



GARCH Family Models vs EWMA: Which is the Best Model to Forecast Volatility of the Moroccan Stock Exchange Market?

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ABSTRACT

Nowadays, modeling and forecasting the volatility of stock markets have become central to the practice of risk management; they have become one of the major topics in financial econometrics and they are principally and continuously used in the pricing of financial assets and the Value at Risk, as well as the pricing of options and derivatives. The aim of this article is to compare the GARCH (Generalised Auto Regressive Conditional Heteroskedasticity) family models –GARCH (1.1), GJR-GARCH, PGARCH, EGARCH, and IGARCH– with the EWMA (Exponentially Weighed Moving Average) model in the hope of finding the best model to forecast the volatility of the Moroccan stock-market index MADEX. We use daily returns covering the period between 01/04/1993 and 30/08/2016. We find that the asymmetric model IGARCH following a normal error distribution yields the best forecasting performance results and therefore, surpasses the EWMA model. Our results could have application in the risk management in Morocco, as well as leading to a better understanding of the Moroccan stock-exchange volatility dynamics, especially with the lack of previous similar studies.

Keywords: Volatility forecasting; volatility modeling; stylized facts; GARCH family models; EWMA.

JEL classification: G11; G17; C13; C52; C53; C58.

MSC2010: 91B84; 62P20; 91B30.

Modelos de la familia GARCH vs EWMA: ¿cuál es el mejor modelo para pronosticar la volatilidad del mercado de valores marroquí?

RESUMEN

Hoy en día, modelar y pronosticar la volatilidad de los mercados bursátiles se ha convertido en un aspecto central para la práctica de la gestión de riesgos; se ha convertido en uno de los temas principales en la econometría financiera y se utiliza principal y continuamente en la determinación de precios de los activos financieros y el valor en riesgo, así como la fijación de precios de opciones y derivados. El objetivo de este artículo es comparar los modelos de la familia GARCH (heterocedasticidad condicional regresiva automática generalizada) –GARCH (1.1), GJR-GARCH, PGARCH, EGARCH e IGARCH– con el modelo EWMA (media móvil ponderada exponencialmente) con la esperanza de encontrar el mejor modelo para pronosticar la volatilidad del índice bursátil marroquí MADEX. Utilizamos los rendimientos diarios que cubren el período comprendido entre el 01/04/1993 y el 30/08/2016. Encontramos que el modelo asimétrico IGARCH, siguiendo una distribución normal del error, produce los mejores resultados de pronóstico y, por lo tanto, supera al modelo EWMA. Nuestros resultados podrían tener una aplicación en la gestión de riesgos en Marruecos, así como llevar a una mejor comprensión de la dinámica de volatilidad de la bolsa de Marruecos, especialmente con la falta de estudios similares anteriores.

Palabras claves: pronósticos de volatilidad; modelización de volatilidad; hechos estilizados; modelos de la familia GARCH; EWMA.

Clasificación JEL: G11; G17; C13; C52; C53; C58.

MSC2010: 91B84; 62P20; 91B30.



1. Introduction

The modeling of volatility in financial markets has become a major issue in the world of finance; due, of course, to the growing and crucial role played by volatility in financial markets. Today, volatility is a central feature of contemporary financial markets, and its uses are broad and diversified. It is an essential component in the process of value at risk, portfolio management, valuation of options and financial assets, among many other uses.

Volatility is the most important variable in the valuation of derivatives; it allows to construct stock market indices, serving as a proxies for measuring the levels of uncertainty among investors and actors in financial markets.

With the 1996 Basel Accord, volatility forecasting has officially become a mandatory task for financial institutions across the globe, demonstrating the importance of volatility in the international financial sphere.

Changing levels of volatility in financial markets could have major repercussions on the global economy, given the way and the extent to which the latter reacts to political and economic shocks, as well as its exponential relationship with the arrival of news, especially when they are bad news.

Being convinced and aware of the increasing role of volatility in the practice of risk management, we have decided to focus this article on the issue of determining and looking for the best model to predict the volatility of the Moroccan stock-market index MADEX.

2. Literature review

The forecasting of volatility, as well as the comparison of the out-of-sample forecast performance of the different models, is a booming subject and several researchers have begun to work on this subject. Akgiray (1989) found that the GARCH model is superior to the EWMA (exponentially weighted moving averages) model, the ARCH model and the historical average model, predicting the monthly volatility of the US stock index. A similar conclusion was obtained by West and Cho (1995) by using the one-step-ahead forecast of the dollar exchange rate.

Despite the fact that there are a number of techniques for modeling volatility in the financial markets, the literature review concluded that the essential elements of the studies are carried out by using models from the GARCH family. This is largely due to their ability to take into account all the stylized facts often observed on financial markets, including:

- Squared returns are positively correlated, meaning that significant changes in the price of a financial asset at time t will imply a significant change in price levels at time $t + 1$.
- The series of financial asset prices is marked by an excess of kurtosis, the equivalent of fat tails. Fama (1965) and Mandelbrot (1963) were the first to point out this non-normality of the financial series.
- Volatility tends to cluster, meaning that periods of high volatility are followed by periods of high volatility, and periods of low volatility are followed by periods of low volatility.
- Leverage: The evolution of financial prices is negatively correlated with volatility. Black (1976) explained that the more than proportional change caused by price volatility can only be explained by leverage. More empirical evidences on this stylized fact were proposed by Engle and Ng (1993).
- Long memory: Volatility is very persistent, especially in the case of high frequency data; there is even evidence of unit root in the process of conditional volatility.

- Correlation of volatility: The observation of several financial assets, and especially the exchange rates, shows the existence of the correlation of volatility between one currency and another.
- Mean reverting: When volatility is disrupted, it tends to return to its mean, which may be itself altered over time.
- Risk premium: The riskiest assets with large variances are the most profitable assets.
- Uncertainty in macroeconomic aggregates implies volatility in financial markets.

Pagan and Schwert (1990) compared GARCH, EGARCH, Markov Regime Switching model and three other nonparametric models to predict the monthly volatility of US stock-market returns. According to this study, the GARCH model, followed by EGARCH, functioned in a moderate manner while the rest of the models performed poorly.

Franses and Van Dijk (1996) compared three models of the GARCH family (GARCH, QGARCH and GJR-GARCH) to predict the weekly volatility of several European stock indices. They found, at the end of this study, that the nonlinear models could not beat the standard GARCH model. Brailsford and Faff (1996) found that the GJR-GARCH models, as well as the GARCH ones, were slightly superior to several simple models to predict the monthly volatility of the Australian stock market.

Engle and Patton (2001) were able to prove the ability of GARCH models to take into account the stylized facts observed on the volatility of the Dow Jones stock index.

Lupu and Lupu (2007) found, working with a daily series covering the period between 03/01/2002 to 17/11/2005, that the EGARCH model is the best model to express the volatility of the Romanian stock index BET-C. Miron and Tudor (2010) worked with several types of asymmetric GARCH models (EGARCH, PGARCH and TGARCH), using stock indices from the US and Romania covering the period between 2002 and 2010. They were able to demonstrate that the estimation of the volatility resulting from the application of the EGARCH model is much more reliable than the estimates made by the other models.

GARCH models represent a generalization of ARCH (autoregressive conditional heteroscedasticity) models; this type of models was developed for the first time by Engle (1982) as an ARCH(q), where conditional volatility was a function of q delays of past squared yields.

The models of the ARCH family have been extensively studied and, in particular, we can cite the works of Bollerslev *et al.* (1992) and Bollerslev *et al.* (1994). GARCH models were an extension of ARCH family models and were proposed by Bollerslev (1986) and Taylor (1986). The main contribution of these models is to allow, in addition to the term ARCH(q), another term GARCH(p) to represent the delays of the conditional volatility h_t itself.

Given the great success of these models, several extensions have been developed to try to perfect this type of models and make it more and more efficient. Among these extensions, we can find the exponential GARCH or EGARCH (Nelson, 1991). For this model, conditional volatility is specified in logarithmic form, which means that there is no need to impose estimation constraints to avoid the problem of negative variance.

This property allows us to take into account the stylized fact that negative shocks imply a greater variation of volatility than positive ones. Another non-symmetric model with characteristics close to EGARCH is TGARCH, also called GJR-GARCH and developed by Zakoian (1994) and Glosten *et al.* (1993), respectively. The main difference between TGARCH and EGARCH is the following: TGARCH models the conditional standard deviation instead of the conditional variance.

While shocks in the volatility series tend to have long memories and, as a result, tend to impact future volatility for a long horizon, the IGARCH model (or Integrated GARCH) was proposed by Engle and Bollerslev (1986) to capture this stylized fact, as well as to make conditional volatility infinite and shocks permanent.

Similarly, Ding *et al.* (1993) proposed the PGARCH (Power GARCH) model, which came to provide another method for modeling the long memory property in volatility. An excellent review of volatility prediction models can be found in Poon and Granger (2003).

3. Data and methodology

The data used in this article consists of 5839 daily price observations of the MADEX Moroccan stock index, from 01/04/1993 to 15/08/2016. This series of daily courses has been downloaded from the CDG CAPITAL BOURSE website. The time series of MADEX prices was divided into two series. The first one was covering the period from 01/04/1993 to 15/08/2015 (i. e. 5,592 observations) and was used to estimate our models of EWMA and those of the GARCH family, as well as to compute the descriptive statistics. However, the second series was covering the period between 15/08/2015 and 15/08/2016 (i. e. 247 observations) and was used to evaluate the out-of-sample forecast performance of each of our models. With this decomposition, we will be able to compare the “future” volatility forecasts while having values that have not been used in model estimation as a reference. Therefore, we will not be limited to the only in-sample observations.

At this stage, it should be noted that our choice of models in the GARCH family is motivated by their great ability to capture the stylized facts often observed on the international financial markets. Similarly, our choice to opt for EWMA is explained by its non-return-to-average property.

To the best of our knowledge, this article represents the first attempt to compare and study several models in order to capture and model the features of the conditional volatility of the Moroccan stock exchange market, producing consequently high-quality forecasts that are necessary for Moroccan risk managers.

In terms of methodology, we have chosen to work with the five main extensions of the GARCH family models: GARCH(1.1), GJR-GARCH, EGARCH, PGARCH and IGARCH, in addition to EWMA model. The mathematical formulation of each of these models is set out below.

3.1. GARCH (Generalized Auto Regressive Conditional Heteroskedasticity)

Bollerslev (1986) and Taylor (1986) developed the GARCH(p, q) model, allowing the conditional variance of the variable to be dependent on previous delays and capturing information and news contained in historical values of the variance. This model is presented as follows:

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2 + \sum_{i=1}^p \beta_i h_{t-i}$$

As the notation shows, the GARCH(p, q) model contains, in addition to the term GARCH(h_{t-1}) or delays in the conditional variance, an squared ARCH (u_{t-1}^2). In the financial literature, the GARCH(1.1) model remains by far the most used model and hence, our choice to use this type of models.

The notation of the GARCH(1.1) model is presented below:

$$h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 h_{t-1}$$

This model has a non-negativity constraint for the coefficients α and β so that the variance is always positive and the coefficient α_0 must be greater than 1.

3.2. GJR GARCH (Glosten-Jagannathan-Runkle GARCH)

The GJR GARCH model is a simple extension of the GARCH model by adding an additional term to account for the asymmetries observed in the financial markets (Brooks, 2008: p. 405). Glosten *et al.* (1993) have developed this model to allow conditional volatility to have different reactions to past innovations based on their signs. This model is presented as follows:

$$h_t = \alpha_0 + \sum_{i=1}^q \alpha_i u_{t-i}^2 + \gamma_i u_{t-i}^2 d_{t-1} + \sum_{i=1}^p \beta_j h_{t-j}$$

where d_{t-1} is a dummy variable:

$$d_{t-1} = \begin{cases} 1, & \text{if } u_{t-1}^2 < 0 \text{ (negative shocks);} \\ 0, & \text{if } u_{t-1}^2 \geq 0 \text{ (positive shocks).} \end{cases}$$

and γ is the coefficient that measures the impact of news arrival. The rest of the parameters in the equation remain the same as those of the GARCH model.

In this model, the effect of good news shows its effect through α_i , whereas the effect of negative shocks is shown by $\alpha + \gamma$. Moreover, if $\gamma \neq 0$, the impact of the arrival of news is said to be asymmetric; and when $\gamma > 0$, then volatility is marked by a leverage effect.

In order to be in line with the condition of non-negativity of the coefficients, it is necessary that $\alpha_0 > 0$, $\alpha_i > 0$, $\beta \geq 0$ and $\alpha_i + \gamma \geq 0$. The model could be still acceptable if $\gamma < 0$ and $\alpha_i + \gamma \geq 0$ (Brooks, 2008: p. 405).

3.4. EGARCH (Exponential GARCH)

For the Exponential GARCH or EGARCH model proposed by Nelson (1991), the conditional volatility specification is given by the following formula:

$$\log(h_t) = \alpha_0 + \sum_{j=1}^q \beta_j \log(h_{t-j}) + \sum_{i=1}^p \alpha_i \left| \frac{u_{t-i}}{\sqrt{h_{t-i}}} \right| + \sum_{k=1}^r \gamma_k \frac{u_{t-k}}{\sqrt{h_{t-k}}}$$

where $\log(h_t)$ represents the logarithm of conditional volatility, $\log(h_{t-1})$ represents the logarithm of the first lag in conditional volatility, and u_{t-i} is the term of the error at time i .

The use of the EGARCH model has the advantage to authorize the effects of information asymmetries to happen. In the EGARCH equation, γ_k represents the leverage parameter used to capture the asymmetry, which is not the case for the basic GARCH model (Thomas and Mitchell, 2005).

The main contribution of this model is that it takes into account the fact that negative shocks have a greater impact on volatility than that of positive shocks.

3.5. Power GARCH(p,d,q)

This model was proposed by Ding *et al.* (1993), and has the advantage of being able to capture and model the long memory property often observed in the series of volatility. It is presented as follows:

$$h_t^d = \alpha_0 + \alpha(|u_{t-1}| + \gamma u_{t-1})^d + \beta h_{t-1}^d$$

where d is a power term, u_{t-1} represents the first lag of the error term (ARCH term), and h_{t-1} is the first lag of the conditional volatility. The power term, denoted d , captures the standard deviation when $d = 1$ and captures the conditional variance when $d = 2$. The asymmetry is counted by the term γ (Carroll and Kearney, 2009).

3.6. IGARCH (Integrated GARCH)

The IGARCH models, introduced by Engle and Bollerslev (1986), have the advantage of providing a statistical response to the problem of the presence of a unit root in the time series of volatility, which makes volatility shocks permanent. It is an integrated model of volatility. The formulation of this model is presented below:

$$h_t^2 = \alpha_0 + \sum_{i=0}^q \alpha u_{t-i}^2 + \sum_{i=0}^p \beta h_{t-i}^2$$

IGARCH models are said to be volatile models, because current information remains valid for forecasting volatility across all horizons.

If $\alpha_0 = 0$, we can say that the series is integrated in variance to the order d . And when $\alpha_0 > 0$, then the series is integrated in the order d with trend; where d is the number of first differences needed in order to render it stationary.

As far as error distributions are concerned, GARCH model theory suggests three assumptions about the distribution of residuals. These three assumptions imply that the residuals of the GARCH regression may follow a normal law, a Student law or a generalized error distribution (GED). Although the vast majority of GARCH models are based on a normal distribution of residuals, the calibration and adequacy of the optimal model remain closely dependent on these distributions; therefore, our choice of sailing through the three distributions, in a way to emphasize the contribution of our study.

3.7. EWMA (Exponentially Weighed Moving Average)

The EWMA model is one of the oldest econometric models and has been mainly developed as a response to the weaknesses of the simple volatility and historical volatility models, which assign the same weight to the past observations.

In fact, the weight of recent information tends to be more important than that of the very old observations. And this is what makes EWMA a very powerful model, despite its relative simplicity. Unlike GARCH models, EWMA has the advantage of a non-return to average, which is considered by many researchers to be a weakness of GARCH models (Ding and Meade, 2010). This is the reason why there is a fairly large amount of works that suggest the ability of EWMA to surpass GARCH models in forecasting and modeling volatility. The EWMA can be presented as:

$$\sigma_n^2(EWMA) = \lambda \sigma_{n-1}^2 + (1-\lambda)r_{n-1}^2$$

where σ_n^2 denotes volatility at time n , σ_{n-1}^2 is the first volatility lag, r_{n-1}^2 is the square of the returns for period $n-1$ and finally, λ is called the smoothing coefficient. Based on recommendations of RISKMERICS, the value of λ was specified at 0.94 when the frequency of observations is daily. The

term $(1-\lambda) r_{n-1}^2$ represents the response intensity of the variance to market news, while $\lambda \sigma_{n-1}^2$ is used to capture persistence in volatility.

The approach followed in this empirical study, is to start by first estimating the conditional volatility of the MADEX index, according to the different GARCH models and according to different error distributions; and then selecting the best models in function of the significant parameters as well as Akaike (AIC) and Schwarz (BIC) information criteria and that of the maximum likelihood estimation. Once we have obtained the best GARCH models, which allow us to better express the volatility of our index, we will compare the forecasting performance of these models with that of the EWMA model, using the following statistics: Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Theil Inequality Coefficient (TIC).

4. Empirical results

In order to properly conduct our study and to be in compliance with the finance theory dealing with these subjects, we have transformed our raw data series into a series of logarithmic returns according to the following function:

$$r = \log\left(\frac{x_t}{x_{t-1}}\right)$$

where X_t represents the price of the sector index at time t , and X_{t-1} refers to the price of the sector index in $t-1$.

Then we applied the Augmented Dickey Fuller (ADF) unit root test to study the stationarity of the return series (see Table 1). Similarly, we used the White test for the purpose of testing the ARCH effect or the heteroskedasticity property of the errors (see Table 2); this test was conducted on the residuals series taken from the following mean model regression:

$$r = c + r(-1) + \varepsilon$$

Variable	ADF Value	t-stat 1%
Returns of MADEX (r)	-57.574***	-3.96

***: values statistically significant at the levels of risk of 1%, 5% and 10%.

Table 1: Results from the ADF test.

Dependent variable	White's statistic	Obs R^2
Returns of MADEX (r)	408,563***	632,12

***: values statistically significant at the levels of risk of 1%, 5% and 10%.

Table 2: Results from the White heteroskedasticity test.

From the results presented in Tables 1 and 2, we conclude that the newly created MADEX return series is a stationary series. Similarly, the statistical significance of the White test led us to reject the null hypothesis of the homoskedasticity of errors and to accept the alternative hypothesis of the heteroskedasticity of errors. So, at this stage, we can safely proceed to the estimation of our models, as the conditions for ARCH and GARCH modeling hold.

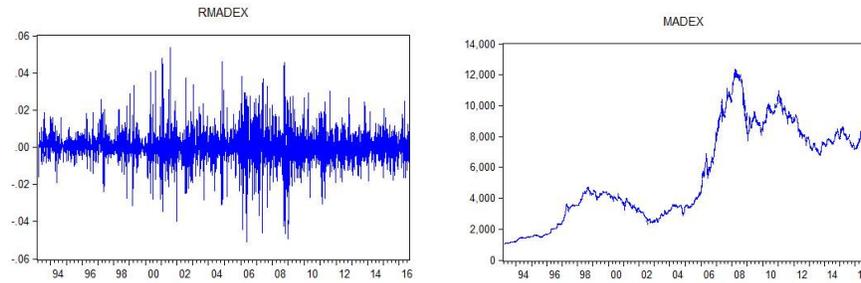


Figure 1: Plots of the evolution of MADEX returns and prices.

Descriptive statistics	Values
Mean	0.000347
Median	0.000157
Maximum	0.0536490
Minimum	-0.050935
Standard deviation	0.007561
Asymmetry	-0.016580
Kurtosis	9.459888
Jarque-Bera	10560.00
Probability	0.000000

Table 3: Summary of descriptive statistics of MADEX returns.

Table 3, including the descriptive statistics for the MADEX returns series, shows a significant difference between the maximum and minimum values, which is synonymous with high volatility in the series; in addition, the existence of a significant difference between the value of the standard deviation and the mean could only reinforce this finding. The kurtosis value, being very large compared with the value of 3, suggests the presence of a fat tail on the right side with respect to the mean and hence, the non-normality of the series. This non-normality is confirmed by the Jarque-Bera test which is significantly different from zero; so the normality hypothesis of the series cannot be accepted.

For the empirical results of the regressions, they will be presented hereafter in function of the error distributions. The AIC, BIC and maximum likelihood criteria are used to find the optimal model, so that AIC and BIC are minimized and the maximum likelihood is maximized independently of the error distributions.

The first observation to be drawn from Table 4 is that the majority of the parameters are significantly different from zero, which underlines the high validity of our models. The sum of the terms α and β for the models GARCH, PGARCH and IGARCH is very close to 1, which is explained by a rather significant presence of persistence in the volatility of the MADEX index. However, the value of α is rather less than that of β , which means that the negative shocks on the conditional volatility of MADEX do not have a greater impact on volatility than those of positive shocks.

For the asymmetric GARCH models, half of the parameters γ are statistically different from zero, which implies that the volatility of the MADEX index is asymmetric and, hence, the existence of leverage effects. The parameter γ of the PGARCH(1.1.1) model, being statistically significant and having a positive value, suggests that the impact of positive shocks on the volatility of the MADEX index is greater than that of negative shocks.

Conditional volatility model	C	ARCH(-1)	GARCH(-1)	Leverage	AIC	BIC	Maximum likelihood
	α_0	α	β	γ			
GARCH(1.1)	3.66E-06 (0.000)	0.280965 (0.000)	0.681808 (0.000)	-	-7.289434	-7.28467	20,316.01
GJR-GARCH	3.61E-06 (0.000)	0.032846 (0.000)	0.684658 (0.000)	0.26247 (0.055)	-7.289476	-7.2835	20,317.13
EGARCH	-1.14839 (0.000)	0.410135 (0.000)	0.915581 (0.000)	-0.0187 (0.0053)	-7.295353	-7.2891	20,333.50
PGARCH(1.1.1)	0.000590 (0.000)	0.245080 (0.000)	0.736882 (0.000)	0.03746 (0.0312)	-7.295313	-7.28936	20,333.39
PGARCH(1.2.1)	0.000444 (0.075)	0.261248 (0.000)	0.750706 (0.000)	0.057984 (0.0777)	-7.431567	-7.42443	20,714.06
IGARCH	-	0.073089 (0.000)	0.926911 (0.000)	-	-7.229333	-7.22695	20,146.54

Note: Values in parentheses represent p -values.

Table 4: Results for the regressions following a Gaussian error distribution.

For the Gaussian distribution, the best model of the conditional volatility of the MADEX index is EGARCH, which presents significant parameters and has the smallest AIC and BIC values while having the greater maximum-likelihood value. This model is closely followed by GARCH (1.1), IGARCH and PGARCH(1.1.1). So, these are the models that will be evaluated later to test and compare their predictive performance. The other models are eliminated due to having non-significant parameters.

Results obtained when following a Student error distribution, are closely related to those of the normal distribution. For this error distribution, we can clearly see that the model GARCH(1.1) is the best to capture and model conditional volatility of our index (see Table 5). The parameters γ , being entirely not statistically significant, imply the non-existence of leverage effects in conditional volatility of MADEX.

Conditional volatility models	C	ARCH(-1)	GARCH(-1)	Leverage	AIC	BIC	Maximum likelihood
	α_0	α	β	γ			
GARCH(1.1)	3.05E-06 (0.000)	0.376660 (0.000)	0.661468 (0.000)	-	-7.417003	-7.41105	20,672.48
GJR-GARCH	3.01E-06 (0.000)	0.345563 (0.000)	0.664075 (0.000)	0.05938 (0.15)	-7.417147	-7.41001	20,673.88
EGARCH	-0.99884 (0.000)	0.471150 (0.000)	0.933200 (0.000)	-0.0223 (0.089)	-7.425269	-7.41813	20,696.51
PGARCH(1.1.1)	0.00041 (0.000)	0.286550 (0.000)	0.748209 (0.000)	0.04001 (0.1798)	-7.427746	-7.42061	20703.41
PGARCH (1.2.1)	3.01E-06 (0.000)	0.374696 (0.000)	0.664039 (0.000)	0.03960 (0.1070)	-7.712	-7.705	21,563.21
IGARCH	-	0.116128 (0.000)	0.883872 (0.000)	-	-7.377404	-7.37383	20,560.14

Note: Values in parentheses represent p values.

Table 5: Results for the regressions following a Student error distribution.

For the case of this distribution, the only models that will be kept for the final study are GARCH(1.1) and IGARCH.

The EGARCH model represents the best way to model the conditional volatility of MADEX in the case of the generalized error distribution (see Table 6). With the exception of the EGARCH model, all the parameters γ are not statistically significant, which implies the non-existence of leverage effects and the asymmetry of the volatility of our stock index.

Conditional volatility model	C α_0	ARCH(-1) α	GARCH(-1) β	Leverage Γ	AIC	BIC	Maximum likelihood
GARCH(1.1)	2.95E-06 (0.000)	0.321129 (0.000)	0.675124 (0.000)	-	-7.42254	-7.41659	20,687.90
GJR-GARCH	2.90E-06 (0.000)	0.289996 (0.000)	0.678385 (0.000)	0.061638 (0.083)	-7.42283	-7.41570	20,689.73
EGARCH	-0.9963 (0.000)	0.422749 (0.000)	0.931471 (0.000)	-0.02703 (0.039)	-7.42954	-7.42240	20,708.42
PGARCH(1.1.1)	0.000444 (0.075)	0.261248 (0.000)	0.750706 (0.000)	0.057984 (0.0777)	-7.43156	-7.42443	20,714.06
PGARCH (1.2.1)	2.90E-06 (0.000)	0.320049 (0.000)	0.678411 (0.000)	0.048255 (0.07)	-7.42283	-7.41570	20,689.73
IGARCH		0.093894 (0.000)	0.906106 (0.000)		-7.38963	-7.38606	20,594.22

Note: Values in parentheses represent p values.

Table 6: Results for the regressions following a generalized error distribution.

In addition to the EGARCH model, the GARCH(1.1) and IGARCH model will also be kept in order to compare their predictive performance in final test.

At this stage and after studying and comparing the models of the GARCH family with the hope to find the best adjustments of these models, we will proceed to the last step which represents the aim and the object of this paper. In this second step, we will present a comparison of the forecasting performances for the following models: GARCH(1.1), GJR-GARCH, EGARCH, PGARCH(1.1.1), PGARCH(1.2.1), IGARCH and, of course, EWMA.

Volatility model	RMSE	MAE	TIC
GARCH(1.1)	0.005872	0.004327	0.954
GARCH(1.1)_t	0.005871	0.004329	0.961
GARCH(1.1)_GED	0.005870	0.004335	0.981
EGARCH	0.005875	0.004326	0.941
EGARCH_GED	0.005870	0.004336	0.983
PGARCH(1.1.1)	0.005873	0.004327	0.951
IGARCH	0.005876	0.004326	0.939
IGARCH_t	0.005872	0.004328	0.955
IGARCH_GED	0.005870	0.004334	0.979
EWMA	0.005882	0.004339	0.943

Table 7: Evaluation table for forecasting performances.

From Table 7, we observe that the ten presented models are very close to each other, but the analysis of the RMSE, MAE and TIC statistics makes possible to conclude that the IGARCH with a

normal error distribution is the best model to forecast the volatility of the MADEX index. This model, compared with the others, presents the best results, by presenting the best values in 2/3 of the forecasting error statistics adopted in this study. Therefore, we can say that the models of conditional volatility are better than those of the exponentially weighted volatility for the case of the MADEX index.

5. Conclusion

Nowadays, the forecasting of volatility in the financial markets is a major subject. Studies aimed at this subject continue to multiply, proposing each time new techniques and new models. Throughout this paper, we have tried to look for the best model to predict and forecast the volatility of the MADEX index. In order to achieve this, we have used GARCH models, which are widely studied and analyzed, and whose performances are largely documented in the financial literature.

The EWMA model was added to our sample models thanks to the interesting number of studies that have proved its superiority to the GARCH models, and to its main property of non-return to average.

Among the results obtained at the end of this study, we found that the GARCH models succeed in modeling and explaining, in a rather satisfactory manner, the volatility of the Moroccan stock index compared with the EWMA model, that has nevertheless succeeded in producing estimates being very close to those of the GARCH family models.

As for the main result –i. e. the best model to forecast the volatility of the Moroccan stock index–, the statistics for the measurement of forecasting errors have declared the IGARCH model with Gaussian distribution of errors as a rightful winner and hence, the superiority of GARCH models in comparison to the EWMA model. The results obtained in this study may basically have uses in the practice of management of financial risks as they may serve as an inspiration for other eager researchers to gain a better understanding of the dynamics of volatility in the Moroccan financial market.

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