives. The student will be aware of the importance of a good use of these techniques, dedicating to it as much time as needed.

## **UNIT 4: Integration**

- 1.- Primitive and Indefinite Integral. Computing Primitives.
- 2.- Definite Integral: Interpretation and Properties. Barrow's Rule.
- 3.- Improper Integrals.
- 4.- Double Integral. Integrals on General Regions.
- 5.- Computer Applications.

<u>Description</u>: Unit 4 deals with simple and multiple integrations. The student must become competent in handling of these integration techniques in order to apply them later to several variables functions or to other subjects such as Microeconomics and Statistics. In these subjects, these tools are very useful to put into practice some concepts: the surplus of the consumer, and the determination of probabilities for bidimensional random variables from the probability density function.

To make a distinction from the computation of derivatives, which is mainly mechanical, the computation of primitives is much more complex. The most common methods, previously described, can help the student to tackle the computation, but only dedication and time will enable the student to face them easily.

#### **UNIT 5: Sequences and Series**

- 1.- Numerical Sequence. Limits of Sequences. Convergent and Divergent Sequences. Computing of Limits.
- 2.- Numerical Series: Series with All their Terms Positive.
- 3.- Addition of Series: Arithmetic Series and Geometric Series.
- 4.- Functional Sequences. Convergence.
- 5.- Computer Applications.

<u>Description</u>: The first part of the unit is devoted to the study of numerical sequences, especially the methods to compute limits of those which are convergent (with a finite limit), as an introduction to the study of numerical series. Arithmetic and geometric series are especially interesting due to their utility in the computation of actual and real values of financial rents, which will be studied further ahead in the subject of Financial Mathematics.

The concepts introduced in this unit are perhaps the most unfamiliar to students. These concepts can seem too abstract and it creates a more difficult challenge for their assimilation. Anyway, we try to deal with them easily and to complete the unit in a practical way, in order to reduce their complexity.



#### 6. METHODOLOGY AND RESOURCES

On-line and on-site teaching will be combined in the development of the subject.

The theoretical part, in which the basic aspects of the subject will be shown, should be taken by the students mainly on-line, using the WebCT platform, according to the schedule specified by the lecturers. On-site classes are mostly intended to solve problems related to Business Sciences, highlight their right understanding and apply the right mathematical theorems and make the subject as practical as possible.

As regards on-site teaching, it will be organized into the following two modules:

## • General/background teaching (GT):

On-site sessions of 1.5 hours each (one per week) will be set throughout the whole term. These classes will be based on formal lessons taught by the lecturer and their main aim will be to introduce, in a schematic and general way, the main theoretical aspects of every unit, so that students can deal with their on-line development by themselves later on. Sessions will be also devoted to solve problems on the blackboard by the lecturer. To do this, skills developed in theoretical lessons and online will be applied.

### • Practical/developmental knowledge-building teaching (PT):

These teachings will be also developed throughout on-site sessions, of 1.5 hours each, during the whole term. Lessons will be mainly practical and they will be devoted to solving problems by students. Some of these sessions will be hold in the computers room and directed by the lecturer. The final aim of these special sessions in the computers room (SSC) consists on providing students with the opportunity to learn how to use the symbolic computing software *Mathematica* so that they will apply it to solve similar problems to those set out during the other sessions. Specific notes about this program will be presented to students prior to SSC taking place, so that students could work on them beforehand.

Finally, personalized tutorials are optional for students. Lecturers will give students some guidance on their personal study, if they need so, to clarify specific doubts related to the subject contents, to correct wrong achieved concepts and habits, to make up some basic knowledge from students with a previous low-level and to provide additional bibliography. Every lecturer will provide some office hours for students, which will be communicated to them at the beginning of the course and they will be published on the WebCT.



#### 7. ASSESSMENT

According to Article 5 of the *Normativa de Evaluación de los Estudiantes de Grado de la Universidad Pablo de Olavide, de Sevilla*, the ongoing assessment system will be considered preferential. This preference will allow the student to reach the competences and knowledge progressively and sequentially. In this sense, the student will complete different activities during the term and all of them will be taken into account. They will be weighted differently in the final assessment depending on how important and difficult they are and how much effort has been put in by the students.

## • Continuous evaluation (ongoing assessment):

Throughout the term, several tests and exercises will be carried out to follow student development in the acquisition of competences proposed in the Course. This ongoing assessment will take up 50% of the final mark, that is, 5 points out of 10. There will be various types of test:

- Theoretical knowledge will be tested at the end of each unit through an on-line, multiple choice questionnaire (with one or more true answers). If technical requirements meet, these exams will be taken through the Blackboard Learn platform on the dates assigned at the time; on the contrary, the corresponding tests will take place on the sessions determined by the lecturers. The total value of these tests will be 1 point (i.e., 10% of the total mark of the course and 20% of the continuous evaluation).
- At the end of each unit, and in the PT session which will be announced, the student will have to solve different exercises corresponding to the unit. These will be corrected and will have a total value of 2 points (that is, 20% of the total mark of the course and 40% of the continuous evaluation).
- Throughout the term, there will be some computer room sessions (SSC). The use of *Mathematica* software will be assessed within these sessions by solving some practical exercises using the computer, as well as practical exercises proposed by the lecturer and related to those given out to students before the computer room sessions. These exercises will have a total value of 2 points (what is 20% of the total mark of the course and 40% of the continuous evaluation).

In order to pass the continuous evaluation, the student has to reach at least 2.5 points (out of 5) in this part (i.e., 25% of the total mark of the course and 50% of the continuous evaluation). In the 1<sup>st</sup> summons, only those students fitting the conditions of the current regulations (i.e., proved illness, labor incompatibility, talented sportsmen and sportswomen, etc.) can resit the ongoing assessment. In the 2<sup>nd</sup> summons, the ongoing assessment can be retaken by those students who



failed the continuous evaluation during the term and by those student who passed but decided to refuse to his/her mark; in this second case, the student must notify his/her decission 10 days prior the 2<sup>nd</sup> summons exam to the course coordinator (ammarcar@upo.es).

## • Final exam (1<sup>st</sup> summons):

According to Article 7 of the *Normativa de Evaluación de los Estudiantes de Grado de la Universidad Pablo de Olavide, de Sevilla*, the evaluation in the *convocatoria de curso* (1<sup>st</sup> summons) will be shaped by the characteristics of the ongoing assessment. This exam will be taken at the end of the term and will make up 50% of the final mark, that is, 5 points out of a total of 10. Theoretical knowledge will be evaluated through short questions or through multiple choice questions with a value of 1.5 points. Practical knowledge will be worth 3.5 points and will be tested by resolving problems.

<u>WARNING</u>: In order to pass the course in this first summons, the two following minimal marks are needed:

- Ongoing assessment: 2.5 points out of 5.
- Total mark (ongoing assessment + final exam): 5 points out of 10.

That is, if the minimal mark in the ongoing assessment is reached, the final mark of this course will be the addition of the marks obtained in the written exam and in the ongoing assessment. A total of 5 points is needed to pass this course.

## • Resitting exam (2<sup>nd</sup> summons):

According to Article 8 of the *Normativa de Evaluación de los Estudiantes de Grado de la Universidad Pablo de Olavide, de Sevilla*, students who failed the curse in the 1<sup>st</sup> summons have a second chance of passing it in the so-called *convocatoria de recuperación de curso* (2<sup>nd</sup> summons) which will take place within June/July.

In this second summons, the students will have to take a final exam. Besides, the mark obtained in the ongoing assessment will also be considered. There are three possible cases here:

- a) If the student reached the minimum in the ongoing assessment during the term (2.5 points out of 5), then his/her mark can be computed by adding the mark in the ongoing assessment with the mark in this final exam (2<sup>nd</sup> summons). This sum has to reach at least 5 points (out of 10) in order to pass the course.
- b) If the ongoing assessment had not been completed during the term, the student will also be tested on them in the 2<sup>nd</sup> summons (so, this second final exam will include an extra part). Please remember that, in this course, and regarding the aspects indicated by Article 8 of the assessment regulations, the continuous evaluation is said to be completed (or passed) during the term when the student have at least 2.5 points (out of 5) in the sum of all the tests involving such ongoing assessment.

The written part of this second final exam will have a maximum mark of 5 points (out of 10), and it will have the same characteristics than the first summons' final exam.



The additional tests regarding the ongoing assessment will take place in the computers romm, and this part of the exam consists of theoretical and practical exercises that will be solved with the aid of Mathematica program.

<u>WARNING</u>: In order to pass the course in this second summons, the two following <u>minimal marks</u> are again needed:

- Ongoing assessment: 2.5 points out of 5.
- Total mark (ongoing assessment + final exam): 5 points out of 10.
- c) The student that reached the minimum in the ongoing assessment has the chance of refusing to his/her mark, in order to sit an exam with a maximum mark of 10 points. In this specific case, all the rules of section b) apply, and the student has to take both tests specified in that section b). This decission must be notified to the course coordinator (ammarcar@upo.es) 10 days prior the 2<sup>nd</sup> summons exam.

When taking exams (either the 'minimal knowledge' or the written exam), students will not be allowed to use reference or support materials, except from those explicitly authorised by the course lecturers. Specifically, mobile phones or any other kind of devices that store, communicate or share information are totally forbidden; its use will imply a failing grade and can bring academic sanctions.

To sit exams, students must be officially identified through their ID or another official form of identification.



### 8. BIBLIOGRAPHY

#### GENERAL READING

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#### **FURTHER READING**

#### PREREQUISITES:

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