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Sobre el agotamiento de recursos no renovables y no duraderos con un proveedor monopolista.

On the depletion of non-durable, non-renewable resources with a monopolistic supplier

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RESUMEN

El objetivo de este artículo es proporcionar elementos para responder a la pregunta de cómo y cuándo se alcanzaría la situación de agotamiento de un recurso no renovable y no duradero. Para ello se utiliza un modelo de programación no lineal bajo el supuesto de que el recurso está en manos de un monopolista que conoce las funciones de demanda correspondientes y busca maximizar el valor actual de sus ingresos netos. Se establecen propiedades generales de las políticas óptimas de extracción y los precios correspondientes y se analizan los casos de funciones de demanda exponenciales negativas y lineales. El estudio muestra que no es posible determinar un patrón de política óptima válido para todos los escenarios, ya que dicha política depende de la disponibilidad inicial del recurso, las funciones de demanda, los costes, el tipo de interés y la duración del horizonte de planificación adoptado por el monopolista.

PALABRAS CLAVE

Recursos no renovables; agotamiento de recursos; políticas óptimas de extracción; monopolio; pico del petróleo; programación no lineal.

ABSTRACT

The aim of this article is to provide elements to answer the question of how and when the situation the situation of depletion of a non-renewable, non-durable resource would be reached. To this end, a nonlinear programming model is used under the assumption that the resource is in the hands of a monopolist who knows the corresponding demand functions and is trying to maximize the present value of their net income. General properties of optimal extractive policies and corresponding prices are established, and the cases of negative exponential and linear demand functions are analyzed. The study shows that it is not possible to determine a pattern of optimal policy that is valid for all scenarios, since such a policy depends on the initial availability of the resource, the demand functions, the costs, the interest rate and the duration of the planning horizon adopted by the monopolist.

KEYWORDS

Non-renewable resources; resource depletion; optimal extraction policies; monopoly; peak oil; non-linear programming.

Clasificación JEL: Q31, L12, C61.

MSC2010: 91B38, 90C30, 49K27.

1. INTRODUCTION

For some time, there has been widespread awareness that certain non-renewable and nondurable resources exist in limited quantities relative to demand, and therefore may be fully exhausted or rendered inaccessible due to rising extraction costs. This has prompted debates and research on when and how, gradually or abruptly, such scarcity might occur. Given that some non-renewable resources play a central role in consumption, production, and distribution, anticipating their availability is crucial for planning measures to manage their depletion.

This paper examines optimal policies for resource exploitation under several scenarios, assuming that the resource is controlled by a monopolist who knows the demand functions and seeks to maximize the present value of net income. The aim is to illustrate how the resulting dynamics depend on the economic context and the initial availability of the resource.

As discussed in Section 2, the issue has been examined in the literature from a variety of perspectives, typically under restrictive assumptions. In Hotelling (1931) was stated what is now known as Hotelling's law, relating to the evolution of the price of an exhaustible resource in perfect competition. Hubbert's theory (Hubbert, 1956), or the peak oil hypothesis, posits that oil production rises until roughly half of total reserves have been extracted and, once the maximum extraction rate (peak oil) is reached, subsequently declines until the resource is exhausted. This framework has been widely used, both within and beyond academia, as a model of the evolution of oil production and consumption, and has also been extended to other non-renewable resources. However, neither theory nor empirical evidence provides sufficient support for applying Hubbert's model to exhaustible resources in general. Rather, the dynamics of production, consumption, and prices are likely to depend on factors such as the initial availability of the resource, the evolution of demand, interest rates, the expected time horizon of resource exploitation, and the market structure governing the supply of the resource.

In this paper we use a discrete-time approach and nonlinear programming as the tool to determine optimal policies. Although the continuous-time approach is more common in the literature, since was used in Hotelling (1931), the discrete-time framework has also been proposed and developed by several authors (Conrad and Clark, 1987; Sweeney, 1993; Conrad, 2010; Corominas and Fossas 2015; Corominas, 2017). Perman et al. (2011) discuss both approaches and provide a detailed treatment of the discrete-time case. A discrete-time formulation, with a very simple model structure, facilitates the derivation of general expressions and the computation of numerical solutions, while allowing considerable flexibility with respect to the underlying assumptions.

The rest of the paper is organized as follows. Section 2 provides a brief review of the literature. Section 3 specifies the assumptions adopted, formulates the optimization model, and presents the general results derived from it. The properties of the optimal solutions when the demand function is negative exponential or linear are analyzed in Sections 4 and 5, respectively. Section 6 concludes with a discussion of the main results and directions for further research.

2. A SHORT LITERATURE REVIEW

The purpose of this section is to provide the most significant references directly related to the issue addressed in the article. For a more detailed study of the relevant literature, see Corominas & Fossas (2015). Comprehensive treatments of the economy of exhaustible resources may be found in Dasgupta and Heal (1979), Conrad and Clark (1987), Sweeney (1993), Conrad (2010) and Perman et al. (2011).

Until well into the second half of the 20th century, the prevalent assumption was that the Earth could indefinitely provide all the resources necessary to perform human activities. However, since then, due to demographic and economic growth, among other causes, there has been growing awareness that non-renewable resources (fossil fuels, phosphates, metals, etc.) are limited; in particular, those that are not durable, which disappear as such when used, may be exhausted in the-not-too distant future. Their duration depends on the rate of consumption, which is the result of the conjunction of supply and demand; this, in turn, depends on public policies that encourage or discourage consumers to use certain resources.

However, in Hotelling (1931) the author had already established as the motivation of his paper that "Contemplation of the world's disappearing supplies of minerals, forests, and other exhaustible assets has led to demand for regulation of their exploitation. The feeling that these products are now too cheap for the good of future generations, that they are being selfishly exploited at too rapid a rate, and that in consequence of their excessive cheapness they are being produced and consumed wastefully, has given rise to the conservation movement".

It is therefore logical to ask, for a given scenario, when and how the exhaustion of the resource could occur, in order to implement measures to either avoid it or to prepare alternative options in the event of exhaustion.

In any case, the first article to explicitly consider the eventuality of depletion is Gray (1914), in which the author establishes that the present value of the marginal net revenue obtained from the resource must be the same for all periods in which extraction is positive. This result, collected in Conrad (2010), is remarkable and anticipates that obtained with the Karush, Kuhn and Tucker conditions (hereafter, KKT) with the model used in this article.

Nevertheless, Hotelling (1931) is considered the seminal article in the economics of non-renewable resources (Solow 1974; Devarajan and Fisher 1981; Arrow 1987; Gaudet 2007, among others). It is in this article that the now famous Hotelling's law is formulated, according to which it is reasonable to expect that, in perfect competition, the price of an exhaustible resource evolves according to the expression $p = p_0 \cdot \exp(\gamma t)$, where γ is "the force of interest". Regarding the possibility of depletion, the author establishes that, within the framework of the assumptions specified in the article, if the demand function is a negative exponential, extraction will continue indefinitely; if it is linear, the resource will be exhausted in a finite time, a result analogous to that

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shown in the present article for the monopoly case. It is clear, however, that in this type of resource it is very difficult for the conditions that characterize perfect competition to be met. Hotelling himself indicates that this law is not applicable to the case of monopoly, which is also analysed in his article.

Between Hotelling's article and the 1970s, there were few publications on the subject (among others, Byé, 1957 and Barnett and Morse, 1963), although the theory of peak oil or Hubbert's peak (Hubbert, 1956) corresponds to this period. Hubbert's theory, succinctly described in Section 1, was initially presented as an empirical result obtained by fitting historic production data to a symmetric bell-shaped function (Cleveland and Kaufmann 1991). Since then, other authors have come up with economic explanations of why the Hubbert curve, or similarly shaped curves, represent the time evolution of non-durable and non-renewable resource production (Menard and Sharman 1975; Cleveland and Kaufmann, 1991; Al-Jarri and Startzman 1999; Reynolds, 1999; Bardi 2005, 2007, 2009; Jakobsson et al. 2012). Debates about whether Hubbert's peak has been reached have become frequent. From an analysis of global production data, Aleklett et al. (2010) concludes that the peak oil was probably reached in 2008 and Campbell (2012) concurs. On the other hand, Lynch (2003) does not give validity to the theory of peak oil itself. Instead, several authors extend the peak theory to other kinds of exhaustible resources (e.g., peak phosphorous, Cordell & White, 2011). In any case, the peak oil theory –beyond transmitting a warning message about the depletion of non-renewable resources, and regardless of whether or not the production curve is bell-shaped and symmetrical– is not operational, given the uncertainty about the moment when the peak has or will occur.

Concern about the scarcity of exhaustible resources was triggered above all by the report on the limits to growth, sponsored by the Club of Rome (Meadows et al., 1973), the oil crisis of 1973 and the relevance that Solow gave to Hotelling's article in the Richard T. Ely Lecture, in December of that same year (Solow, 1974). Since then, the number of publications on the economics of exhaustible resources has grown steadily.

The unit cost of resource extraction and distribution has a varying impact on the configuration of the optimal policy. In general, it can depend, for each resource, on the quantity available at any given time (and therefore on the accumulated production; Malueg & Solow, 1990), on the extraction rate (Levhari & Pyndick, 1981) and on the available technology, which can be considered as a function of time.

Some publications dealing with optimal policies of exhaustible resources in a monopolistic market include a formulation of the optimization problem, the necessary and sufficient conditions of optimality and the solution to particular cases, such as linear demand curve. However, Corominas & Fossas (2015) show general properties and conclusions derived from a non-linear programming formulation and KKT conditions under diverse assumptions regarding the demand curve, discount rates and length of the planning horizon. Corominas (2017) extends the approach to the study of partially or totally durable exhaustible resources.

This article builds on and develops the results shown in Corominas & Fossas (2015), summarized in the following section, and analyses in detail the implications of the model proposed there, in order to explore the issue of how and when a non-renewable and non-durable resource can be expected to become exhausted.

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3. THE MATHEMATICAL PROGRAMMING MODEL AND THE GENERAL RESULTS DERIVED FROM IT

We consider the problem of determining the extraction rate of a non-durable and non-renewable resource with a known available amount, R>0, to optimise the present value for its monopolistic supplier in a planning horizon of T periods of equal or different duration.

We assume that the supplier's production capacity is sufficient to meet the demand and that the amount of resource extracted in a period must be consumed during the same period (i.e. there is no possibility of stockpiling the resource in one period for its sale in subsequent periods). We also assume as known:

1. The inverse demand function of each period, $p_t(q_t)$, giving the price associated with quantity q_t. It is defined for $0 \leq q_t \leq Q_t$, with $p_t(Q_t) = 0$ and $p'_t(q_t) < 0$. Moreover, $p_t(0) = P_t$, where P_t denotes the choke price (the price at which the quantity demanded falls to zero). If the choke price is unbounded above, we assume instead that $\lim_{q_t \to 0} p_t(q_t) \cdot q_t = 0$ for all t.
   We will deal with inverse demand functions such that $p_t(q_t) \cdot q_t$, the gross income of period t, is a concave function (it is straightforward to check that this is equivalent to $p''_t(q_t) \leq -2 \cdot p'_t(q_t)/q_t$). Given that the gross income is null for q_t=0 and for $q_t = Q_t$, the concavity implies that it reaches its maximum at some intermediate point in $(0, Q_t)$.
   Note that we allow for the possibility of different demand functions throughout the planning horizon. Demand for an exhaustible resource may increase or decrease for various reasons, such as changes in demography, technology, or the availability and prices of alternative resources.

2. The costs of extracting and distributing a unit of resource, c_t, where $c_t < P_t$.
   Therefore, as the gross income of every period is a concave function, so are the corresponding net incomes.

3. The discount coefficients α_t (such that $0 < \alpha_t \leq 1$ for all t and $\alpha_t \geq \alpha_{t+1}$, $t = 1, \ldots, T-1$) are applied to the net income of each period to obtain its present value.
   Although usually α_t is equal to $(1+i)^{-t}$, where i is the interest rate (assumed to be non-negative), our formulation is more flexible and allows the interest rate to vary along the planning horizon.

Taxes are not explicitly considered in the model, but, depending on their nature, they can be incorporated as either a fixed amount per product unit or a percentage of the price.

Therefore, in order to obtain the optimal extraction policy, we can formulate a non-linear mathematical program as follows:

$$\underset{\{q_t\}_{t=1}^{T}}{\text{maximize}} \sum_{t=1}^{T} f_t(q_t) = \sum_{t=1}^{T} \alpha_t \cdot [p_t(q_t) - c_t] \cdot q_t$$

s.t.

$$\sum_{t=1}^{T} q_t \le R$$

$$0 \le q_t \le Q_t \quad t = 1, \dots, T \tag{1}$$

As the constraints are linear and define a set with interior points, and the functions f_t are concave, the KKT conditions are necessary and sufficient for optimality and, thus, can be used to analyse the properties of the optimal solutions. Given the particularly simple structure of the mathematical program, the KKT conditions also make it possible to determine the numerical values that define the optimal policy for a specific demand function.

As R increases, optimal production rises and the corresponding prices fall.

Let $u(T) \ge 0$ be the KKT multiplier corresponding to eq. (1), the resource availability constraint. This multiplier is the shadow price of the resource, which is equal to zero when this constraint is not active $(\sum_{t=1}^{T} q_t < R)$ in an optimal solution, i.e., when the optimal policy does not exhaust the resource.

Let q_t^* be the values of the variables in an optimal policy and $\hat{q}_t$ the optimal quantities in each period under the assumption that there is no stock limitation.

Depending on the characteristics of the functions involved, the numerical values of the parameters and the duration on the planning horizon, T, there may be active $(q_t > 0)$ and inactive $(q_t = 0)$ periods.

Regarding the structure of the optimal solutions we must distinguish two cases, as indicated below, depending on whether or not T is sufficiently long for the optimal policy to exhaust the resource.

3.1 T too short to exhaust the resource

$T | \sum_{t=1}^{T} \hat{q}_t \le R$: all periods are active and the optimal production of each period is the same as if the resource stock were unlimited.

Therefore, the optimality conditions are:

$$f_t'(q_t) = \alpha_t \cdot [p_t'(q_t) \cdot q_t + p_t(q_t) - c_t] = 0; \text{ i.e., } p_t'(q_t) \cdot q_t + p_t(q_t) - c_t = 0 \ \forall t$$

Note that this result is independent of the discount coefficients as well.

3.2 T long enough to exhaust the resource

If *T* is such that $\sum_{t=1}^{T} \hat{q}_t > R$, the unconstrained optimal policy would be infeasible. Therefore, the resource will be exhausted $(\sum_{t=1}^{T} q_t^* = R)$ and there may be inactive periods.

Since the availability constraint is active in this case, we have for all active periods:

$$f_t'(q_t) = \alpha_t \cdot [p_t'(q_t) \cdot q_t + p_t(q_t) - c_t] = u(T) > 0 \tag{2}$$

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where $u(T)$, the shadow price of the resource, is positive (because $q_t^* < \hat{q}_t \; \forall t$), and increases monotonically with T (the shadow price of the resource cannot decrease when the planning horizon increases, since this enlarges the space of feasible solutions).

Instead, for the inactive periods we have:

$$f_t'(0) = \alpha_t \cdot (P_t - c_t) \leq u(T)$$

where $P_t - c_t$ is the net choke price (hereafter, for the sake of simplifying the notation, we will use $\tilde{P}_t = P_t - c_t$).

That is, under an optimal policy the discounted marginal net revenues are identical across all active periods and equal to the shadow price of the resource, and no smaller than the discounted marginal net revenue at the origin of any inactive period.

Determining which are the active periods under an optimal policy is not straightforward, but to do so the following proposition (which holds if $p_t'(0)$ is finite or $\lim_{q_t \to 0} p_t'(q_t) \cdot q_t = 0$) is useful (see Corominas & Fossas, 2015 for the formal proof of this proposition):

$$(q_t^* > 0 \wedge q_{t'}^* = 0) \longrightarrow \left(f_t'(0) = \alpha_t \cdot \tilde{P}_t \geq f_{t'}'(0) = \alpha_{t'} \cdot \tilde{P}_{t'}\right) \quad (3)$$

Indeed, if the derivative at the origin of the term associated with period t in the objective function exceeds that for another period t', a solution with t'active and t inactive cannot be optimal, as income would increase by shifting extraction from t' to t.

Note that, if $\tilde{P}_t$ is not bounded above, t will be active in all optimal solutions. If this condition is fulfilled for all t, all periods will be active.

Hence, if the periods are arranged in the non-increasing order of the products $\alpha_t \cdot \tilde{P}_t$, there is always an optimal policy so that the active periods are either all those of the planning horizon or a certain number of them occupying the first positions in the sequence thus defined, which we will call σ-sequence. The subscript $[x]$ will hereafter be used to indicate the position of the period x in the σ-sequence.

The order of the periods on the σ-sequence does not depend, therefore, on the available amount of the resource nor on the shape of the demand functions, but only on the net choke prices.

Let $\tilde{q}_{[\tau]}[u(T)]$ denote be the quantities extracted that satisfy equation (2) for a given value of $u(T)$. For some demand functions, including the negative exponential and the linear (see Sections 4 and 5), $\tilde{q}_{[\tau]}[u(T)]$ admits a closed-form expression depending on the parameters and on $u(T)$.

Let Ω denote the number of active periods in an optimal policy. Then, starting with the first ω_0 periods of the σ-sequence, where ω_0 is such that $\sum_{\tau=1}^{\omega_0-1} \hat{q}_{[\tau]} \leq R$ and $\sum_{\tau=1}^{\omega_0} \hat{q}_{[\tau]} > R$, one can calculate the values $u(\omega) | \sum_{\tau=1}^{T} \tilde{q}_{[\tau]} [u(\omega)] = R$ for $\omega = \omega_0, \omega_0 + 1, \ldots, T-1$ or until $\omega = \tilde{\omega} < T - 1 | u(\tilde{\omega}) \geq \alpha_{[\tilde{\omega}+1]} \cdot \tilde{P}_{[\tilde{\omega}+1]}$. If this condition is met, the first and only the first $\tilde{\omega}$ periods of the σ-sequence are active in an optimal solution, that is, $\Omega = \tilde{\omega}$; if it is not fulfilled for any value of $\omega \leq T-1$, $\Omega = T$.

In the case where the inverse demand function is the same for all periods, (i) it is clear that if $\alpha_t = 1$ for all t, all periods will be active under the optimal policy, with the same value of $q_t^* = R/T$; (ii) on the other hand, if the sequence of discount coefficients strictly decreases, so will that of optimal production, while that of prices will strictly increase. This trend of decreasing quantities and increasing prices can be reinforced or compensated, partially or totally, by a sustained decrease or increase in demand.

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4 THE CASE OF NEGATIVE EXPONENTIAL DEMAND FUNCTIONS

We assume in this section that the demand functions are:

$$q_t = Q_t \cdot exp(-\lambda_t \cdot p_t), \text{ for } t = 1, \dots, T.$$

Therefore, the inverse demand functions are:

$$p_t = -\frac{1}{\lambda_t} \cdot ln\frac{q_t}{Q_t}, \text{ where } 0 \le q_t \le Q_t, \text{ for } t = 1, \dots, T.$$

And:

$$f_t(q_t) = \alpha_t \cdot \left(-\frac{1}{\lambda_t} \cdot ln\frac{q_t}{Q_t} - c_t\right) \cdot q_t,$$

which are concave functions for all t.

4.1 T too short

We have:

$$q_t^* = \hat{q}_t = \frac{Q_t}{e} \cdot exp(-\lambda_t \cdot c_t)\,;\; p_t^* = \frac{1}{\lambda_t} + c_t$$

Hence, for $\lambda_t = \lambda, c_t = c\ \forall t$:

$$q_{t'}^*/q_t^* = Q_{t'}/Q_t\ \forall t', t$$

$$p_t^* = \frac{1}{\lambda} + c$$

Then, under the above assumption concerning λ and c, as long as the condition that T is too short holds, production will increase or decrease proportionally to the sequence of Q_t but the price will be the same throughout the planning horizon.

4.1 T long enough

As the choke prices are not bounded above, all periods will be active.

The simplest setting is defined by the assumptions that, for all $t, \alpha_t = 1,\ \lambda_t = \lambda, c_t = c$. In this case, $q_t^* = \frac{R}{T}$ and $p_t^* = -\frac{1}{\lambda} \cdot ln\frac{R}{T \cdot Q}$: production quantities and prices would be the same in all periods.

However, if the assumption $\alpha_t = 1$ for all t does not hold, as is usually the case, it follows from the KKT conditions that:

$$q_t^* = Q_t \cdot exp\left[-\lambda_t \cdot \left(\frac{u(T)}{\alpha_t} + c_t\right) - 1\right]\ \forall t \tag{4}$$

$$\sum_{t=1}^{T} q_t^* = \sum_{t=1}^{T} Q_t \cdot exp\left[-\lambda_t \cdot \left(\frac{u(T)}{\alpha_t} + c_t\right) - 1\right] = R \tag{5}$$

This equation allows calculating $u(T)$ and, from it, the optimal productions —using (4)—and the corresponding prices:

$$p_t^* = -\frac{1}{\lambda} \cdot ln\frac{q_t^*}{Q_t} = \frac{u(T)}{\alpha_t} + \frac{1}{\lambda_t} + c_t\ \ \forall t \tag{6}$$

Therefore, for $Q_t = Q,\ \lambda_t = \lambda, c_t = c\ \forall t$, and given that the sequence α_t is assumed to be decreasing strictly, so will the sequence of extracted quantities and consequently prices will be increasing strictly (if $T \to \infty$, production will gradually tend to 0 and prices will rise without limit). Under these assumptions, the steeper the decline in the sequence of discount factors, the faster production falls and the price rises.

It is straightforward to find closed form expressions for comparing production and prices corresponding to two different periods. For instance, if $\alpha_t = 1/(1+i)^{t-1}$; $\lambda_t = \lambda, c_t = c\ \forall t$:

$$\frac{q_{t+\tau}^*}{q_t^*} = \frac{Q_{t+\tau}}{Q_t} \cdot exp(-\lambda \cdot u(T) \cdot ((1+i)^\tau - 1) \cdot (1+i)^{t-1}) \quad (7)$$

$$\frac{p_{t+\tau}^*}{p_t^*} = \frac{\lambda \cdot u(T) \cdot (1+i)^{t+\tau-1} + 1 + \lambda \cdot c}{\lambda \cdot u(T) \cdot (1+i)^{t-1} + 1 + \lambda \cdot c} \quad (8)$$

$$p_{t+\tau}^* - p_t^* = ((1+i)^\tau - 1)(1+i)^{t-1} \cdot u(T) \quad (9)$$

These equations allow analysis of the properties of optimal policies under specific assumptions. For instance, for $\lambda_t = \lambda$, c_t=0 for all t and a given sequence of the discount coefficients, the production corresponding to optimal policies is the same for all values of λ , since the value of the product $\lambda \cdot u(T)$ that fulfils equation (5) is the same for all, and the optimal production depends only on this product and the corresponding discount coefficients. Under the specified assumptions, prices depend on λ, according to (6); however, the ratio between the prices of two different periods does not, according to (8), take into account that $c = 0$.

In general, the evolution of production and prices throughout the planning horizon depends on the values of Q_t and on the discount coefficients (see numerical examples in Table 1).

Table 1

Scenario	*R*	Q_t	λ	*c*	*i*	q_T^*/q_1^*	p_T^*/p_1^*
E1	1,000	500	>0	0	0.025	0.873	1.088
E2	1,000	$500 \cdot 1.02^{t-1}$	>0	0	0.025	1.023	1.096
E3	1,000	500	0.01	25	0.025	0.923	1.051
E4	100	500	>0	0	0.025	0.521	1.180
E5	100	500	0.01	25	0.025	0.551	1.163
E6	100	500	0.10	25	0.025	0.912	1.024
E7	1,000	500	0.10	25	Any	1.000	1.000

Numerical examples for the case of negative exponential demand function (with $T = 10$))

The comparison between scenarios E1 (with the same demand function for all periods) and E2 (with exponentially growing demand for all prices) shows that the increase in demand, even if its growth rate (2.0%) is lower than the interest rate (2.5%), can compensate the declining trend in production induced by the discount. In both cases final prices exceed initial prices (in E2 because the increase in demand compensates for the effects of increased production on the price). In E3 the cost attenuates, relative to those in E1, the decrease in production and the corresponding price increase.

In E4, with a resource stock 90% lower than that of E1, the production drop and the price rise are more pronounced than in E1. But the price, as in preceding scenarios, evolves more slowly than production. The comparison between E4 and E5 is analogous to that of E1 and E3: cost attenuates the fall in production and the price rise. In E6 the effect of a higher λ goes in the same direction and notably strengthens that of cost.

The shape of the optimal solution in scenario E7 is quite different. Since $\hat{q}_t = 15.10$ for all t, T is too short for the optimal policy to exhaust the resource. Production and price are the same for all periods.

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5 THE CASE OF LINEAR DEMAND (AND INVERSE DEMAND) FUNCTIONS

In this Section we consider the following demand functions:

$$q_t = Q_t \cdot \left(1 - \frac{p_t}{P_t}\right),\ 0 \le p_t \le P_t, \text{for } t = 1, \dots, T,$$

From which the corresponding inverse demand functions are derived:

$$p_t = P_t \cdot \left(1 - \frac{q_t}{Q_t}\right), 0 \le q_t \le Q_t,\ \text{for } t = 1, \dots, T.$$

Therefore:

$$f_t(q_t) = \alpha_t \cdot \left(P_t \cdot \left(1 - \frac{q_t}{Q_t}\right) - c_t\right) \cdot q_t, \text{for } t = 1, \dots, T,$$

which are concave functions for all t.

5.1 *T* too short

As we have:

$$q_t^* = \hat{q}_t = \frac{Q_t}{2 \cdot P_t} \cdot \tilde{P}_t;\ p_t^* = \frac{P_t + c_t}{2}.$$

The net present value of the income is:

$$\sum_{t=1}^{T} \alpha_t \cdot \frac{\tilde{P}_t^2 \cdot Q_t}{4 \cdot P_t}.$$

5.2 *T* long enough

When T is long enough to exhaust the resource some periods may be inactive. Therefore, the periods have to be arranged in the non-increasing order of their net choke prices.

Then:

$$q_{[\omega]}^* = \frac{Q_{[\omega]}}{2 \cdot P_{[\omega]}} \cdot \left(\tilde{P}_\omega - \frac{u(\Omega)}{\alpha_{[\omega]}}\right) \quad \omega = 1, \dots, \Omega \tag{10}$$

$$p_{[\omega]}^* = P_{[\omega]} - \frac{1}{2} \cdot \left(\tilde{P}_{[\omega]} - \frac{u(\Omega)}{\alpha_{[\omega]}}\right) \quad \omega = 1, \dots, \Omega. \tag{11}$$

And, given that $\sum_{\omega=1}^{\Omega} q_{[\omega]}^* = R$:

$$u(\Omega) = \frac{\sum_{\omega=1}^{\Omega}\left(\frac{Q_{[\omega]}}{P_{[\omega]}} \cdot \tilde{P}_{[\omega]}\right) - 2 \cdot R}{\sum_{\omega=1}^{\Omega} \frac{1}{\alpha_{[\omega]}} \cdot \frac{Q_{[\omega]}}{P_{[\omega]}}}. \tag{12}$$

Therefore, $u(\Omega)$ can be replaced in eqs. (10) and (11) with the right-hand side of eq. (12), thus obtaining closed form expressions for computing the optimal productions and prices when there are Ω active periods in the optimal policy.

Then, it is straightforward to obtain the following relationships, which may be useful for sensitivity analysis of the optimal policy:

$$\frac{\partial u(\Omega)}{\partial R} = -\frac{2}{\sum_{\omega=1}^{\Omega} \frac{1}{\alpha_{[\omega]}} \cdot \frac{Q_{[\omega]}}{P_{[\omega]}}} < 0 \tag{13}$$

$$\frac{\partial q_{[\omega]}^*}{\partial R} = \frac{1}{\alpha_{[\omega]}} \cdot \frac{Q_{[\omega]}}{P_{[\omega]}} \cdot \frac{1}{\sum_{\omega=1}^{\Omega} \frac{1}{\alpha_{[\omega]}} \cdot \frac{Q_{[\omega]}}{P_{[\omega]}}} > 0 \tag{14}$$

$$\frac{\partial p^*_{[\omega]}}{\partial R} = -\frac{1}{\alpha_{[\omega]}} \cdot \frac{1}{\sum_{\omega=1}^{\Omega} \frac{1}{\alpha_{[\omega]}} \cdot \frac{Q_{[\omega]}}{P_{[\omega]}}} < 0 \,. \tag{15}$$

And for $c_t = c \; \forall t$:

$$\frac{\partial q^*_{[\omega]}}{\partial c} = \frac{Q_{[\omega]}}{2 \cdot P_{[\omega]}} \left(\frac{1}{\alpha_{[\omega]}} \cdot \frac{\sum_{\omega=1}^{\Omega} \frac{Q_{[\omega]}}{P_{[\omega]}}}{\sum_{\omega=1}^{\Omega} \frac{1}{\alpha_{[\omega]}} \cdot \frac{Q_{[\omega]}}{P_{[\omega]}}} - 1 \right) \tag{16}$$

$$\frac{\partial p^*_{[\omega]}}{\partial c} = -\frac{1}{2} \cdot \left(\frac{1}{\alpha_{[\omega]}} \cdot \frac{\sum_{\omega=1}^{\Omega} \frac{Q_{[\omega]}}{P_{[\omega]}}}{\sum_{\omega=1}^{\Omega} \frac{1}{\alpha_{[\omega]}} \cdot \frac{Q_{[\omega]}}{P_{[\omega]}}} - 1 \right). \tag{17}$$

When $Q_t = Q, P_t = P, c_t = c \; \forall t$, from (10) and (12) we have:

$$q^*_{[\omega+\tau]} - q^*_{[\omega]} = -\frac{1}{2} \cdot \frac{Q}{P} \cdot \left[\frac{1}{\alpha_{[\omega+\tau]}} - \frac{1}{\alpha_{[\omega]}} \right] \cdot u(\Omega) \tag{18}$$

$$p^*_{[\omega+\tau]} - p^*_{[\omega]} = \frac{1}{2} \cdot \left[\frac{1}{\alpha_{[\omega+\tau]}} - \frac{1}{\alpha_{[\omega]}} \right] \cdot u(\Omega) \,. \tag{19}$$

Therefore, if $\alpha_t = 1/(1+i)^{t-1}$ and the order of periods in the σ-sequence coincides with the chronological one, the drop in production from one period to the next increases progressively, as does the increase in price from one period to the next.

Eq. (12) shows that the multiplier u is a stepwise linear concave function of R. The values of R that determine the linear segments are those in which u is equal to each of the successive values of $\alpha_{[\omega]} \cdot \tilde{P}_{[\omega]}$. These critical values of R are the available amounts of the resource, above which the corresponding periods are active in an optimal policy or, alternatively, below which the period will be inactive, because the resource will have been depleted in the preceding period of the σ-sequence (see example in Table 2).

The properties stated in the preceding paragraph and illustrated in Table 2 derive essentially from the concavity of net revenues in each period. When the available resource stock is very small, it is optimal to exhaust it entirely in the first period of the sequence, as long as the discounted marginal net revenue in that period remains greater than the discounted marginal net revenue at the origin of the second period. At the stock level for which these two discounted marginal net revenues become equal, the second period becomes active. Additional resource availability is then allocated between the first and the second periods until discounted marginal net revenues equal the discounted marginal net revenue at the origin of the third period, and the process continues in the same way for subsequent periods.

Table 2

ω	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Critical values of R $\alpha_t = 1.025^{(t-1)} \; \forall t$	0	6.10	18.14	35.99	59.51	88.54	122.97	162.65	207.47	257.28
Critical values of R $\alpha_t = 1.100^{(t-1)} \; \forall t$	0	22.73	66.12	128.29	207.53	302.30	411.18	532.90	666.27	810.24

Critical values of R for $c_t = 0,\; Q_t = 500,\; P_t = 200,\; c_t = 0 \;\; \forall t$
In these examples, the order of periods in the σ-sequence coincides with the chronological one

Table 3 shows some numerical examples. The most important difference regarding the exponential demand function is that with linear demand functions, depending on the availability of the resource, the evolution of the demand throughout the planning horizon and the discount coefficients, some periods may be inactive and may be located anywhere within the planning horizon (for example, at the beginning, at the end or at the beginning and the end, as can be seen in the examples in Table 3).

Table 3

Scenario	R	Q_t	P_t	i	c	t_F	t_L	t_{qM}	t_{pM}	t_{qm}	t_{pm}	$q^*_{t_L}/q^*_{t_F}$	$p^*_{t_L}/p^*_{t_F}$
L1	1,000	500	$0{,}4 \cdot Q_t$	0.025	0	1	10	1	10	10	1	0.713	1.087
L2	1,000	500	$0{,}4 \cdot Q_t$	0.025	25	1	10	1	10	10	1	0.766	1.068
L3	200	500	$0{,}4 \cdot Q_t$	0.025	0	1	8	1	8	8	1	0.116	1.085
L4	1,000	$500 + 100 \cdot (t-1)$	$0{,}4 \cdot Q_t$	0.025	0	4	10	10	10	4	4	11.707	1.463
L5	100	$500 + 100 \cdot (t-1)$	$0{,}4 \cdot Q_t$	0.100	0	4	9	7	9	4	4	2.844	1.618
L6	1000	$500 - 50 \cdot (t-1)$	$0{,}4 \cdot Q_t$	0.025	0	1	9	1	1	9	9	0.033	0.326
L7	$\geq 2{,}500$	500	$0{,}4 \cdot Q_t$	Any	0	1	10	-	-	-	-	1.000	1.000

Numerical examples for the case of linear demand function (with $T = 10$)
Subscripts F and L denote the first and last active periods. Subscripts qM and qm denote the periods of maximum and minimum production, respectively, while pM and pm denote the periods of maximum and minimum prices.

In scenarios L1, L2 (both with the same value of *R*) and L3 (where *R* is 80% lower) the demand function is the same for all periods. In all three scenarios, production decreases and price increases (but, both in terms of production and price, the change is less pronounced in L2 than in L1, due to the impact of the higher unit cost of production and distribution in L2). In L3, the decline in production is much more pronounced and the resource is depleted before the planning horizon ends, in period 8, when the production reaches its minimum and the price, its maximum. However, the price increase is not very different from that of scenarios L1 and L2.

Scenarios L4 and L5 both show strong growth in demand throughout the planning horizon. In both cases, exploitation of the resource does not start until period 4. The shape of the optimal policies is basically due to the large expected increase in demand, which makes it advisable to delay the start of production.

In L4, all subsequent periods until the end of the planning horizon are active, but in L5, period 10 is inactive. In L4, the increase in production is sustained throughout the interval of active periods, the quantity extracted at *t=10* is almost twelve times that of *t=4*, and there is also a sharp increase in price.

In scenario L5, where resource availability is only 10% of that in L4 and the interest rate is much higher (0.100 versus 0.025), in the last active period (*t=9*) output and price are much higher than in the first active one (t=4). Prices increase in all periods and the total relative increase is the

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biggest of all the scenarios considered. Instead, production reaches a maximum at t=7, $q_7/q_4 = 5.078$. and $q_7/q_9 = 1.785$.

In L6, characterized by declining demand, the resource is exhausted at *t*=9, with very sharp declines in production and prices throughout the active periods.

Finally, in L7 the greater availability of the resource allows production to be optimized in each period disregarding the stock constraint and, since the demand function is assumed to be the same for all periods, so are production and prices.

As indicated in 4.2, the interest rate, through its effect on the discount coefficients, has an impact on the trajectories of production and prices in the optimal policies. As the interest rate increases, the monopolist becomes more impatient and production shifts toward the earlier periods of the planning horizon.

In this respect, the comparison of optimal policies when considering different interest rates in a scenario with the same assumptions regarding stock, demand, and costs as in L1 is illustrative. As shown in Table 4, the increase in the interest rate strongly affects the decline in production from the initial period to the final one, which in turn also leads to an increase in price, although this increase is not as pronounced as the decrease in production.

Table 4

i (%)	$\frac{q_{10}^*}{q_1^*} \cdot 100$	$\frac{p_{10}^*}{p_1^*} \cdot 100$
0,00	100	100
2,50	71	109
5,00	50	118
7.50	32	127
10,00	18	137

Effect of the interest rate on production and prices under optimal policies when $R = 1{,}000, Q = 500, P = 200, c = 0$

6 CONCLUSIONS AND PROSPECTS

The proposed models make it possible to study the behaviour of the sole supplier of a nonrenewable, non-durable resource over the planning horizon under various scenarios, defined by the available stock of the resource, and the nature and evolution of the demand functions, as well as that of the interest rate.

Analysis of the implications of the models and their application to various scenarios shows that production and prices may follow diverse trajectories over the planning horizon. The active periods can be all those of the planning horizon or only a subset of them, and the inactive periods, if any, may occur at different points within that horizon.

The configuration of the optimal policy, within the framework of the adopted assumptions, depends on the initial stock of the resource, the demand functions, the discount coefficients (and therefore on the interest rate), the production and distribution costs, and the length of the planning horizon chosen by the monopolist.

Optimal production and price trajectories vary considerably across scenarios, as illustrated in the preceding sections. In some cases, production and prices remain constant throughout the entire planning horizon and the resource is not exhausted. In others, production declines until the resource is depleted (either at the end of the horizon or earlier), with the consequent increase in prices. Likewise, production may increase but be abruptly interrupted before the end of the planning horizon. It may also occur—mainly as a result of growing demand and high interest rates—that production reaches a maximum in an intermediate period of the planning horizon, while prices increase steadily throughout all active periods.

On the other hand, the results of this study do not suggest that the moment of maximum production (the peak) must coincide with the moment when the cumulated production reaches approximately half of the total stock of the resource. The optimal policy from any given moment can depend on the past only to the extent that it affects production costs.

From this, it follows that no universal exploitation pattern leading to depletion or non-profitability can be considered valid for all non-renewable, non-durable resources.

Since the depletion of non-renewable resources is considerably more complex than suggested by the proposed models, several directions for future research can be identified. These include, among others, the analysis of oligopolistic markets, the use of demand functions different from those considered in this work and based on data, the introduction of capacity constraints in production, and production and distribution costs that depend on the quantities expected to be extracted in each period. Finally, a particularly promising line of research would be to analyse the impact of taxes on the production and prices of these non-renewable resources.

REFERENCES

Aleklett, K., Höök, M. K., Jakobsson, K., Lardelli, M., Snowden, S., & Söderbergh, B. (2010). The peak of the oil age - Analyzing the world of oil production reference scenario in World Energy Outlook 2008. *Energy Policy*, *38*, 1398-1414. https://doi.org/10.1016/j.enpol.2009.11.021

Al-Jarri, A. S., & Startzman, R. A. (1999). U.S. oil production and energy consumption. A Hubbert modelling approach to forecast long-term trends in various components of U.S. energy consumption with an emphasis on domestic oil production. *Advances in the Economics of Energy and Resources*, *11*, 37-58.

Arrow, K. J. (1987). “Hotelling, Harold”, in Eatwell, J. et al. *The New Palgrave. A Dictionary of Economics*. Macmillan.

Bardi, U. (2005). The mineral economy: a model for the shape of oil production curves. *Energy Policy*, *33* (1), 53-61. https://doi.org/10.1016/S0301-4215(03)00197-6

Bardi, U. (2007). Energy prices and resource depletion: lessons from the case of whaling in the nineteenth century. *Energy Sources*, Part B, *2* (3), 297-304. https://doi.org/10.1080/15567240600629435

Bardi, U. (2009). Peak oil: The four stages of a new idea. *Energy*, *34*, 323-326. https://doi.org/10.1016/j.energy.2008.08.015

Barnett, H. J., & Morse, C. (1963). *Scarcity and growth: The economics of natural resource availability*. Johns Hopkins Press.

Byé, M. (1957). Marché imparfait et relations internationales. L’autofinancement de la grande unité interterritoriale et les dimensions temporelles de son plan. *Revue d’Économie Politique*, *LXVII*, 269–312.

Campbell, C. J. (2012). Recognition of peak oil. *WIREs Energy and Environment*, *1*, 114-117. https://doi.org/10.1002/ wene.7

Cleveland, C. J., & Kaufmann, R. K. (1991). Forecasting ultimate oil resources and its rate of production: incorporating economic forces in the models of M. King Hubbert. *The Energy Journal*, *12* (2), 17-46. https://doi.org/10.5547/ISSN0195-6574-EJ-Vol12-No2-3

Conrad, J. M. (2010). *Resource Economics, 2nd ed*. Cambridge University Press. https://doi.org/10.1017/CBO9780511781087

Conrad, J. M., & Clark, C. W. (1987). *Natural resource economics. Notes and problems*. Cambridge University Press. https://doi.org/10.1017/CBO9781139173575

Cordell, D., & White, S. (2011). Peak phosphorous: clarifying the key issues of a vigorous debate about longterm phosphorous security. *Sustainability*, *3* (10), 2027-2049. https://doi.org/10.3390/su3102027

Corominas, A. (2017). Using discrete-time mathematical programming to optimise the extraction rate of a durable non-renewable resource with a single primary supplier. *Operations Research Perspectives*, *4*, 118-122. https://doi.org/10.1016/j.orp.2017.09.002

Corominas, A., & Fossas, E. (2015). Optimising the extraction rate of a non-durable non-renewable resource in a monopolistic market: a mathematical programming approach. *SpringerPlus*, *4*, 503. https://doi.org/10.1186/s40064-015-1276-0

Dasgupta, P. S., & Heal, G. M. (1979). *Economic Theory and Exhaustible Resources*. Cambridge University Press. https://doi.org/10.1017/CBO9780511628375

Devarajan, S., & Fisher, A. C. (1981). Hotelling's economics of exhaustible resources: fifty years later. *Journal of Economic Literature*, *21*, 1045-1051.

Gaudet, G (2007). Natural resource economics under the rule of Hotelling. *Canadian Journal of Economics / Revue Canadienne d'Économique*, *40*, 1033–1059. https://doi.org/10.1111/j.1365-2966.2007.00441.x

Gray, L.C. (1914). Rent under the Assumption of Exhaustibility. *Quarterly Journal of Economics*, *28*, 466-489. https://doi.org/10.2307/1884984

Hotelling, H. (1931). The economics of exhaustible resources. *The Journal of Political Economy*, *39*, 137–175. Reprinted in Bulletin of Mathematical Biology (1991), *53*, 281-312. https://doi.org/10.1086/254195 Hubbert, M. K. (1956). Nuclear energy and the fossil fuel. *Shell Development Company*.

Jakobsson, K., Söderberg, B., Snowden, S., Li, C.-Z., & Aleklett, K. (2012). Oil exploration and perceptions of scarcity: The fallacy of early success. *Energy Economics*, *34* (4), 1226-1233. https://doi.org/10.1016/j.eneco.2011.11.003

Levhari, D., & Pindyck, R. S. (1981). The pricing of durable exhaustible resources. *The Quarterly Journal of Economics*, *96*, 365–377. https://doi.org/10.2307/1882678

Lynch, M. C. (2003). The new pessimism about petroleum resources: debunking the Hubbert model (and Hubbert modelers). Minerals and Energy – Raw Materials Report, 18 (1), 21–32. https://doi.org/10.1080/14041040310001966a

Malueg, D. A., & Solow, J. L. (1990). Monopoly production of durable exhaustible resources, *Economica*, New Series, *57*, 29–47. https://doi.org/10.2307/2554079

Meadows, D. H., Meadows, D. L., Randers, J., & Behrens, W. W. (1972). *The limits to growth: a report for the Club of Rome's project on the predicament of mankind*. Universe Books, N.Y. https://doi.org/10.1349/ddlp.1

Menard, H. W., & Sharman, G. (1975). Scientific uses of random drilling models. *Science*, *190* (42), 337-343. https://doi.org/10.1126/science.190.4212.337

Perman, R., Ma, Y., Common, M., Maddison, D., & McGilvray, J. (2011). *Natural Resource and Environment Economics, 4th ed*. Pearson, Addison Wesley.

Reynolds, D. B. (1999). The mineral economy: how prices and costs can falsely signal decreasing scarcity. *Ecological Economics*, *31* (1), 155-166. https://doi.org/10.1016/S0921-8009(99)00098-1

Solow, R. M. (1974). The economics of resources or the resources of economics. *The American Economic Review*, *64* (2), 1-14. Reprinted in *Journal of Natural Resources Policy Research* (2009), *1* (1), 69-82. https://doi.org/10.1057/9780230523210_13

Sweeney, J. L. (1993). Economic theory of depletable resources: an introduction, chapter 17 in Kneese, A. V., & Sweeney, J. L. (eds). *Handbook of Natural Resource and Energy Economics*, Volume *3*, 759-854. https://doi.org/10.1016/S1573-4439(05)80004-1