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Strategic Allocation of Pension Reserve Funds: Application of ALM Model and LDI Technique

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ABSTRACT

This article will focus on the research for the strategic allocation of reserve fund of the Moroccan pension scheme in order to ensure and improve its solvency. The first aim of this paper is to construct and test an Economic Scenario Generator (ESG) based on a model inspired of the Ahlgrim approach (2005) and adapted to the specificities of asset-liability management (ALM) and liability-driven investing (LDI).

In our study, we will also develop the ALM technique based on the maximization of the reserve under the criterion of maximization of solvency ratio (since the fund is in deficit).To do this, we consider a recent strategic asset allocation approach based on the "constant weight" strategy, or Fixed-Mix, Kouwenberg (2001). Indeed, we will implement the LDI strategies based on the Sharpe and Tint model (1990). For that, we will first try to find the desired weightings of the asset classes in an asset context only. Afterwards, we try to build a hedge portfolio (LHP) and a performance research portfolio (PSP).

Keywords: solvency, reserves fund, economic scenario generation (ESG), Monte Carlo simulation, ALM model, strategic allocation, LDI strategy, Moroccan civil pensions regime.

Clasificación JEL: G11; G12; G17; G22; G63.

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Asignación Estratégica de Fondos de Reserva de Pensiones: Aplicación del Modelo ALM y LDI Técnica

RESUMEN

Este artículo se centrará en la investigación para la asignación estratégica del fondo de reserva del plan de pensiones marroquí para garantizar y mejorar su solvencia. El primer objetivo de este documento es construir y probar un generador de escenarios económicos (ESG) basado en un modelo inspirado en el enfoque Ahlgrim (2005) y adaptado a las especificidades de la gestión de activos y pasivos (ALM) y la inversión basada en pasivos (LDI)

En nuestro estudio, también desarrollaremos la técnica ALM basada en la maximización de la reserva bajo el criterio de maximización de coeficiente de solvencia (ya que el fondo está en déficit). Para hacer esto, consideramos un enfoque de asignación estratégica de activos reciente basado en el "peso constante "estrategia, o Fixed-Mix, Kouwenberg (2001). De hecho, implementaremos las estrategias LDI basadas en el modelo de Sharpe y Tint (1990). Para eso, primero intentaremos encontrar las ponderaciones deseadas de las clases de activos solo en un contexto de activos. Luego, tratamos de construir una cartera de cobertura (LHP) y una cartera de investigación de rendimiento (PSP).

Palabras clave: solvencia, fondo de reservas, generación de escenarios económicos (ESG), simulación de Monte Carlo, modelo ALM, asignación estratégica, estrategia LDI, régimen de pensiones civiles marroquíes.

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1. Introduction

The investment of reserve funds plays a major role in the survival of pension plans. An in-depth study of asset allocation and regular monitoring should optimize the financial returns generated by the reserves and thus improve and ensure the solvency of the plan retirement.

The pension reserve therefore remains a very good indicator for the financial management of pension plans since its level and the projection of its behavior over time make it possible to see the number of years that the fund is able to honor its commitments and define the sustainability horizon of the plan. In addition, the level of the pension reserve makes it possible to calculate the financing ratio, ie. the degree of representativity of the fund's liabilities.

The long-term financial management of pension fund reserves is based on two important points. On the one hand, Economic Scenario Generators (ESGs) used to model fluctuations in macroeconomic and financial variables over the long-term then. On the other hand, the models and strategies proposed for the elaboration of the asset allocation.

The generation of economic scenarios is a crucial phase in asset-liability management (ALM) and liability-driven investing (LDI) of an insurance company or a pension fund. The choice of the strategic allocation comes in a second time to reflect the attitude towards the risk of the long-term investor (Meucci, 2005).

The needs of pension plans for economic scenarios have increased in recent years. The evolution of these needs is mainly due to two factors: the evolution of prudential regulation and the evolution of financial communication, particularly in the valuation of insurance and pension companies.

The economic crisis, which began with a financial crisis in the United States in 2007 has strongly affected the performance of an insurer's asset portfolio, particularly pension funds. In particular, it highlighted the shortcomings of certain existing generators, in particular in terms of the occurrence of extreme events and the structure of dependency in the tails of distributions and imposed on insurance and pension funds no longer benefiting from this tool, to put it in place. The financial and economic crisis of 2008 also had a negative impact on the CMR portfolio of assets. Thus, in the context of the insolvency of our civil regime, the creation of an economic scenario generator proves to be an indispensable tool for risk management and decision support. Our ESG must therefore be in an environment that includes probabilities of crises, which will increase the number of prudent and rational decisions.

The subject of asset allocation is at the center of concern for institutional investors. On the one hand, the financial crisis had a profound impact on the modeling of liabilities and assets, the characteristics of which are no longer consistent with long-term assumptions. On the other hand, the current conditions of historically low rates raise questions about their traditional investment approach, namely stable returns secured by more or less long-term rate products. In these circumstances, it is difficult for them to perform consistently and to manage extreme risks in parallel.

The asset allocation of the reserve funds is a strategic step to guarantee the financial stability of the pension funds and the achievement of the objectives set by them. Indeed, the pension fund is supposed to provide pension benefits to retirees throughout their lives and possibly to their beneficiaries. This is why some countries have even opted for the creation of institutions specialized in the allocation of reserve funds of pension funds.

The strategic asset allocation of a pension plan in general is often defined as a step in a broader asset / liability management process, particularly as a downstream step in risk apprehension and upstream of tactical asset allocation. In this sense, the strategic asset allocation aims either at confirming the optimality of the reserve's existing asset structure or at proposing an optimal asset structure for this reserve that will allow the fund to achieve a certain financial performance objective while respecting its commitments with a given level of confidence.

The strategic allocation must take into account the evolution over time of asset returns and their dependency structure. This involves selecting the dynamics of each variable and then estimating the parameters of the chosen model (the calibration of the model). This leads to the establishment of a long-term economic scenario generation model that allows the projection of both asset value and liability.

The most widely used reference tool for determining the ability of a pension fund to take risks and therefore to define its strategic benchmark is Asset Liability Management (ALM). This strategy (ALM) has emerged in recent years for pension funds as a risk management approach that takes into account the assets, liabilities and also the different interactions that exist between these two parts (Adam, 2007). Fund managers must determine which eligible strategies provide sufficient assurance that the solvency of the fund is assured (taking into account the expected benefits).

Asset-liability management generally can be based on optimizing the value of the reserve, taking into account the constraints related to the liabilities it must meet. It is in this context that the choice of an effective strategic allocation plays a key role in the asset-liability management of a pension plan and the optimization of its solvency, since reserves contribute to the viability and the durability of this scheme.

In the 1990s, asset management for the majority of pension funds was done by examining the expected return and volatility of the assets with little regard to actuarial liabilities.The fall in interest rates in recent years, combined with stock market volatility and changes in regulations and accounting standards has led to the development of several reflections for the implementation of a strategic approach that manages the risk in a consistent manner according to the liabilities of the pension funds, notably the "liability-driven investments" or LDI. The objective of the LDI is to meet current and future obligations, hedge liabilities and reduce the volatility. LDI strategies are considered the most recent and sophisticated form of ALM. This model aims to generate performance once the commitments are honored. For this reason, this model is based on two portfolios: a hedge portfolio and a performance research portfolio.

In this sense, this study is aimed at seeking the best possible allocation of civil reserve portfolio of the CMR within the framework of Asset-Liability management and the LDI model.To achieve our goal, we will answer the central question: *how to optimize the strategic allocation of reserve funds under ALM management and LDI?*

This paper will be composed of four sections, after an introduction. The purpose of the second section is to present a review of a Generator of Economic Scenarios within our civil pension system financed by a pay-as-you-go, asset allocation models as well as the ALM models in the context of the management of a pension plan and the LDI strategy. The third section will be devoted to the presentation of the data and the methodology adopted for our study. The last section will present the different results and conclusions.

2. Literature review

The literature on ESG is rich. David Wilkie (1986) marked a turning point in the design of ESG by publishing 'A stochastic investment model for actuarial use'. Not only are the works realized the first to integrate the interaction of the different variables in order to make the projections more coherent; but it also takes into account all financial and macroeconomic variables (inflation, equity, interest rates and real estate).

In order to characterize the structure of dependency, Wilkie uses a cascade approach, which can be applied in a wide field, particularly to insurance and pension funds. The first version of Wilkie's model (widely used in the Anglo-Saxon countries) was applied as part of the solvency measurement of an insurance company by the Faculty of Actuaries (1986). Similarly, one of the first areas of application of Wilkie's actuarial model was the valuation of equity-linked commitments. In general, this model is rather consistent with the logics of capital requirement and value projection (case of asset-liability management for example) than with pricing logics.

Several English actuaries have strongly criticized Wilkie's model, notably Kitts (1990), Daykin and Hey (1990), Huber (1995), Geoghegan and al. (1992). The main criticisms concern several elements: some of the parameters are unstable over time and a significant cross-correlation between the residuals of the projected variables is observed. The model of price indices for inflation does not allow the projection of periods with irregular shocks with high values of inflation. Similarly, the likelihood of having negative inflation values with this model is high. Indeed, Wilkie's model is above all empirical and two central assumptions of financial theory are not respected: the efficiency of financial markets and the absence of arbitrage opportunity.

Brennan and Xia (2000) discuss the modeling of inflation-linked assets. They choose a structure that places inflation and the short-term interest rate net of inflation (or real interest rate) at the center of modeling. The number of parameters of the Brennan and Xia model (2000) is lower than that of Wilkie (1986) and the parameters selected by these authors in 2000 are economically interpretable. Nevertheless, in an asset allocation objective, the approach of Brennan and Xia (2000) may seem limited because it does not include real estate, which can represent up to 40% of an insurer's portfolio.

Campbell and Viceira (2001) present an approach based on the correlation structure, the application of which was carried out in the context of determining the strategic asset allocation for a long-term investor, in particular pension funds. On the other hand, Kouwenberg (2001) adopted this structure to develop a scenario generation model based on a tree diagram for scenario projection. It compares the effect of choosing the projection scheme on optimal asset allocation in the context of asset-liability management of a German pension fund. The tree structure chosen by Kouwenberg (2001) is more suited to a series of dynamic models of asset-liability management based on stochastic programming techniques.

In the context of Dynamic Financial Analysis (DFA) or ALM non-life research, supported by two American professional associations, the Casualty Actuarial Society (CAS) and the Society of Actuaries (SOA). Ahlgrim, D'Arcy and Gorvett (2005) are essentially based on the two-point criticism of Wilkie's model (1995): the relationship between inflation and interest rates is considered incoherent and the treatment of stock returns by an autoregressive approach seems too simplistic of the observed history.They propose alternative processes by justifying their choices by backtesting on deep historical data. The model of Ahlgrim et al. (2005) follows the model of Hibbert et al. (2001) as a model of long-term value projection and risk management.

In the article 'A framework for the Actuarial Profession', Ahlgrim et al. (2005) put the inflation rate and the short-term interest rate at the center of their model. The model of Ahlgrim et al. (2005), which seems to be the most appropriate in the context of asset portfolio management, attaches particular importance to the correlation between the different economic and financial series. Nevertheless, this correlation does not introduce tail dependence and is therefore limited especially in times of crisis. Armel, Planchet and Kamega (2010) improves this aspect by using the theory of copulas, a function that allows modeling the dependency structure between random variables.

It is possible to represent the evolution over time of financial and macroeconomic variables in two schematic forms: the linear projection structure on the one hand Ahlgrim et al. (2005) and the structure of tree (or tree) projection on the other hand Kouwenberg (2001).The value of the nodes can be determined by different methods. Faleh, Planchet and Rullière (2009) group them into four main families: methods based on sampling, those based on matching statistical properties, Bootstrapping techniques, and the use of Principal Component Analysis.

For Hibbert et al. (2001), the properties that a ‘good’ ESG must have are: economic plausibility, parsimony, representativeness, transparency and evolution. The importance of asset allocation on portfolio performance has been established in the literature. Using the US investment data, Brinson, Hood and Beebower (1986) found that approximately 90% of the variability in returns over time of a typical portfolio is explained by the policy of the allocation of assets. A similar study using data for an Australian fund manager came to the same conclusion (Santacruz, 2013). In addition, a study by Brinson et al. (1986) found that about 40% of the variation in returns in several portfolios is explained by the asset allocation (Ibbotson & Kaplan, 2000). Previous research (eg, Exley & Mehta (1996) and Exley, Mehta & Smith (1997)) suggested that there is no optimal allocation of assets for a defined benefit pension plan.

Pension funds can adopt high-risk, high-yield strategies or low-risk, low-yield strategies. No strategy is optimal because optimal strategies need to be considered in the context of the overall asset and liability profiles of members and investors (Exley & Mehta, 1996).

From larger pension funds, the modern concepts of portfolio theory have been increasingly used to derive strategic asset allocation (SAA) and manage investment risk. Indeed, pension funds carry out an efficient frontier analysis based on the Markowitz mean-variance model, which aims to improve the efficiency of their investments.

This Markowitz risk-return model has revolutionized the solvency system approach by providing a new foundation for prudential investment analysis. Solvency II, which is considered the "revolution" of insurance solvency standards, is only an "evolution" since its vision is always "mono factorial", concentrated on a single objective: risk control.

Modern portfolio theory has some limitations. In particular, the risk measure used by Markowitz is variance, which may be limited in general. Moreover, this model does not take into account the existence of a liability, which makes it limited in certain insurance issues. The shortcomings of these analyzes are concentration on assets only and the short-term investment horizon (Campbell & Viceira, 2006).

In the literature, it is even indicated that the application of a single period model repeatedly for more than one period results in suboptimal dynamic decisions (Dupacová et al., 2003).

Research on optimal dynamic allocation began with Fama (1970) as part of a discrete-time model, Merton (1969 and 1971) and Samuelson (1969) in continuous time. These models make it possible to obtain closed formulas for optimal allocation over time - with a definite advantage for continuous time models - under assumptions about asset dynamics and utility function, Wachter (2002) or Chacko and Viceira (2005).

Following a review of the multi-period mean-variance approach based on scenario trees, Steinbach (2001), a more recent paper proposes a discrete time-series model that takes into account expected performance and the variance from one period to the next. Dynamic programming using objective functions that depend on the expected return and the variance of the final value of the portfolio is then used to determine the optimal portfolios (Çelikyurt & Özekici, 2007).

By adopting a long-term investment horizon, it is estimated that portfolio returns can be better optimized (Brennan, Schwartz, & Lagnado, 1997). However, differences in growth rates between different asset classes tend to move the portfolio away from the optimal allocation in the case of a buy and hold approach.

Even small changes in asset allocation weights are often statistically significant (Christie, 2005). One way to keep the portfolio near the efficient frontier in multi-period scenarios is to rebalance each time the asset weights deviate from the optimal allocation by more than the predetermined thresholds. This approach is called a "rebalancing" or "fixed-mix" management rule.

Multi-period strategies involving rebalancing were judged to outperform a buying and retaining strategy without rebalancing (Yu & Lee, 2011). However, it is observed that current rebalancing practices are characterized by suboptimal timing rules and other heuristics. An algorithm is proposed to determine when rebalancing is statistically desirable (Michaud, Esch & Michaud, 2012). The theoretical properties of the Fixed-Mix strategy are discussed in Merton (1990), Dempster et al. (2003), Infanger (2007, Brinson et al. (1991) and Kouwenberg (2001).

After the crisis of 1987, the Brady Commission strongly criticized the pro-cyclical dynamic strategies implemented as part of the replication of portfolio insurance strategies: they were accused of helping to accelerate the fall in financial markets 1987[1]. Mark Rubinstein concluded in 1988 in his article "Portfolio Insurance and the Market Crash" that the contribution of these strategies was only 12% and predicted as an oracle: "Those who proclaim the death of preferential motives for dynamic strategies presuppose an unrealistic reconstruction of natural human desires" (Rubinstein, 1988, p. 41). Another subgroup of models, and the most recent one, is mainly inspired by Merton's choice of consumption and portfolio theory (1971). These are dynamic or inter-temporal allocation models. These recent models come up against the problem of implementation considering the complexity of the mathematical tools used, Hainaut and Devolder (2005), Rudolf and Ziemba (2004) and Yen and Hsu (2003).

Asset-liability management or 'ALM' dates back to Frederick Macaulay in Berkeley (1938) and John Hicks in Oxford (1939). These two financial economists were interested in finding (the duration) a measure of the approximate change in the price of liabilities following a change in the performance of these commitments. Paul Samuelson (1945) and F. M. Redington (1952) rediscovered the duration measure in their analysis of the sensitivity of financial institutions' assets and liabilities to changes in interest rates. Redington was the first to associate this concept with the assets and liabilities of insurers.

Kim and Santomero (1988), Sharpe and Tint (1990), Leibowitz, Kogelman and Bader (1992) aim to minimize the risk of loss of surplus (measured by the variance of surplus profitability) assets. They are mono-periodic models, which limits their utility in practice for problems of allocation over the long term. The chosen risk measure can then be the variance or the Value at Risk of the surplus of the asset in relation to the liabilities.

Bauer, Hoevenaars and Steenkamp (2006) of the Dutch pension research community defines ALM as a study of the impact of investment, contribution and indexation decisions on the different fund actors (employees, employers, pensioners and future generations).

In a definition from the United Kingdom, Blake (2003) defines ALM as a quantitative technique used by some pension funds to structure their asset portfolio with due regard to the structure of their liabilities. Indeed, Blake (2006) summarizes the utility, advantages and disadvantages of asset-liability management. He argues that asset-liability management is a way

[1] These methods applied by a large number of players, on significant volumes (US pension funds) led the managers to reduce their positions all at the same time, and over very short periods.

of protecting oneself against the uncertainty associated with the economic and demographic assumptions used to value the present value of assets and liabilities.

The limitations of classical models of asset-liability management and the development of computerized media in the 1980s have recently led to the transition to other types of models: dynamic models and ALM stochastic models. Recent years have also seen the emergence of methods known as stochastic programming.

Rudolf and Ziemba (2004) include liabilities in their model and maximize the utility preferences of a pension fund relative to the inter-temporal surplus. In their model, surplus returns are assumed to depend on other state variables such as monetary returns or exchange rates.

Detemple and Rindisbacher (2008) also develop a dynamic model of asset allocation in the presence of liabilities for the management of pension funds. However, unlike Rudolf and Ziemba (2004), they argue that surplus maximization is problematic in circumstances where liabilities exceed assets. In this case, the surplus is negative leading to a poor specification of the objective - function.

Often, based on simulations because of its complexity, stochastic programming provides a flexible and powerful tool for ALM. Its importance lies in its ability to combine many types of characteristics within a common framework. In addition, assets and liabilities are all influenced by many sources of risk and risk aversion is taken into account. Another advantage of stochastic programming (SP) is that it can be extended directly from an asset analysis to an asset and liability management (ALM) problem only. The inclusion of balance sheet liabilities is an essential requirement for a financial planning model by most institutional investors: pension funds and insurance companies (Zenios & Ziemba, 2007; Ziemba, 2003).

Stochastic dynamic programming is provided as an alternative method for proposing decision policies for dynamic asset allocation strategies (Zenios & Ziemba, 2007). The main difference between stochastic programming models and stochastic dynamic programming is the solution concept. In stochastic programming, emphasis is placed on the decision of the first period, whereas in stochastic dynamic programming the aim is to establish decision rules which could be applied in the system, Dupacová et al. (2003). Stochastic programming models require rigorous scenarios as inputs to optimize and develop decision-making policies. The optimization results depend entirely on the scenarios on which they are optimized.

On the other hand, dynamic stochastic programming models incorporated in the system automatically cover current portfolio allocations against future uncertainties over a longer time horizon, leading to more robust decisions and insights into possible future problems and benefits. It is this characteristic and its ability to integrate different attitudes to risk that make dynamic stochastic optimization the most natural framework for the effective solution of pension fund management problems (Rudolf and Ziemba, 2004; Gao, 2008).

In dynamic stochastic optimization, the uncertain future is represented by a large number of future scenarios and contingent decisions are taken in stages according to the tree representations of future data and decision processes (Boender, 1997).

Large-scale applications were developed in the 1990s, such as the Russell-Yasuda Kasai model (1998) for a Japanese insurance company (Cariňo et al., 1994), or the scenario-based optimization model of Dert for pension funds (Dert, 1998). Boender, Van Aalst and Heemskerk (1998) developed and described the ORTEC model, which again uses multi-stage stochastic programming.

The Towers Perrin-Tillinghast ALM system in Mulvey (1996, 2000) has three components: a scenario generator (CAP: Link), an optimization simulation model (OPT: Link) and a financial reporting module (FIN: Link). Kouwenberg (2001) focuses on comparing scenario generation methods for a multi-stage stochastic programming model of a Dutch pension fund. Hoyland and Wallace (2001) analyze the implications of regulation in Norwegian life insurance companies using a stochastic ALM model at several stages. Their results show that certain regulations (the guaranteed annual rate of return, for example) do not coincide with the best interests of the insured.

To overcome the disadvantages of simulation, stochastic linear programming (SLP) models have been used to solve ALM problems. Instead of exogenous variables, as in simulation, decisions become endogenous. Kusy and Ziemba (1986) developed a multi-period stochastic linear programming model for the Vancouver City Savings Credit Union for a five-year planning period.

Consigli and Dempster (1998) presented the Computer Assisted Liability (CALM) model, which maximizes wealth at the end of the time horizon. These authors present the CALM model, which was designed to deal with uncertainty affecting both assets (in either the portfolio or the market) and liabilities (in the form of scenario-based payments or borrowing costs). Their work suggests that their stochastic ALM model has given good results compared to deterministic models of 5 years. The Watson model is a specific example of the CALM model for a pension fund.

The stochastic control approach captures uncertainty by allowing a continuum of states that can be characterized by a small number of state variables that follow a joint Markov process. Bielecki and Pliska (1998) proposed a new stochastic control approach for pension funds that explicitly incorporates the underlying economic variables into the model as well as an objective function of the infinite risk-sensitive horizon.

The academic literature with respect to the application of LDI is still rather small. Amenc et al. (2006, 2007) apply the LDI concept to insurance companies and private wealth management. They argue that the LDI concept can be applied successfully in this case and give examples of the effectiveness of both static and dynamic LDI strategies. They show that, given a surplus optimization perspective, more efficient asset mixes can be found by introducing a liability-hedging portfolio (LHP) in the menu of asset classes. LDI solutions thus consist of three basic building blocks (cash, LHP, and a performance portfolio), as opposed to the allocation to standard asset classes, as in the context of regular surplusoptimization techniques.

Mindlin (2006) discusses the relevance of LDI solutions for pension funds. He argues that the LDI concept is inadequate for open-ended pension plans since matching assets for on-going plans rarely exist. For example, if the matching asset for an on-going plan existed, it would contain bonds indexed to wage inflation with maturities of 50 years or more. Mindlin therefore concludes that the LDI approach is more appropriate for terminated plans or plans for which termination is likely. He also stresses that the impact of other risk factors (contribution risk, solvency risk, accounting risk, etc.) is not weighted appropriately by LDI proponents.

Boender (2007) points out that LDI is not a completely new concept at all: asset and liability management (ALM) for pension funds has given the liabilities a central role a long time ago (see for example Zenios and Ziemba (2006, 2007) or Ziemba and Mulvey (1998) for excellent overviews). Boender also demonstrates, with an example from practice that most LDI solutions for open-ended pension funds lead to solutions that are too risk-averse and, as a consequence, too expensive. He remarks that it is common practice that, given an agreed-upon asset allocation, sponsors contribute more in case of a decreasing funded ratio.

Studies focusing on this liability driven investment (LDI) approach of pension funds constitute a growing area within the portfolio choice literature (e.g., Sharpe & Tint, 1990; Rudolf & Ziemba, 2004; Hoevenaars, Molenaar, Schotman &Steenkamp, 2008; Berkelaar & Kouwenberg, 2010; Ang, Chen & Sundaresan, 2013; and Van Binsbergen & Brandt, 2014).

3. Methodology and data.

Optimizing the financial management of reserves is very important to ensure the solvency of pension funds.

3.1. Presentation and description of data

As stipulated by the law n° 43-95 reorganizing the Moroccan pension fund, the financial management of the reserve funds aims to contribute, in the long term, to the financial equilibrium of the pension plans managed by the CMR. As regards the use of reserves, the article 14 of this law defines the investment universe and establishes the list of asset classes in which the CMR can make use of its established reserves. Thus, surpluses generated by the civil pensions scheme may be invested in government securities and those with a guarantee of 65%, listed shares and any securities traded on a regulated market up to 30%, and 5% of real estate, after authorization from the Tutorship ministry.

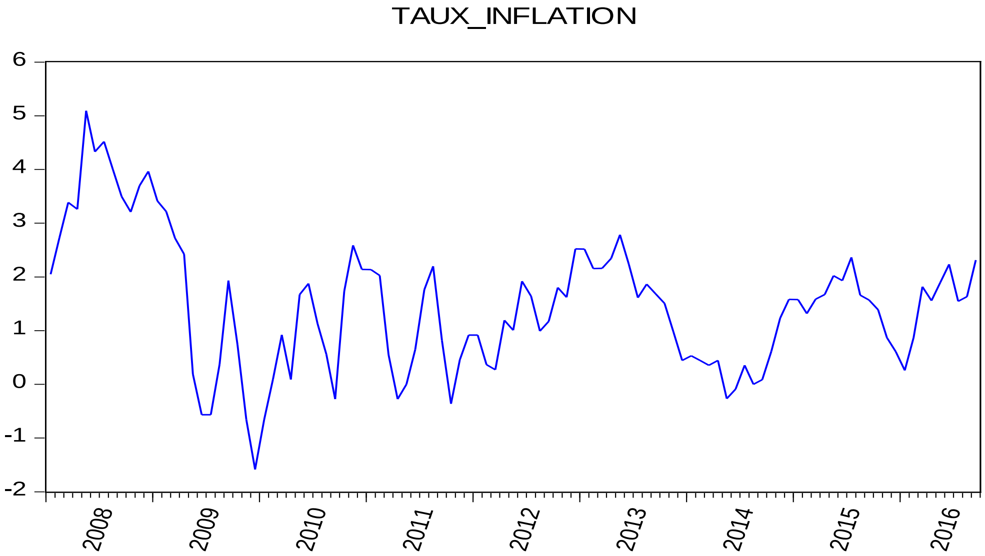
As we have already mentioned, we will focus on the generation of economic scenarios whose financial and macroeconomic variables are the inflation rate, the real long-term interest rate and the short-term interest rate, an investment in stocks, in bond and the return on an investment in real estate. Moreover, the modeling of these variables must integrate the long-term perspective of the management of these variables. This long-term perspective has consequences for the estimation of the parameters of the model chosen: the estimation must be made on series whose depth must be consistent with the envisaged horizon of projection. In this context, we have selected series of monthly data[2] with the exception of the real estate indices for which they are quarterly and dating from January 2004 to December 2016, thus covering a period of about 13 years, which is coherent with a projection horizon of 20 years. This period seemed appropriate to take into account changes in political and economic context. Moreover, having defined this period, the construction of these data series was based on the sources (available on the CMR sites, Casablanca Stock Exchange, HCP, the Bank of Maghreb BAM and DAPS). Particular attention will be paid to inflation, mainly because of the central role it plays in the asset-liability model ALM and LDI. The historical inflation rate is based on the Consumer Price Index (CPI) published on the HCP site. The CPI is calculated and published monthly only in 2006, to replace the cost of living index (LCI) introduced in 1991. More precisely, the annual inflation rate in month m is: $q_m = \ln(\frac{IPC_m}{IPC_{m-12}})$.

In particular, the high level of inflation rate was observed in 2008, despite state control measures and a sharp drop to negative levels in 2009 following the effects of the 2007 financial crisis (Figure 1). In order to avoid obtaining negative rates and therefore the deflation hypothesis, we applied a lower bound equal to zero.

The nominal long-term rate (Figure 2) is the long-term government bond rate published by BAM (maturing 10 years).

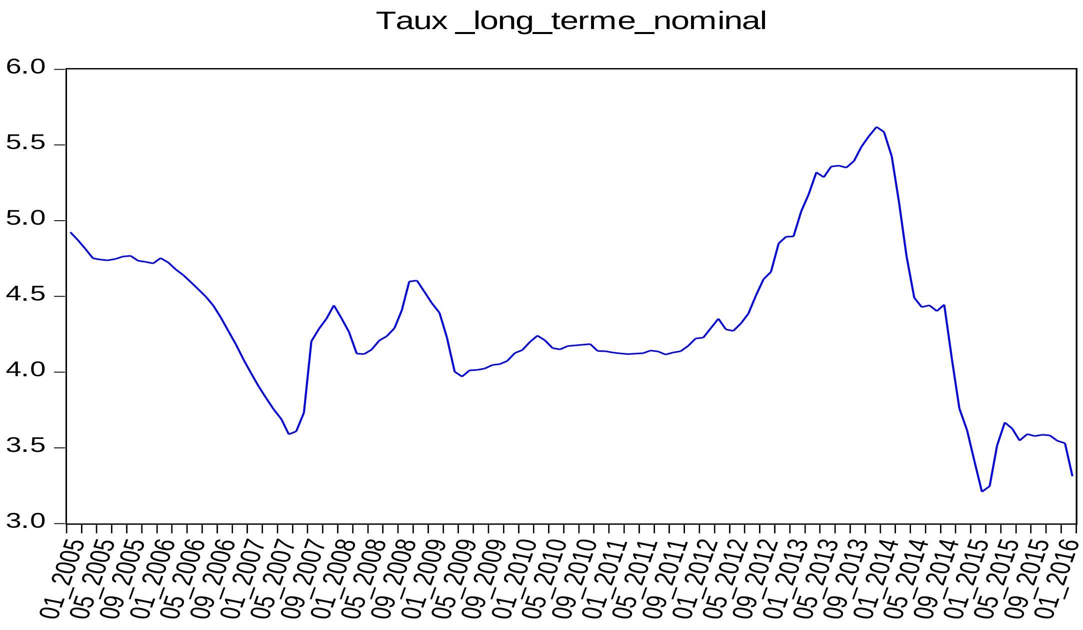
[2] Indeed, annual or quarterly data would give us a depth of history too low for our modeling to be reliable.

Figure 1. Evolution of the inflation rate between 2006 and 2016.



Source: Own elaboration.

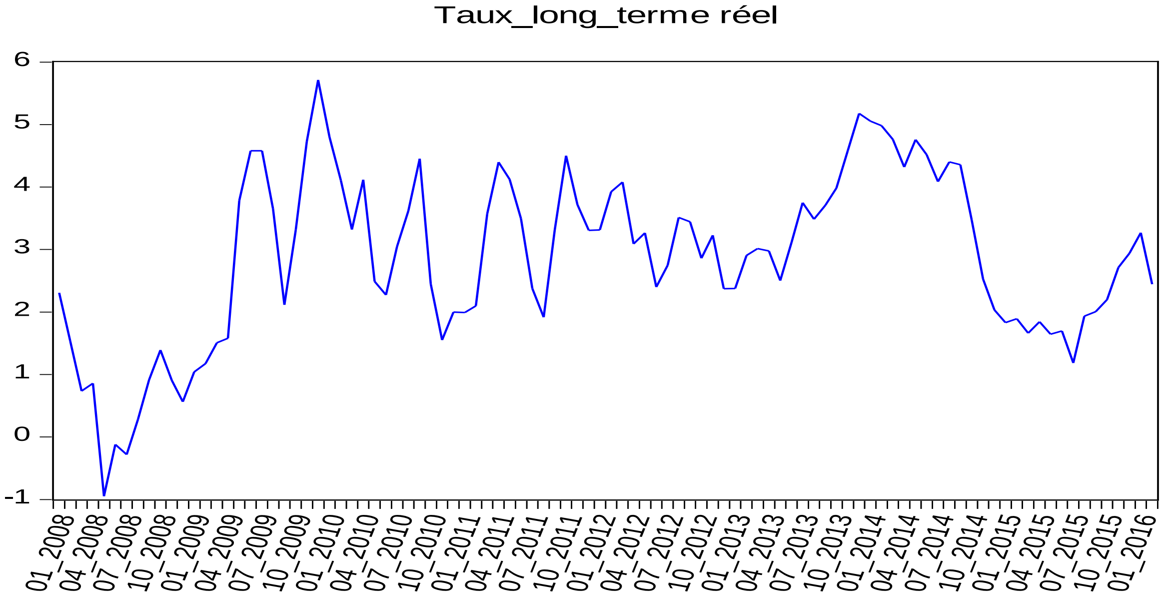
Figure 2. Evolution (percentage) of the nominal monthly long-term interest rate.



Source: Own elaboration.

Long-term real interest rate history is constructed from the historical values of nominal long-term interest rates to which the historical values of the inflation rate are subtracted (Figure 3).

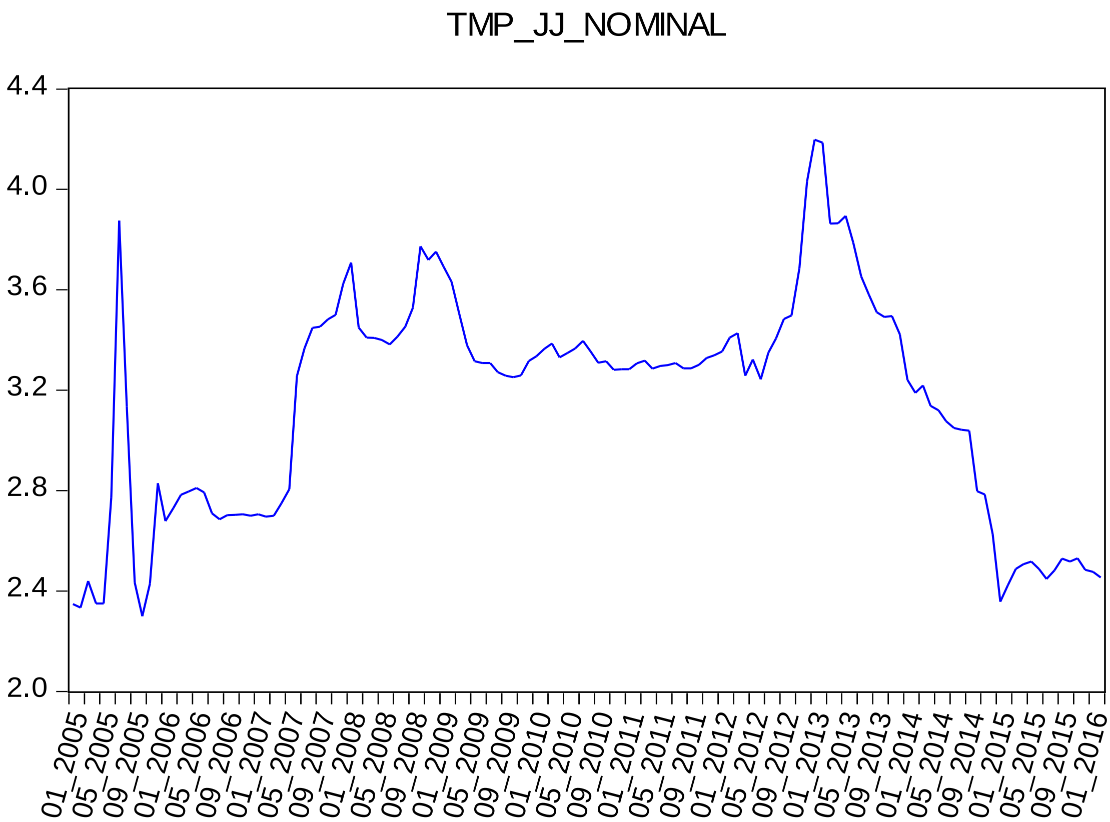
Figure 3. Evolution (percentage) of the real long-term interest rate.



Source: Own elaboration.

The nominal short-term interest rate (Figure 4) used corresponds to the average daily money market rate (TMPJJ) published by Maghreb Bank between 2006 and 2016.

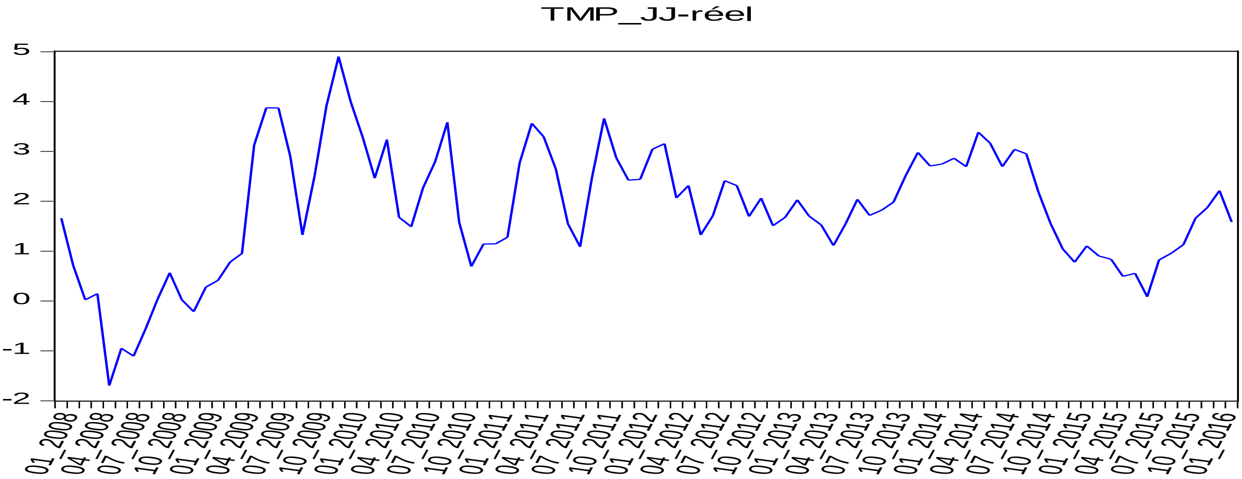
Figure 4. Evolution (percentage) of the short-term nominal monthly interest rate.



Source: Own elaboration.

The history of short-term real interest rates is constructed in the same way as real long-term interest rates.

Figure 5. Evolution of the short-term real interest rates.



Source: Own elaboration.

Concerning real short-term and long-term rates, they are calculated at any time t, taking into account the previously determined inflation, namely: $r_t = \left(\frac{r_{nom,t}}{1+q_t}\right) - 1$ et $l_t = \left(\frac{l_{nom,t}}{1+q_t}\right) - 1$

It should be noted, however, that the generation of negative interest rates may be a problem for the calculation of market values of bonds in the general modeling of pension funds. We have therefore minimized the rates to zero to avoid any inconsistencies in the model.

The Moroccan pension fund has a much diversified portfolio of equities that provide a significant return. However, there is a regulatory limit capped in 25% of the total value of the CMR portfolio.The main reason is obviously to avoid assets that are too risky given that there are commitments to be met in the medium and long term. The CMR equity portfolio reflects 70% of the MASI index. Thus, in order to construct the history of stock returns, the MASI index with dividends reinvested (total return) was retained.

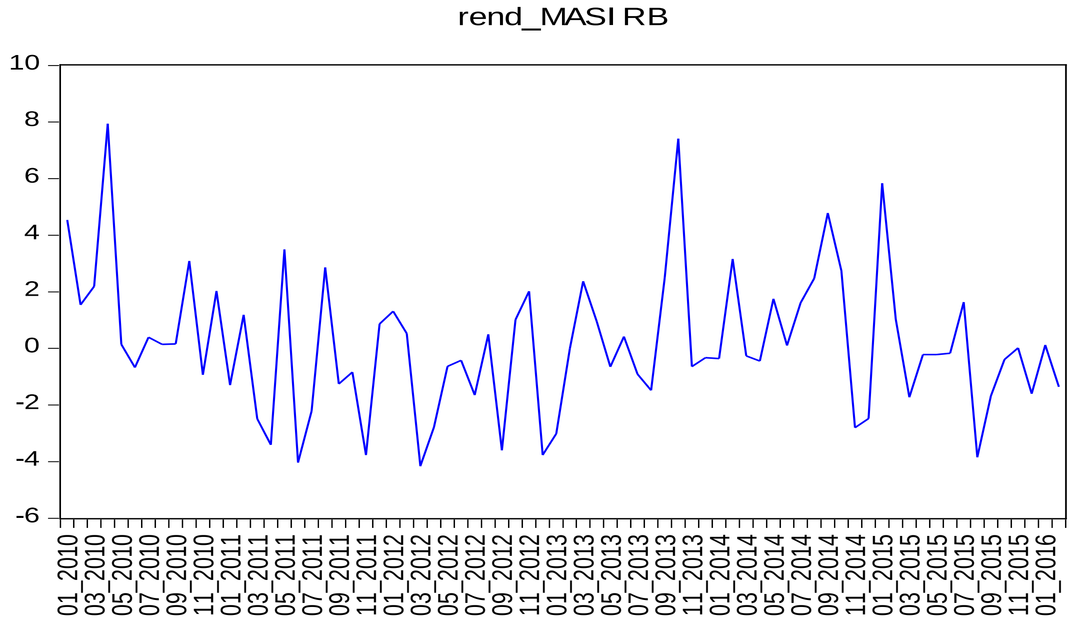
The MASI index is a capitalization index that traces the overall evolution of the market and takes into account all listed securities, which number 75 in 2015. The choice of this index responds to the constraint of coherence and that of diversification. The monthly return is calculated using the following formula: $x_m = \ln({}^{S_m}/_{S_{m-1}})$, where S_m represents the MASI index with dividends reinvested in month m.

The return of equities with reinvested dividends evolves according to the formula below:

$$x_t = i_{r,t} + e_t$$

- $i_{r,t}$: the nominal short-term interest rate at any time t.
- $e_t = x_t - (r_t + q_t)$: the excess return of the shares relative to the risk-free rate.

Figure 6. Evolution of the monthly return of MASI between January 2010 and February 2016 (percentage).



Source: Own elaboration.

However, unlike the stock rate of return, calibrating the performance of the excess return on the stock over the history of January 2010 to February 2016 poses a problem in the average. Indeed, this value is negative and if it is retained for projections and simulations, we will end up with a yield that diverges in the long term. The cause of this problem is the sharp decline in excess returns during the month of November 2011, representing a return of (-7.33%) and the month of March and December 2012 respectively of (-7.58%) and (-7.80%), as well as the first month of 2013 with a return of -7.21%.

Sovereign bonds represent the largest share of the CMR portfolio with a minimum regulatory level of 60% of the total value of the portfolio.This is the least risky asset class. The civil regime of the Moroccan pension fund, invested throughout the yield curve. In this sense, the performance of its sovereign bond portfolio will be calculated from the global MBI[3] fixed income index[4]. The monthly return of the MBI in month m is calculated using the following formula:

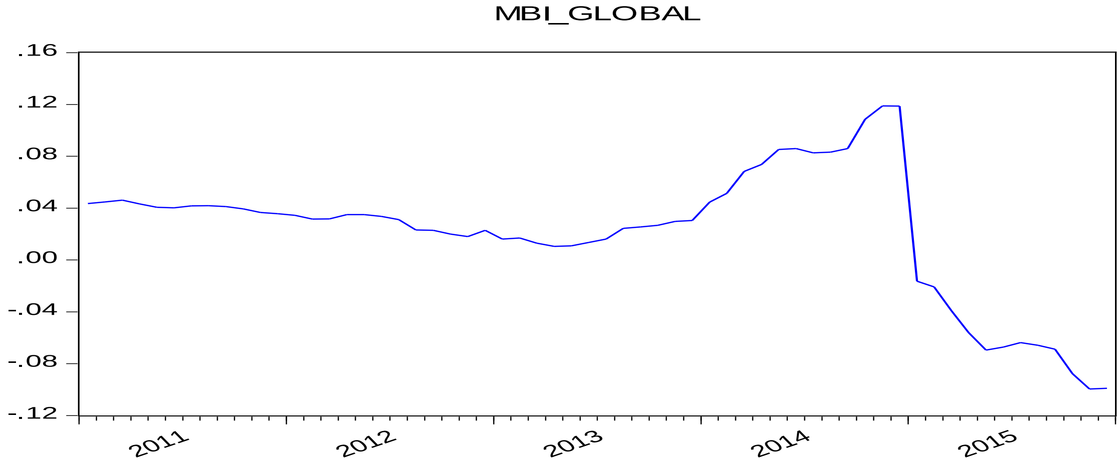
$y_m = \ln({}^{B_m}/_{B_{m-1}})$, where $B_m$represents the index MBI with coupons reinvested in m.

Given the importance of real estate in financial investments, more and more academic publications encourage investment in this type of asset, as some research has shown that real estate could have replaced bonds in a portfolio, and offers higher yields. As regards CMR, the regulations do not grant it the right to exceed 5% of the total value of the investment portfolio. The main reason for this limitation is the heaviness of the risks associated with this asset class such as (natural disasters that can destroy buildings, liquidity risk, the offer and demand).

[3] The MBI index was created by BMCE Capital. It consists of several hundred fixed-rate government bonds, mainly treasury bills. This index reflects the actual yield of the Moroccan bond market.

[4] For bonds, we based our composition on bond indices for simplicity of handling. The Manager will then be responsible for including any obligations he may wish.

Figure 7. Evolution of the return of the MBI index between January 2010 and February 2016.



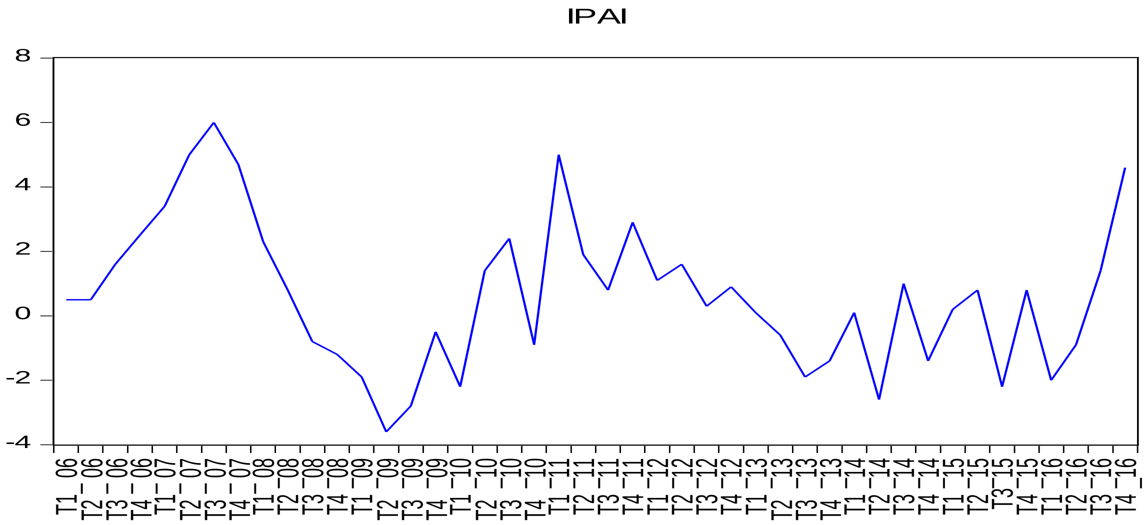
Source: Own elaboration.

We now have to project the value of real estate held by the CMR, but there is no history of the value of the buildings held, so it will be necessary to use a proxy for the underlying risk index real estate. The Property Price Index (IPAI) was developed jointly by Bank Al-Maghreb and the National Agency for Land Conservation, Cadastre and Cartography (ANCFCC). On a quarterly basis, this index, based on 100 in 2006, is calculated using the method of repeated sales, which makes it possible to remedy the problem of the heterogeneity of real estate. It should be noted that the only indicators of the real estate sector published regularly in Morocco are the IPAI and the index IMMO published by the Casablanca Stock Exchange. As we are interested in an index that traces the price evolution of the property itself, rather than the performance of the listed companies attached to the real estate sector, we decided to choose the IPAI that is more appropriate to the philosophy of our model.

More specifically, the annual yield of real estate in quarter m is written at any time t:

$$Im_m = \ln(^{IPAI_m}/_{IPAI_{m-4}})$$

Figure 8. Evolution of the annual quarterly performance of the IPAI between January 2006 and December 2016.



Source: Own elaboration.

The graphic above illustrates the evolution of the annual quarterly rate of return of real estate between January 2006 and December 2016. In particular, there is the sudden fall in 2009 due to the financial crisis.

The calibration of the yield of our property net of inflation on the history dating from the first quarter of 2006 to the fourth quarter of 2016 poses a problem at the level of long-term rate of return. Indeed, this value is negative (-0.487%) and if it is chosen for projections and simulations we will find (for certain values of real estate returns) with a yield that diverges in the long term. In order to remain consistent with the modeling of the other variables of the economic scenario generator, the calibration is made on the series of gross return of inflation.

The CMR reserve fund portfolio includes other assets that will not be taken into account in the development of our ESG; private equity, monetary and private bonds. We then present the basic descriptive statistics of the set of study series.

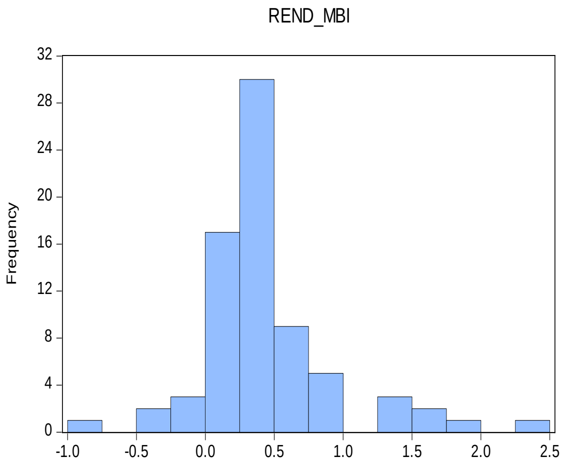
Table 1. Descriptive Statistics of ESG Components.

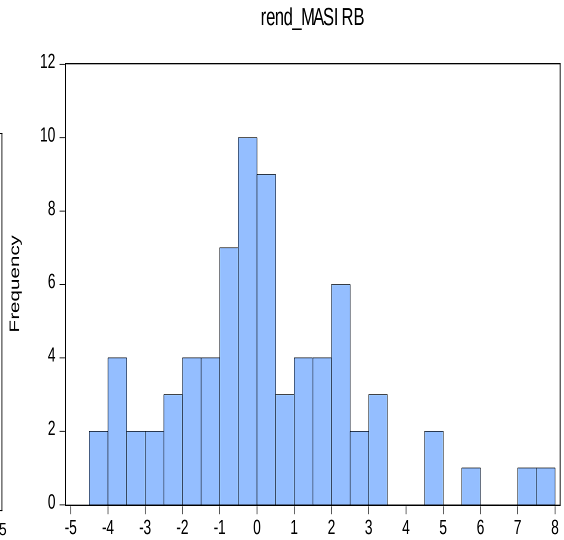
	Inflation	Nominal long-term	Nominal short-term	Equities	bond	Real-estate
Mean	1.464856%	4.332467%	3.1510%	0.1671%	0.447914%	0.629545%
Volatility	1.2422%	0.516883%	0.4466%	2.5020%	0.51249%	2.285707%
Median	1.577596%	4.239005%	3.2920%	-0.0898%	0.35755%	0.65%
Maximum	5.0948%	5.620259 %	4.1978%	7.9411%	2.47000%	6%
Minimum	-1.5866%	3.209706%	2.3000%	-4.1553%	-0.9200%	-3.6%
Skewness	0.373206	0.379064	-0.222584	0.6983	1.33419	0.442091
Kurtosis	3.146694	3.062950	2.274811	3.8990	6.68281	2.697879
Jarque-Bera	0.282014	0.198773	0.132474	0.01421	0.00000	0.449195

Source: Own elaboration.

Apart from stock returns and bond yields, the other series positively respond to the Jarque-Bera normality test. These early statistics suggest that the sample could approximate a normal distribution.

Figure 9. Histograms of the returns of the MASI RB and MBI indices.





Source: Own elaboration.

Most models rely mainly on the linear correlation matrix to model the interdependence between ESG variables. This is due to several factors; the simplicity of the models with linear dependence (correlation).

This correlation matrix between the gross inflation ESG variables is estimated on annual data between 2010 and 2015. The differences observed in the correlation coefficients suggest a great diversity of the dependency structures of the bi-varied series.On the gross inflation matrix below, we note, the positive correlation between the rate of inflation and the return of equities. Indeed, this class should theoretically protect the investor against inflation. Similarly, the correlation between short and long-term interest rates and bond yields is strongly positive. Intuitively, in times of uncertainty and turbulence in the markets, investors seek security and turn to sovereign bonds. Similarly, the correlation between nominal short-term and long-term interest rates and equities is positive but not strong enough.

We note that in this system, real estate is negatively correlated with other variables. This can be explained by the assumption made on real estate as a safe haven in the event of a crisis.

Table 2. Correlation matrix (gross inflation) over the entire history of ESG components.

	Inflation	Real-estate	Nominal long-term rate	Nominal short-term rate	Equities	Bond
Inflation	1.000000	-0.080176	0.27517	0.522136	0.183656	0.38081
Real-estate	-	1.000000	-0.433763	-0.120329	-	-
Nominal long-term	0.27517	-0.433763	1.00000	0.877658	0.252228	0.97656
Nominal short-term	0.522136	-0.120329	0.877658	1.000000	0.228202	0.94247
Equities	0.183656	-0.731682	0.252228	0.228202	1.000000	0.3348
Bond	0.38081	-0.373266	0.976566	0.942478	0.3348	1.00000

Source : Own elaboration.

The occurrence of a technical deficit of DhM 937 million from 2014 and the second deficit in 2015 has led to a mechanical increase in the contribution rate needed to balance the scheme. The contribution rate must be of the order of 45.75% in 2045, which shows the serious deficit incurred by the civil regime of the CMR.

Given the increase in benefits and the reduction in contributions, it is clear that the coverage rate is falling. Thus, it rose from 85.5% in 2015 to 32.25% in 2045. From the year 2014, the scheme began to draw on its reserve fund to finance the gap between the revenue and expenditure of the scheme. However, this situation would gradually lead to a depletion of reserves which would become negative in 2023 with an amount of about DhM 15 995 billion if no action is taken (Aitoutouhen & Faris, 2016).

3.1. Construction and test of a ESG.

In this study, the choice of a diffusion model relates to the following financial variables: term structure of interest rates (including nominal, real and inflation interest rates), return on equities, bonds and the yield of real estate. These variables are selected because of their importance in the context of a long-term strategic allocation of our pension fund (Campbell et al. (2001).

The inflation model is assumed to follow the Uhlenbeck Ornstein process. There are two reasons for choosing such a process. Empirically, there is a "mean reverting" effect of inflation rates. Institutionally, central banks have an inflation target, which suggests that this "mean reverting"should continue to exist in the future.

The inflation variable will be denoted by q_t at any time t and corresponds to the following stochastic differential equation with $q_t = q_0$ for $t = 0$ and $Z_{q,t}$ the Brownian motion at any instant t.

$$dq_t = \kappa_q(\mu_q - q_t)dt + \sigma_q dZ_{q,t}$$

The exact discretization of processes:

$$q_{t+\delta} = q_t e^{-\kappa_q\delta} + \mu_q(1 - e^{-\kappa_q\delta}) + \varepsilon_{q,t}\sigma_q\sqrt{\frac{1 - e^{-2\kappa_q\delta}}{2\kappa_q}}$$

We used the two-factor Hull & White model (1994) to describe the dynamics of the long real interest rate denoted l_t and the short real interest rate denoted r_t, in order to deduce there from the laws of nominal rates. The stochastic differential equations corresponding to this modeling are the following:

$$dr_t = \kappa_r(l_t - r_t)dt + \sigma_r dZ_{r,t}$$
$$dl_t = \kappa_l(\mu_l - l_t\ )dt + \sigma_l dZ_{l,t}$$

We obtain after discretization:

$$r_{t+\delta} = r_t e^{-\kappa_r\delta} + l_t(1 - e^{-\kappa_r\delta}) + \varepsilon_{r,t}\sigma_r\sqrt{\frac{1-e^{-2\kappa_r\delta}}{2\kappa_r}}$$
$$l_{t+\delta} = l_t e^{-\kappa_l\delta} + \mu_l(1 - e^{-\kappa_l\delta}) + \varepsilon_{l,t}\sigma_l\sqrt{\frac{1-e^{-2\kappa_l\delta}}{2\kappa_l}}$$

In our study, we choose to model stock returns using the Black-Scholes formula; the basic assumption is that the price follows a geometric Brownian motion with constant volatility. We then have the following formulation for the dynamics of the instant return of the equity portfolio under the real probability: $dx_t = \mu_x x_t dt + \sigma_x x_t dZ_{x,t}$.

The resolution of the equation of return of the equity index is based on the lemma of Itô:

$$x_{t+\delta} = (\mu_x - \frac{{\sigma_x}^2}{2})\delta + \sigma_x\sqrt{\delta}\varepsilon_{x,t}$$

This model has the advantage of being easy to calibrate, simple to implement and the estimation of the different parameters by the method of maximum likelihood is direct.

In our case, we will retain a benchmark index on the market MBI and adopt the same modeling used for stock returns.The difficulty of holding a portfolio of bonds with different maturities and the uncertainty of the availability of similar bonds in the future can be overcome by the use of bond indices, which represent a class of bonds. Fund managers often use these bond indices to compare their portfolio of bonds (Vassiadou-Zeniou & Zenios, 1996).

The performance of the bond index with reinvested coupons evolves according to the Black-Scholes formula. Thus, the following formulation for the dynamics of the instant yield of the bond portfolio under the real probability is: $dy_t = \mu_y y_t dt + \sigma_y y_t dZ_{y,t}$

The resolution of the yield equation of the bond index is based on the Itô lemma:

$$y_{t+\delta} = \left(\mu_y - \frac{{\sigma_y}^2}{2}\right)\delta + \sigma_y\sqrt{\delta}\varepsilon_{y,t}$$

The diffusion model for real estate yield follows the same logic as that of inflation, namely the following stochastic differential equations:

$$dIm_t = \kappa_{Im}(\mu_{Im} - Im_m)dt + \sigma_{Im}dZ_{Im,t}$$

The following discretization follows:

$$Im_{t+\delta} = Im_t e^{-\kappa_{Im}\delta} + \mu_{Im}\left(1 - e^{-\kappa_{Im}\delta}\right) + \varepsilon_{Im,t}\sigma_{Im}\sqrt{\frac{1-e^{-2\kappa_{Im}\delta}}{2\kappa_{Im}}}$$

Our choice of a process characterized by the mean-reverting for real estate is justified by the supposed presence of a real estate bubble.

Calibration is a fundamental step following the implementation of financial models. It allows adjusting the parameters of the model in order to make the results coherent.For this purpose, two main types of estimation methods for stochastic process parameters should be distinguished. On the one hand, the Vasicek models (1977), which are similar to autoregressive models of order 1 after discretization (models on inflation, real interest rates and real estate yields and on the other hand the total return model for equities and bonds, in which the maximum likelihood should be estimated for average returns and volatility.

The relevance of the model will be assessed on the basis of its quality of fit and its overall significance. Adjustment quality will be assessed from the coefficient of determination R^2 and the overall significance will be measured from a Fisher test (1954). In addition, it will be necessary to evaluate the individual significance of the coefficients from the Student test.

Particular attention should be given to residue analysis, and in this context, the first step is to ensure that the residues are of zero mean.In a second step, it will be necessary to detect the possible auto-correlation of the errors, conventional in the time series models. For this purpose, the Durbin-Watson test for first order autocorrelations and the Breusch-Godfrey test for autocorrelations of order $p \geq 1$, Godfrey (1979). The test based on the ARCH models on the residuals verifies the non-heteroskedasticity of the model. In a fourth step, it will be necessary to carry out an error of normality. Indeed, we will retain the Jarque and Bera (1980).

The generation of economic scenarios will be realized in a real - world, in which the behavior of our variables is projected as observed in historical contexts.The projection of our variables consists in generating yield trajectories (for each asset class) thanks to discretization formulas presented above.

As a result, these projections are carried out using the Monte Carlo technique on the basis of the technical calibration of our model from a model close to that of Ahlgrim. In this work, this Monte Carlo simulation method consists in generating 10,000 economic scenarios to obtain various indicators. Moreover, it allows us to observe the distribution of the macroeconomic and financial variables of our model. These projections are presented in the case where the dependency structure of residuals and yields on equity and bond is described by the correlation. This is equivalent to saying that the dependency structure of the financial and macroeconomic indices studied is also described by the correlation (and is therefore Gaussian) since the relations which link these indices to residues and yields are linear.

In general, the diffusion processes used are characterized by residues following normal laws. Projecting the residues thus amounts to simulating Gaussian random variables correlated with each other by the estimated correlation matrix via the factorization of Cholesky (Campbell & Viceira, 2001). Moreover, for all projections, it is assumed that the initial date, t = 0, corresponds to the end of February 2016.

Table 3. Initial Values of ESG Variables.

ESG Variables	Initial Values
Inflation	0.6084%
Nominal long-term rate	1.9616%
Nominal short-term rate	0.9339%
Rate of return on real estate	0.8%

Source: Own elaboration.

The overall approach is therefore to first project the residuals of the inflation rate, real interest rates and real estate as well as the returns of MASI RB and MBI. Then the projections of the financial and macroeconomic variables can be deduced from the formulas presented above.

To test the robustness of our ESG, we will analyze its different properties: representativeness, economic interpretation, parsimony, transparency and evolution (Hibbert et al., 2001).

The quality of this forecast is measurable through a comparison between the empirical distribution of the ESG variable resulting from market observations and that of the projected variable resulting from the modeling.

Backtesting is the validation of a model on historical data. In our context, we will first estimate the model with a history up to December 2013 and then project the ESG variables from the 10000 simulations over the next 10 years, that is to say from 2014 to 2024. Establishing a backtesting for our model is equivalent to verifying that it is capable of making realistic predictions that follow the observed data trend.

3.2. Study of strategic allocation criteria in the framework of ALM.

We will focus on the application of asset-liability management based on the Fixed-Mix strategy to our pension fund that is currently in deficit. The implementation of an ALM strategy makes it possible to determine the best allocation of assets in the medium or long term. The objective of the optimal allocation sought is to optimally guarantee the solvency of the scheme while preserving assumptions of reserves and coverage ratio.

First, plan management always involves some kind of modeling of pension fund assets and liabilities for a given time horizon (usually at least 10 years). In our case, we have chosen to value assets and liabilities according to their market value, while striving to remain consistent with the results of modern financial economic theories.

The valuation begins with an appropriate estimate based on rules that take into account all relevant actuarial factors. Then, the present value of these cash flows is calculated in each forecast year using a discount rate[5]. The present value of liability flows is done with the values of the nominal long-term interest rates simulated in each forecast year by the ESG that we will build.

$$L_t = \sum_{i=t+1}^{t+h} \frac{\mathrm{flow}_i}{\prod_{j=t+1}^{i}(1+r_j)}$$

[5] As we consider a defined benefit plan, the total liability is the discounted value of future predefined payments. At some point t, it represents the amount the fund must repay if it is to close at that time.

with r_j the interest rate over period j (real rate in the case of real flows and nominal rate in the case of nominal flows) and $h = 20$ years.

This liability is also revalued at the rate of revaluation or indexation by the inflation rates simulated by the ESG developed. In this case, the real flux at time t is multiplied by the indexing coefficient (ie 1/3 of the inflation value for the civil regime case) simulated over the periods between the initial time and t:

$Indexed\ flow_t = Realflow_t \times \prod_{j=1}^{t}(1 + i_j)$, with i_j the inflation rate over period j.

$$L_t = \sum_{i=t+1}^{t+h} \frac{Indexed\ flow_i}{\prod_{j=t+1}^{i}(1 + r_j)}$$

During the course of our study, the value of the asset changes over time, depending on the contributions made to the fund, the evolution of the reserve on the basis of the returns on the financial assets constituting the portfolio and the choices for rebalancing the portfolio on the various financial assets. Future contributions will therefore be assumed known:

$$A_t = R_{t-1}(1 + \rho_t) + C_t$$

The definition of eligible asset classes in the modeled portfolio primarily depends on the regulatory, operational, modeling constraints and desired level of finesse in the portfolio representation. This will make it possible to reduce the size of the allocation matrix and thus substantially reduce the computation time. Consistent with our perceptions of risk and our long-term strategies, we have taken into account the financial risk factors (bond, real estate and equities) without considering the risk of longevity (Juillard et al., 2008) or the spread risk (Planchet et al., 2012). The distribution of the resources of the Moroccan Pension Fund between the above-mentioned uses is determined by regulation, the law of May 2010[6].

Another type of constraint is systematically taken into account in the context of pension funds: the sum of the weights of each asset is equal to the unit.

When the initial allocation is defined, ie the weight of each asset class, we consider the management rule which consists in recomposing the portfolio at the end of each period in order to preserve the initial proportions 'rebalancing'. This approach controls the weight of each asset and the risk associated with the portfolio.

The amount of the total reserve evolves according to the following dynamics:

$$R_t = R_{t-1}(1 + \rho_t) + C_t - P_t$$

We determine the value of the 'objective' function for each of these allocations. Then, those that maximize the objective function and respect the constraints fixed will be retained.

[6] The major part of the portfolio is invested in favor of the State in order to finance the Treasury. This rate was 80% as a minimum requirement of the total value of the portfolio. On the other hand, it was not until 2010 that the State gave the green light to the CMR to invest up to 30% of the portfolio in equities. These figures raise the question of the effective independence of WRC from the management of its reserve funds. Similarly, the CMR introduced new regulatory limits by asset class:
- Extension of the class of private debt to commercial paper and unlisted bonds.
- Extension of the share class to Venture Capital Funds and Securitization Funds.

For our regime, which is in the depletion phase, maximizing the coverage or funding ratio can remain a good goal. The optimization system can be formulated as follows:

$$\text{Max } R_{t-1}(1+\rho_t) + C_t - P_t \text{ , under constraint}$$

$$\text{Max} \frac{\text{Asset}_t}{\sum_{t=t_0+1}^{t_0+20} \frac{\text{Indexed flows}_t}{\prod_{j=t_0+1}^{t}(1+r_j)}}$$

In practice, the theoretical optimal allocation (i.e. resulting from the optimization program) is not always selected by the decision makers, especially when they call on their expert opinion. In this last approach, it is necessary to understand the behavior of the balance sheet in relation to the allocation chosen by these decision-makers, in particular the exposure to the ruin that the selected allocation implies.

Furthermore, it should be noted that the Fixed-Mix strategy involves periodic rebalancing of assets, to keep the composition of the financial portfolio constant at the beginning of each subperiod. Although the Fixed-Mix strategy has the merit of taking into account the randomness of the economic and financial scenarios, this strategy can be criticized for not assuming flexibility in future market positions in relation to the various scenarios projected performance. More precisely, this approach does not allow the full use of information that happens over time to optimize the choice of strategic allocation (Zenios, 2007).

In this context, the objective of this work is to propose a procedure for determining the specific strategic allocation of pension schemes integrating the particularities of insurance, without the need to fix a priori the probability of ruin, which is simply controlled ex post.

We measure the risk associated with the strategy at Value at Risk and CVaR over the one year horizon at 90%, 95% and 99.5%. Indeed, exposure to ruin based on the insurer's VaR calculated from projections from the developed generator is compared to that obtained from the CVaR model.

The most important risk management tool is the stress test imposed by the regulator.To realize these stress tests, we will be inspired by the rules applied to the insurance industry to stress the optimal portfolio of funds on the reserves of the RPC. This exercise consists of simulating extreme financial conditions and ensuring the portfolio capacity of the FDR (funds on the reserves) to resist a systemic risk materialized by several scenarios of negative developments on the financial markets. To do this, four market development scenarios were chosen to carry out the stress tests: a scenario of falling stocks (-30%), real estate (-20%), a scenario of rising inflation rates (+ 2%) and a combined scenario.

3.3. LDI model implementation.

In order to optimize the financial management of the CMR's civil reserve fund, we will try to put in place a Liability Driven Investment Policy (LDI) that will try to find a better allocation of the reserve funds of the CMR under the regulatory constraints imposed on it and taking into account the evolution of liabilities.

In this sense, the LDI model treats the fund's liabilities as a state variable and specifies an 'objective' function by relating assets and liabilities. The investor considers the correlation between liabilities and assets to determine the optimal allocation of the portfolio.

In what follows, we will determine the optimal portfolios, which correspond to the risk accepted by the pension fund manager and the desired level of performance according to the first criterion 'optimization Asset-only'.

In the long term, our pension plan does not take short positions, so we are spreading short sales by imposing non-negativity constraints on the proportion of assets. We also do not allow borrowing money. In addition, we will not use derivative instruments since they do not exist in the case of Morocco. In addition, our LDI model includes upper and lower limits on the proportion of assets invested to exclude solutions that would be unacceptable for the plan.

We start by determining a portfolio over a period of 10 years. We emphasize the static and long-term nature of this projection, which should answer the question of the expected average rate of return and the level of risk for each asset class, if we averaged over 10 years.

The simple method of strategic allocation that we will present is based on the modern theory of portfolio choice, Markowitz (1959). The construction of model portfolios is based on the portfolio constructed by the MASI RB Index, the MBI Bond Index and the IPAI Real Estate. The resulting optimization program is written as follows:

$$\text{Min } \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

$$\begin{cases} \sum_{j=1}^{N} x_j \mu_j = \rho \\ \sum_{j=1}^{N} x_j = 1 \\ x_j \geq 0 \quad j = 1, \ldots\ldots\ldots N \\ \text{Regulatory constraints} \end{cases}$$

In an LDI approach, we will try to apply the surplus optimization method to our civil pension plan using the Sharpe and Tint (1990) approach. This model based on the notion of surplus applies to a portfolio made up of several asset classes. The aim of the model is to minimize the risk of loss of surplus (measured by the variance in return of the surplus) for a level of return of the surplus given the integration of contractual constraints on liabilities and regulatory constraints.

$$\text{Min } [\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} - \frac{1}{r_{f_0}} \sum_{i}^{N} \sigma_{i,L} x_i]$$

$$\begin{cases} \sum_{j=1}^{N} x_j \mu_j \geq \mu_P \\ \sum_{j=1}^{N} x_j = 1 \\ x_j \geq 0 \quad j = 1, \ldots\ldots\ldots N \\ \text{Regulatory constraints} \\ Cov(\tilde{R}_i, \tilde{R}_j) = \sigma_{ij}, Cov(\tilde{R}_i, \tilde{R}_L) = \sigma_{i,L}, \text{ et } r_{f_0} \text{ is the ratio of funding of pension funds.} \end{cases}$$

Next, we rely on the Sharpe and Tint model (1990) to determine the strategic allocation and the LHP and PSP portfolios (Markowitz & van Dijk, 2006). Alex Keel and Heinz H. Müller (1995) of the University of St. Gallen proposed the most practical approach. Recall our surplus optimization problem presented above:

$$\text{Min } [\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij} - \frac{1}{r_{f_0}}\sum_{i}^{N}\sigma_{i,L} x_i]$$

$$\text{Such that:} \begin{cases} \sum_{j=1}^{N} x_j \mu_j \geq \mu_P \\ \sum_{j=1}^{N} x_j = 1 \end{cases}$$

Keel and Müller used the Kuhn-Tucker theorem (a generalization of Lagrange multipliers, taking into account the constraints of inequality) to find the optimal conditions.

The portfolio x^* solution of the optimization problem is given by the following relation: $x^* = x^{min} + \lambda z^*$, λ depends on the level of profitability of the surplus.

- $x^{min} = \left(\frac{\sigma_{i,j}{}^{-1}e}{e'\sigma_{i,j}{}^{-1}e}\right) + \frac{1}{rf_0}(\sigma_{i,j}{}^{-1}\sigma_{i,L} - \frac{e'\sigma_{i,j}{}^{-1}\sigma_{i,L}}{e'\sigma_{i,j}{}^{-1}e}\sigma_{i,j}{}^{-1}e)$ represents the minimum variance portfolio.

- $z^* = \sigma_{i,j}{}^{-1}\mu + \frac{e'\sigma_{i,j}{}^{-1}\mu}{e'\sigma_{i,j}{}^{-1}e}\sigma_{i,j}{}^{-1}e.$

In order to test the resilience of our LDI fund design, we tested a stress scenario (down 30% from the MASI RB index and 20% from the real estate index).

4. Results.

Result 1

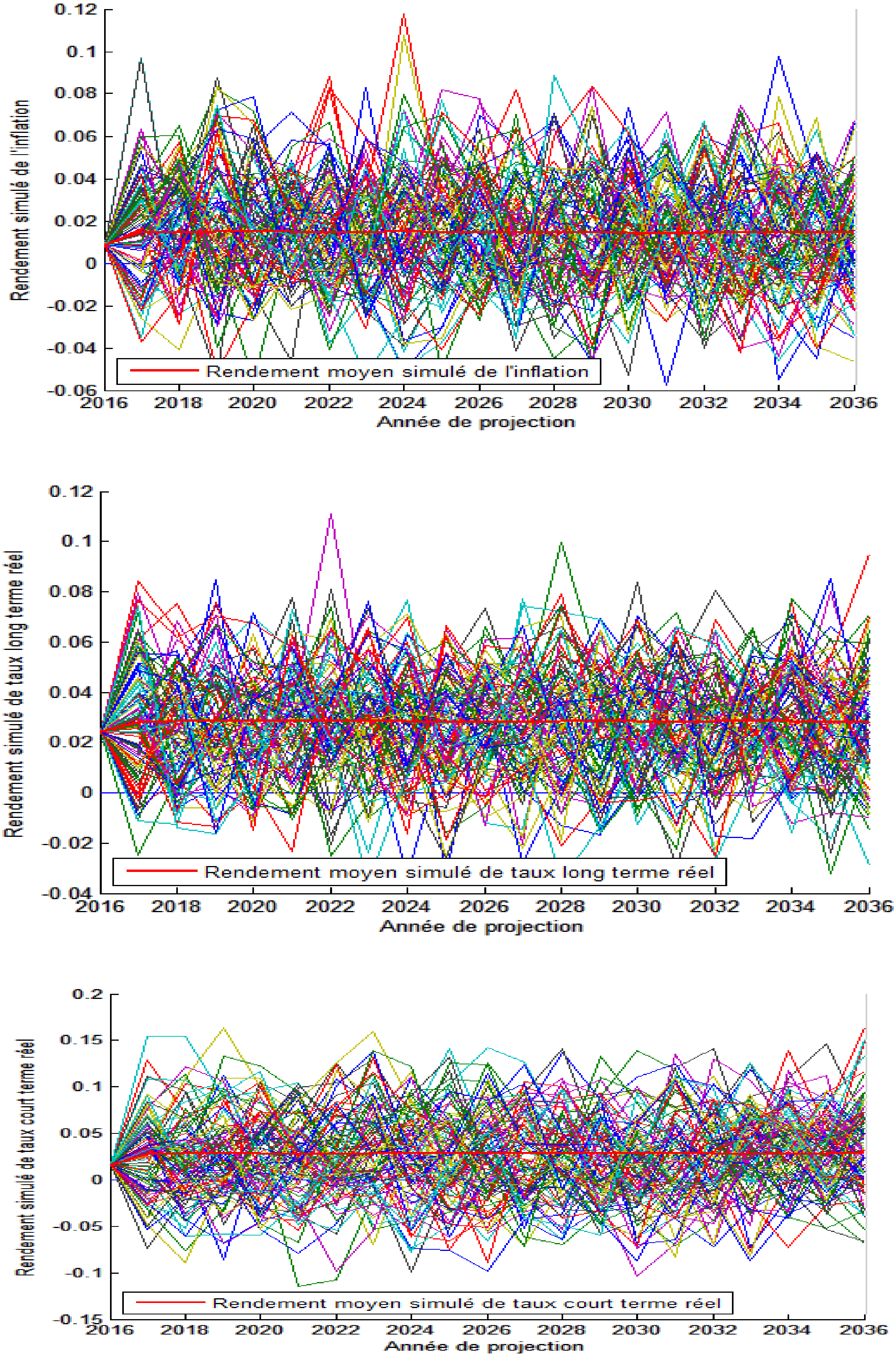
Based on the initial values presented below, calibration, statistical tests and for 10 000 simulations over a 20-year time horizon, we present the graphs relating to the projection of the components of our ESG by highlighting some properties theoretical models of the GSE model being studied.

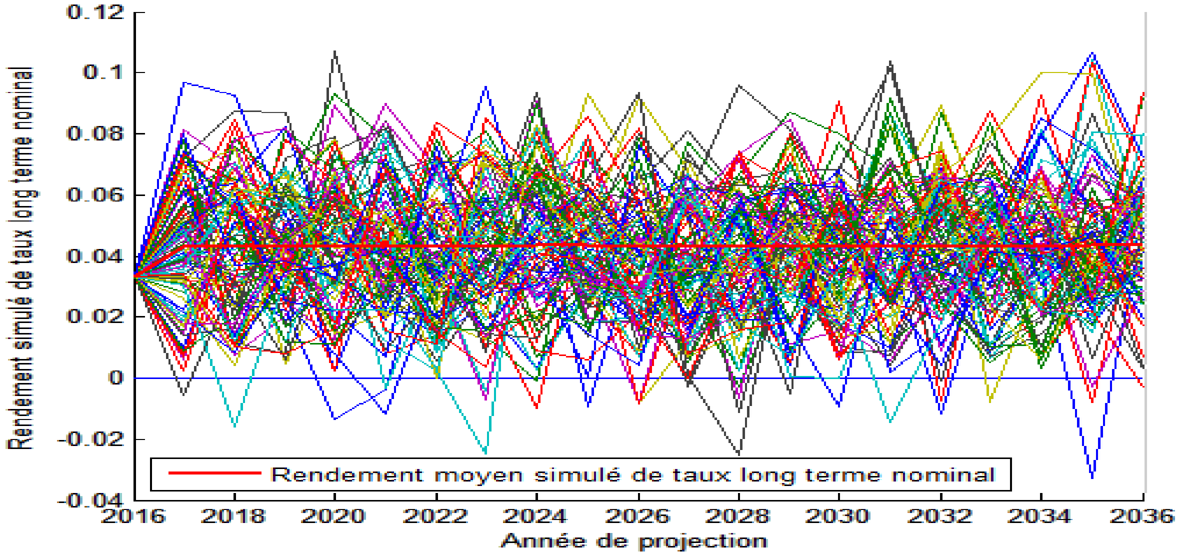
Table 4. Matrix of correlation between residuals and returns of the share index and bond.

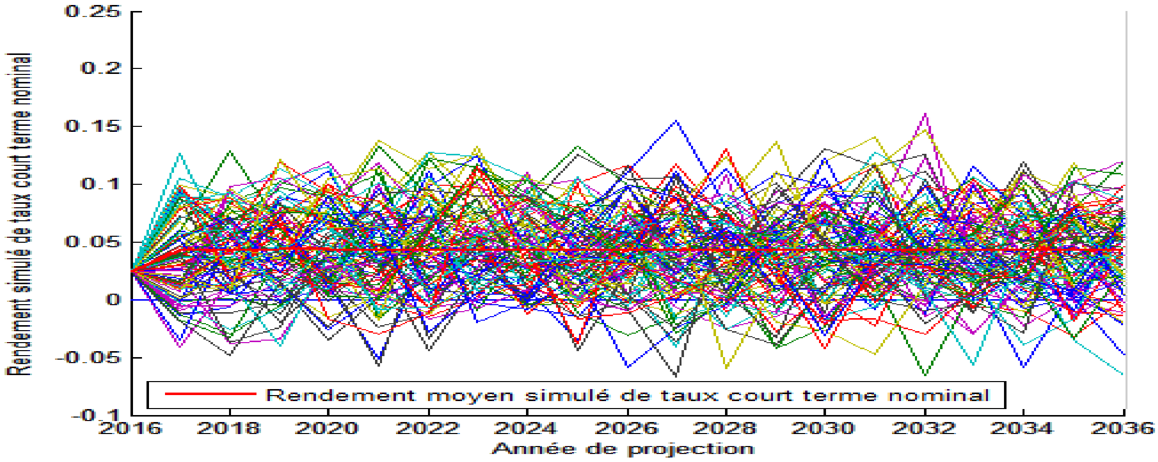
	Equities	Real-estate	Inflation	Bond	Real ST rate	Real LT rate
Equities	1.000000	-0.215066	0.259798	-0.126602	-0.018864	0.044033
Real-estate	-0.215066	1.000000	-0.175596	-0.288786	0.158413	-0.076115
Inflation	0.259798	-0.175596	1.000000	-0.203141	-0.644874	-0.643463
Bond	-0.126602	-0.288786	-0.203141	1.000000	0.142084	0.301494
Real ST rate	-0.018864	0.158413	-0.644874	0.142084	1.000000	0.889294
Real LT rate	0.044033	-0.076115	-0.643463	0.301494	0.889294	1.000000

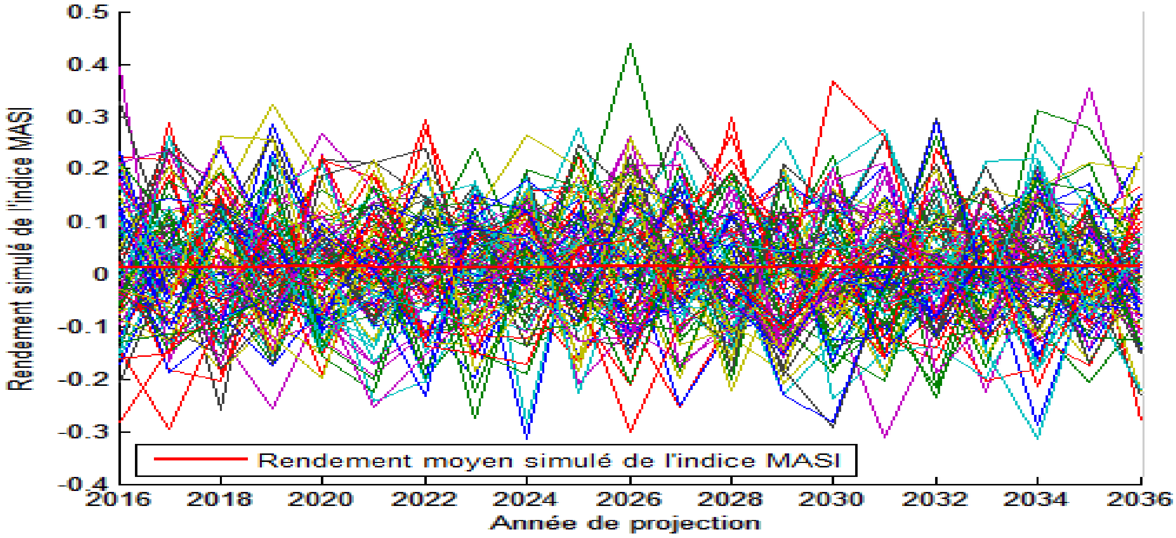
Source: Own elaboration.

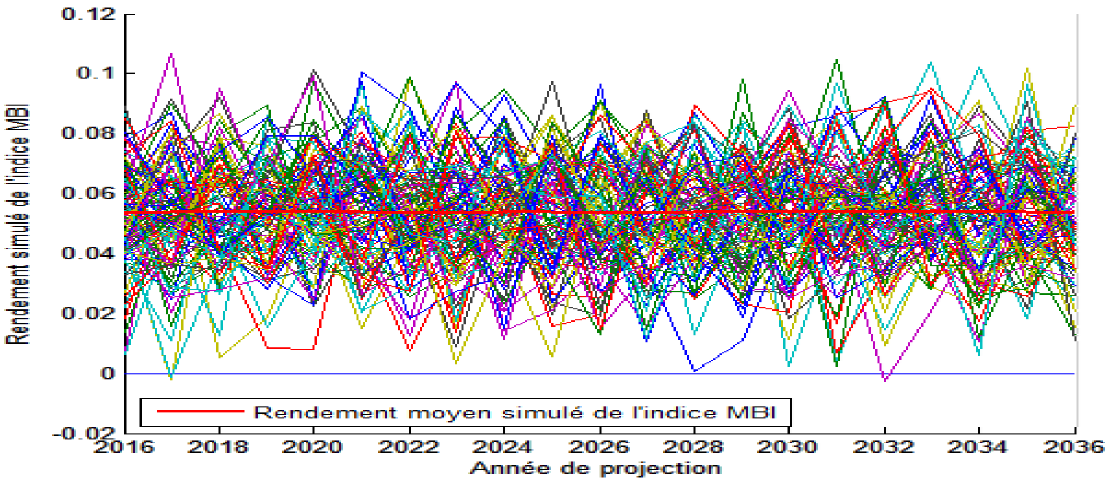
Figure 10. Simulated Variable Rate and / or Yield Variables of GSE.

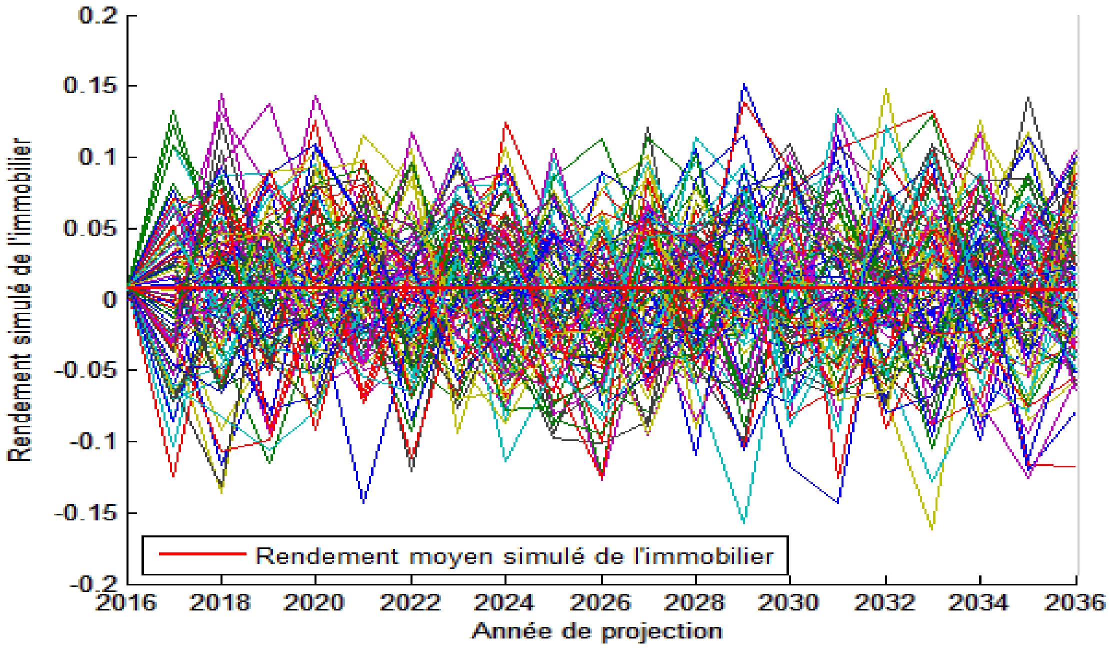



Rendement simulé de taux long terme nominal
Rendement moyen simulé de taux long terme nominal
Année de projection


Rendement simulé de taux court terme nominal
Rendement moyen simulé de taux court terme nominal
Année de projection


Rendement simulé de l'indice MASI
Rendement moyen simulé de l'indice MASI
Année de projection





Source: Own elaboration.

We observe that the average returns curves of the variables (inflation rate, real and nominal interest rate) start from their initial values to reach the level of the long-term averages with a speed equal to the rate of return to the average.

In the case of our ESG, we have selected models that reflect the reality of our market and the economic context of the ESG components, including the process of mean-reversion and the Black & Scholes model.

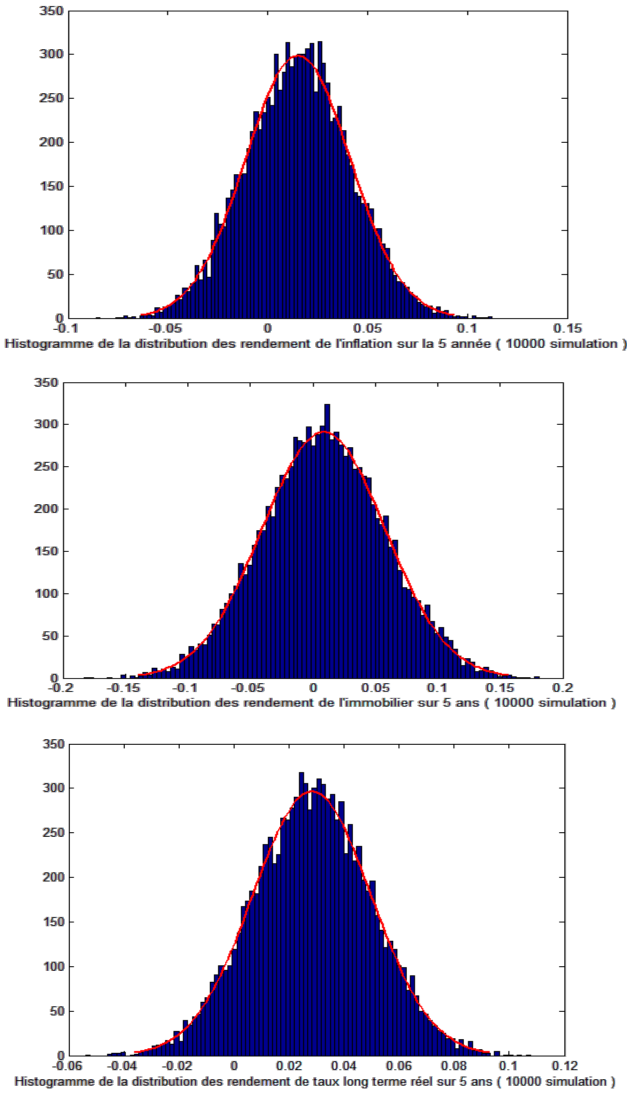
The projections of our ESG variables are economically interpretable, ie consistent with the Market data and based on generally accepted principles in the financial market place. For example, the model should be implemented in the absence of an arbitrage opportunity.This assumption is essential in the use of stochastic processes to model the behavior of financial assets such as the Hull and White model for interest rates and the Uhlenbeck model for inflation and real estate.

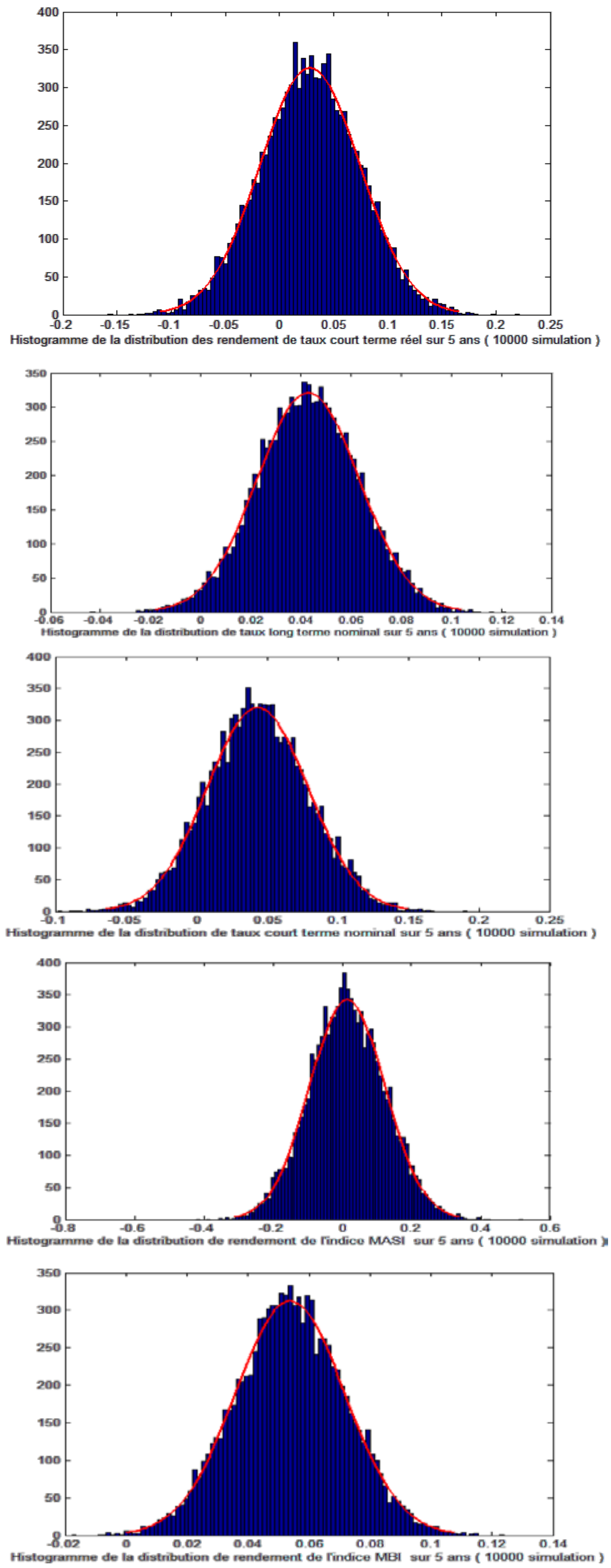
Simplicity is preferred in the choice of models: it is the principle of parsimony. In this case, we preferred a model similar to that of Ahlgrim et al. (2005) to that of Wilkie (1986). Indeed, the parameters of the model of Ahlgrim and al are economically interpretable, while those of Wilkie (1986) are difficult to interpret. Moreover, a complex model that perfectly imitates all the characteristics of the assets could give the illusion to the expert to be able to model the real world, which can lead to a loss of objectivity and critical analysis.

Our built ESG is transparent since it can be explained and justified in a simple way to a large public (such as managers and administrators, for example), for financial communication purposes but also within the current regulatory framework, development of future reforms. For example, the scenarios generated by the model are used to make long-term risk factor projections for Asset-Liability and LDI.The adaptability of ESG to the evolution of the market is also a desired characteristic. Our ESG can be used, with its recalibration, whatever the financial and macroeconomic environment.

The graphs below represent, in the form of histograms, the distributions obtained from different rate and / or yield variables.

Figure 11. Histograms of Variable Rate and / or Yield Variables of ESG.



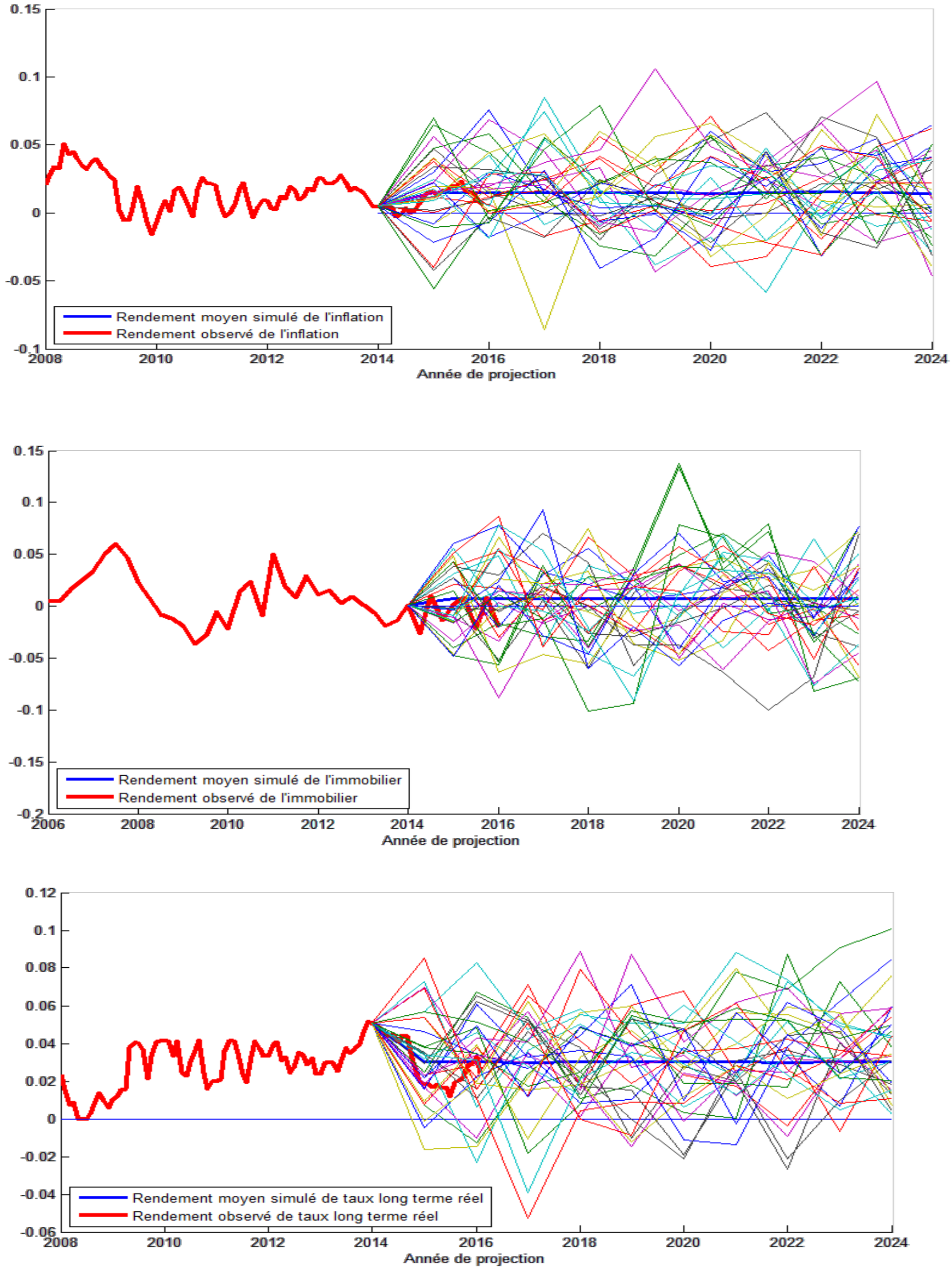


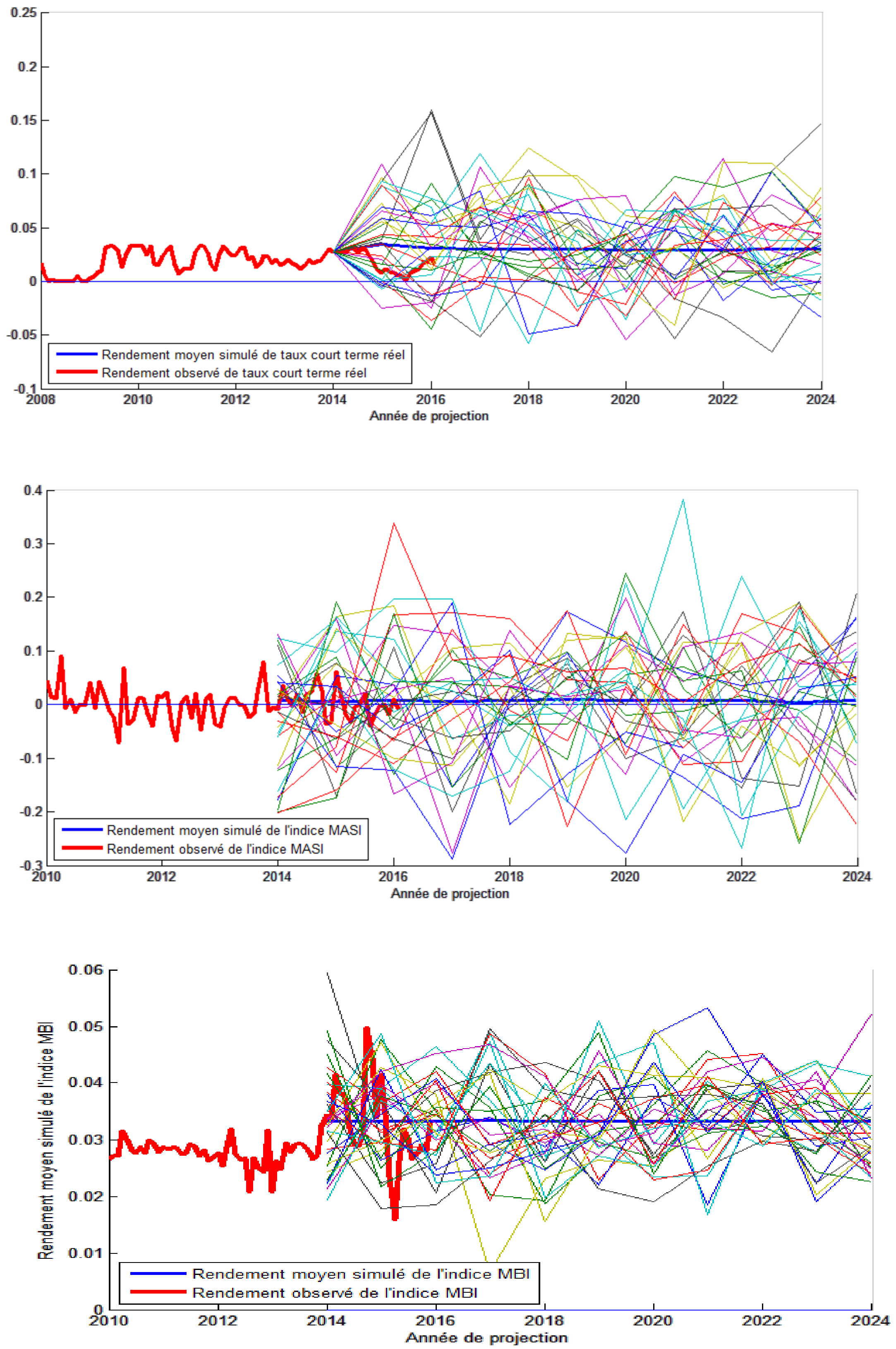
Source: Own elaboration.

We note that these histograms of simulated yields provide a good estimate of the probability density of the normal distribution.

The graphs below present the backtesting of the rates and / or returns of the components of our ESG by comparing the historical reality with the projections obtained.

Figure 12. Backtesting of the rates and / or returns of the components of our ESG.





Source: Own elaboration.

Based on the backtest results, we find that the model provides a fairly good estimate of the variation of the different components of ESG: inflation rate, short and long term real interest rates, MASI RB index, the MBI index and the IPAI index.This remark emphasizes the robustness of our built ESG model, which can only reflect the historical evolution of our variables.

Result 2

- **Optimal Central Allocation**

Based on the assumptions proposed, we tested 1000 asset allocations between the minimum and maximum limits that we set by article 14, respecting the constraint on the sum of the shares of the classes and setting a mesh step of 2.5%. As a result, we obtain 39 allocation options for our three asset classes (stock, bond and real estate).

These simulations made it possible to determine a cloud of points, each point corresponds to a well defined allocation. From these points, the strategic allocation is determined according to the decision criteria presented above.Thus, of the 39 eligible allocations obtained, we choose 10 allocations that will allow us to have a maximum investment reserve and a maximum funding ratio. It should be noted that in order to have a reliable and relevant analysis, we did not retain allocations whose weight of real estate or share is zero even if the latter offer maximum values.

Table 5 shows the top 10 allocations selected from the 39 allocations tested with the central scenario (without economic shock) under regulatory, operational and prudential constraints.

We note that in the 10 maximum allocations presented above, the weight of bonds occupies a prominent place followed by that of equities and real estate, which is consistent with the investment strategy adopted by the CMR.

Table 5. Asset allocation by category for tested allocations.

	Alloc_1	Alloc_2	Alloc_3	Alloc_4	Alloc_5	Alloc_6	Alloc_7	Alloc_8	Alloc_9	Alloc_10
Bond	0.875	0.925	0.925	0.9	0.875	0.85	0.825	0.85	**0.95**	0.9
Equities	0.1	0.05	0.025	0.075	0.075	0.1	0.15	0.125	**0.025**	0.05
Real Estate	0.025	0.025	0.05	0.025	0.05	0.05	0.025	0.025	**0.025**	0.05

Source: Own elaboration.

It should be noted that the maximum reserve and the funding ratio are calculated over a horizon equal to the allocation horizon (10 years). Portfolio performance is the average value calculated over the entire projection (20 years).

The funding ratio and the average return are collected for each of these portfolios; we therefore retain the portfolio with the best combination of performance, reserve and hedging rate as the strategic allocation of the pension fund.

Table 6. List of best allocations obtained by the central scenario.

	Alloc_1	Alloc_2	Alloc_3	Alloc_4	Alloc_5	Alloc_6	Alloc_7	Alloc_8	Alloc_9	Alloc_10
Reserve	7.08E+09	8.69E+09	8.45E+09	7.88E+09	6.85E+09	6.06E+09	5.51E+09	6.29E+09	**9.50E+09**	7.65E+09
Funding ratio	0.12	0.1254	0.1246	0.1227	0.1192	0.1166	0.1148	0.1174	**0.1281**	0.1219
Average Return	0.0569	0.059	0.0587	0.0579	0.0566	0.0555	0.0548	0.0558	**0.0601**	0.0576

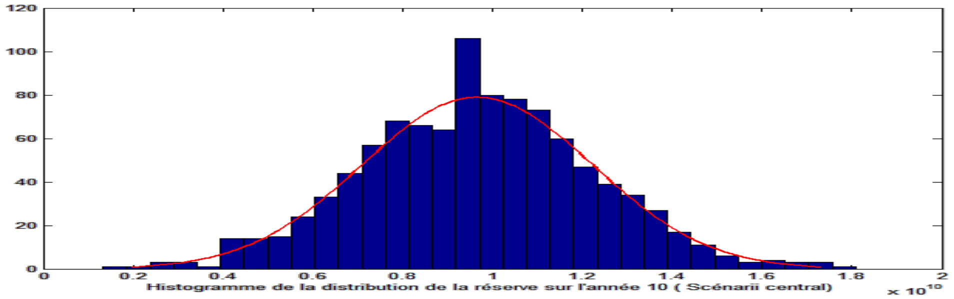
Source: Own elaboration.

We see in the chart above that the maximum funding ratio is about 12.81% for allocation 9 and 12.45% for allocation 2, indicating that in 10 years' time, the Plan Assets will not fully cover the discounted liabilities (approximately 13% of total liabilities).

The strategic portfolio with the best preservation of the investable financial reserve with a hedging of the liabilities by the assets of the reserves and a maximum return of portfolio is the portfolio corresponding to the alloc_9. Its optimal composition includes 0.025% of shares, 0.025% of real estate and 0.95% of bonds. These results are obtained assuming that the economic situation will remain similar in the coming years.

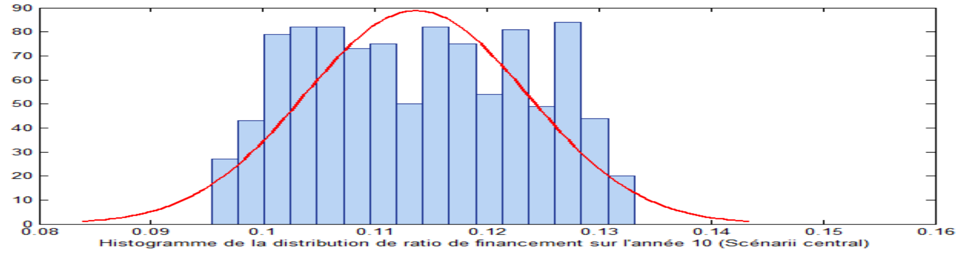
Knowledge of the distribution of the R_t and the funding ratio makes it possible to obtain a first view of the behavior of the balance sheet and a predetermined allocation at a given instant.

Figure 13. Histograms of the maximum reserve over the 10-year horizon in the 'central scenario'.



Source: Own elaboration.

Figure 14. Funding ratio histograms over the 10-year horizon and the central scenario.



Source: Own elaboration.

As a result, we can conclude that the optimal allocation sought has increased the level of the reserve and the funding ratio, which has made it possible to delay the date of depletion of reserve funds and thereby improving the solvency of the fund.

We can also look for an allocation of our reserve fund over the medium term (5 years) and compare it to that of long-term (10 years already determined). The results show that the composition of different portfolios is preserved, with the optimal allocation corresponding to alloc_9. Similarly, there is a significant increase in the funding ratio (from 12.81% to 39.37%), which will ensure a certain level of solvency of the fund. Thus, it is desirable for the civil regime to start optimizing its portfolio as soon as possible to benefit from the improvement of these indicators.

Table 7. List of the ten best allocations obtained by the central scenario over 5 years.

	Alloc_1	Alloc_2	Alloc_3	Alloc_4	Alloc_5	Alloc_6	Alloc_7	Alloc_8	Alloc_9	Alloc_10
Reserve	7.010E+10	7.096E+10	7.089E+10	7.05E+10	7.003E+10	6.961E+10	6.92E+10	6.967E+10	**7.139E+10**	7.046E+10
Funding ratio	0.3884	0.39197	0.39169	0.390217	0.38817	0.38643	0.384974	0.386715	**0.393745**	0.389932
Average Yield	0.0562609	0.058695	0.05850	0.057478	0.056061	0.054848	0.053825	0.055043	**0.059912**	0.057283

Source: Own elaboration.

- **Determination of stressed optimal allocation**

The tests on our allocations are carried out to ensure their robustness in case of stress scenarios: this is to ensure that these 10 best allocations check the constraints in the complementary scenarios selected.

In the context of these tests, it appears that the allocations presented as the best in the central scenario satisfy the constraints in the complementary scenarios. The conclusion of these tests confirms the robust nature of the ten allocations tested. The recommended allocation is then the same as that found in the central allocation, ie alloc_9, composed of 2.5% share, 95% bond and 2.5% real estate.

We note that downward movements in real estate and equity returns have very little impact on reserves and financing ratios. On the other hand, the 2% increase in inflation led to an increase in the value of the plan liabilities, which led to a decline in the funding ratio.

By observing the movement of the top 10 allocations from the central case (without economic stress) to the combined stress scenario, we can see a very small decrease in the reserve value of 0.0137% as well as a decline in the funding ratio which increased from 0.1281 to 0.1264, a percentage of 1.327088%.

Table 8. Stress Scenarios on tested allocation.

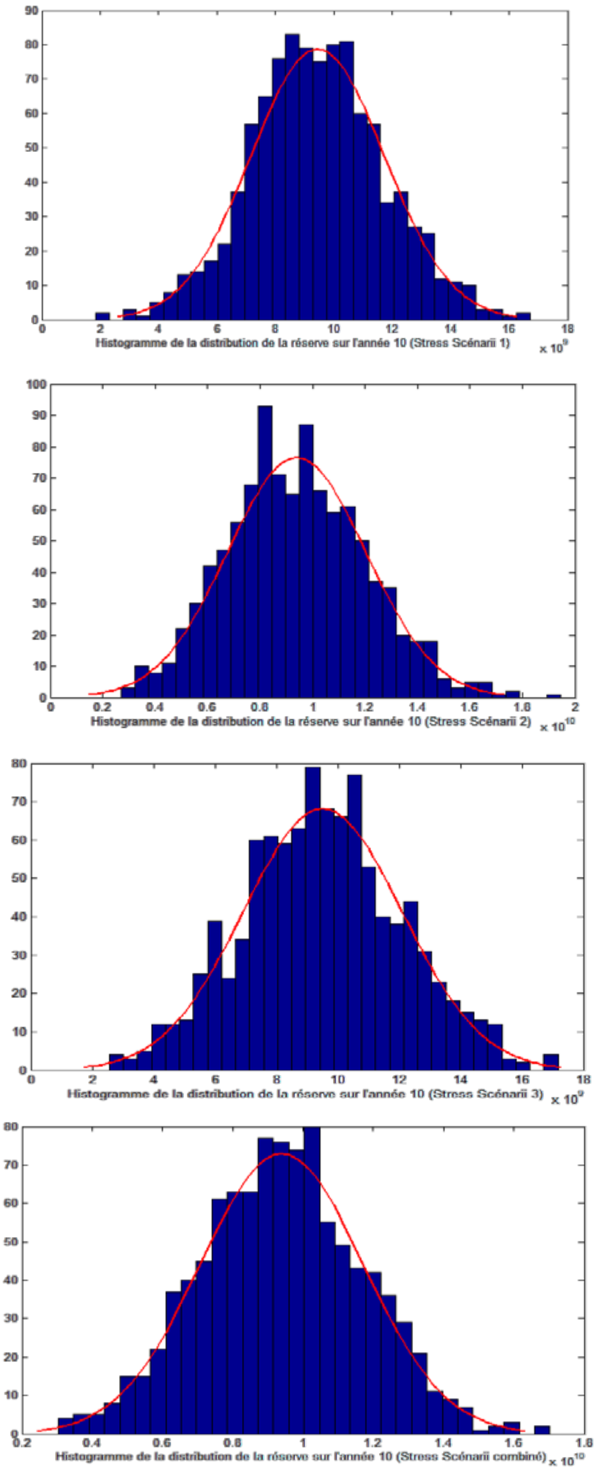
Stress Scenarios	**Stress Scenario_1**		**Stress Scenario _ 2**		**Stress Scenario _ 3**		**Stress Combined scenario**	
Proposed Allocations	Reserve	Tx _ coverage	Reserve	Tx _ coverage	Reserve	Tx _ coverage	Reserve	Tx _ coverage
Alloc_1	6.63E+09	0.1186	7.07E+09	0.12007	7.066E+0	0.1183	6.64E+09	0.1174
Alloc_2	8.45E+09	0.1247	8.66E+09	0.1253	8.67E+09	0.1236	8.45E+09	0.1234
Alloc_3	8.33E+09	0.1243	8.39E+09	0.1244	8.44E+09	0.1229	8.29E+09	0.1228
Alloc_4	7.53E+09	0.1216	7.86E+09	0.1227	7.87E+09	0.121	7.54E+09	0.1204
Alloc_5	6.51E+09	0.1182	6.81E+09	0.1191	6.84E+09	0.1176	6.48E+09	0.1169
Alloc_6	5.62E+09	0.1152	6.03E+09	0.1165	6.05E+09	0.115	5.60E+09	0.1139
Alloc_7	4.85E+09	0.1126	5.51E+09	0.1148	5.49E+09	0.1132	4.87E+09	0.1115
Alloc_8	5.73E+09	0.1156	6.29E+09	0.1174	6.28E+09	0.1157	5.75E+09	0.1144
Alloc_9	**9.38E+09**	**0.1278**	**9.47E+09**	**0.128**	**9.49E+09**	**0.1263**	**9.37E+09**	**0.1264**
Alloc_10	7.42E+0	0.1212	7.60E+0	0.1218	7.63E+0	0.1202	7.38E+0	0.1198

Source: Own elaboration.

In comparison with the new scatter plot, the 10 best allocations of the central case always seem to be very suitable. In terms of strategy, the conclusions that can be made in the central case remain valid in the stressed scenarios.

To get an idea about the behavior of the civil balance sheet as well as that of the predetermined allocation at a given moment. We present the empirical distribution of the financial reserve R_t obtained, corresponding to the optimal strategic allocation over 10 years for the four stress scenarios defined previously.

Figure 15. Reserve histograms over the 10-year horizon according to the four Scenarios.



Source: Own elaboration.

We note that over the projection horizon 10 years, more, the portfolio structure is risky, the thicker the tails, which increases the risk of ruin. Thus, risk takes place at the price of an increase in the probability of ruin.

- **Analysis of the results of the ex-post probability of the ruin**

As for the central scenario, we can see that the VaR increases with the level of risk taken on the projection horizon considered.

Table 9. Value-at-Risk (in billions of DH) for the Central Scenarios.

Risk level	Alloc_1	Alloc_2	Alloc_3	Alloc_4	Alloc_5	Alloc_6	Alloc_7	Alloc_8	Alloc_9	Alloc_10
90%	1.35E+10	6.797E+09	4.445E+09	9.902E+09	9.52E+09	1.197E+10	1.881E+10	1.411E+10	**4.62E+09**	6.707E+09
95%	1.66E+10	8.663E+09	5.418E+09	1.227E+10	1.207E+10	1.573E+10	2.34E+10	1.884E+10	**5.87E+09**	8.759E+09
99.50%	2.58E+10	1.534E+10	8.992E+09	2.174E+10	2.045E+10	2.677E+10	3.908E+10	2.806E+10	**9.21E+09**	1.292E+10

Source: Own elaboration.

Depending on its degree of risk aversion, the civil regime may choose between the two strategic allocations Alloc_3 having a low VaR value at risk compared to the other portfolios and Alloc_9 presenting maximum wealth and financing ratio values.

Table 10. Conditional Value-at-Risk (in millions of DH) for the central scenario.

Risk level	Alloc_1	Alloc_2	Alloc_3	Alloc_4	Alloc_5	Alloc_6	Alloc_7	Alloc_8	Alloc_9	Alloc_10
90%	1986232004	1.041E+09	**651504825**	1.497E+09	1.453E+09	1.903E+09	2.774E+09	2.168E+09	**697561298.6**	1025526923
95%	1087129535	584895058	**357471301**	840604492	810784902	1.073E+09	1.531E+09	1.174E+09	**383498377.9**	568236764.9
99.50%	116938665.1	66414227	**42741726**	101370701	94745264	137744929	169284367	110924309	**43310531.03**	67487800.48

Source: Own elaboration.

This comparison shows that the VaR approach underestimates the probability of occurrence of extreme events in relation to the CVaR approach for the given horizon.

Result3

- **Assets-Only optimization**

The projection horizon will be fixed at 10 years and in order to make the calculations more fluid, we place ourselves at the end of the year closest to our study, ie 31/12/2016.

Table 11. Weight of Optimal 'Assets-Only' Portfolio without constraints.

Return	0.0275	0.03	0.0325	0.035	0.0375	0.04	0.0425	0.045	0.0457	0.05	0.0525	0.055
Risk	5.982	6.9091	7.8422	8.7791	9.7188	10.661	11.604	12.549	13.4943	14.441	15.388	16.336
Proportion												
IPAI	0.606	0.5624	0.5191	0.4758	0.4321	0.3851	0.338	0.291	0.2439	0.1969	0.1498	0.1028
MASI	0.019	0.0143	0.0094	0.0045	0	0	0	0	0	0	0	0
MBI	0.375	0.4233	0.4715	0.5197	0.5679	0.6149	0.662	0.709	0.7561	0.8031	0.8502	0.8972

Source: Own elaboration.

For an expected level of profitability of 4.75% and a minimum risk of around 13.4943%, the optimal portfolio consists of 24.39% of the real estate and 75.61% of the bond. We note that the resulting portfolio does not contain the equity class.

Table 12. Weight of Optimal 'Assets-Only Portfolio' with Regulatory Constraints.

Return	0.045	0.0475	0.05	0.0525	0.055	0.0575	0.06
Risk	13.17%	13.736%	14.5799%	15.4569%	16.3616%	17.2896%	18.2371%
Proportion							
IPAI	0.05	0.05	0.05	0.05	0.05	0.05	0.0084
MASI	0.3	0.2565	0.1942	0.1319	0.0696	0.0073	0
MBI	0.65	0.6935	0.7558	0.8181	0.8804	0.9427	0.9916

Source: Own elaboration.

The optimal structure corresponding to a return level of 4.75% for a minimum risk of 0.13736 is as follows: 5% of the real estate, 25.65% of the equity and 69.35% of the bond. We note that the weight restriction favored equities (which increased from 0% to 25.65%) and increased the portfolio risk by 1.79%. This increase in the risk value is mainly due to the increase in the share of the equity class (risky assets) as well as the decline in the fraction of the bond and real estate class, which are less risky than that of the share.

The return that will allow us to have the wealth necessary to cover our commitments at the end of the tenth year is 4.8582%. This critical return, which guarantees that the pension fund will be fully hedged at the end of 10 years, remains above the calculated returns (4.5% and 4.75%) by the mean-variance model. Intermediate funding ratios are not taken into account at all. In fact, if our portfolio had a stable return of 4.8582% over the 10 years, the pension fund will remain underfunded for the next few years as our benefits continue to increase in future years.In the end, deciding on a critical return reduces the choice of eligible portfolios, as all lower yield portfolios will no longer be considered.

- **Surplus optimization: LDI approach**

Using the mean-variance approach, Sharpe and Tint (1990) considered surplus optimization for portfolios of assets and liabilities.

Table 13. Weight of portfolio asset classes 'optimization-surplus' without constraints.

Return	0.03	0.0325	0.035	0.0375	0.04	0.0425	0.045	0.0457	0.05	0.0525	0.055
Risk	4.8848	5.5437	6.2054	6.8689	7.5341	8.2004	8.8677	9.5358	10.204	10.874	11.543
Proportion											
IPAI	0.566	0.5229	0.4798	0.4328	0.3857	0.3387	0.2917	0.2447	0.1977	0.1507	0.1037
MASI	0.0102	0.0051	0	0	0	0	0	0	0	0	0
MBI	0.4238	0.472	0.5202	0.5672	0.6143	0.6613	0.7083	0.7553	0.8023	0.8493	0.8963

Source: Own elaboration.

We note from the table above that the optimal portfolios obtained consist of a very large part of bonds 'assets less risky' and correlated with the liability followed by the real estate whose proportion decreases with the level of return as well as a zero proportion of the equity. Similarly, we find that risk increases even in the absence of risky assets. Naturally, this increase is accounted for by the liability risk, which we consider to be zero in the asset model only.

Table 14. Weight of portfolio asset classes 'optimization-surplus' with regulatory constraints.

Return	0.045	0.0475	0.05	0.0525	0.055	0.0575	0.06
Risk	9.3161	9.7123	10.3049	10.9206	11.5556	12.2069	12.8754
Proportion							
IPAI	0.05	0.05	0.05	0.05	0.05	0.05	0.0102
MASI	0.3	0.257	0.1951	0.1332	0.0714	0.0095	0
MBI	0.65	0.693	0.7549	0.8168	0.8786	0.9405	0.9898

Source: Own elaboration.

In the results summarized in the table above, we see that, the introduction of regulatory constraints has favored the bond class, which is supposed to mimic the liabilities. We have also seen a significant increase in the equity share relative to the portfolio without constraints. Nevertheless, this asset class experienced declines with the tolerated yield levels until it reached zero for a return of 6% and a risk of 12.875%.

We have also noticed that the proportion of the real estate class has become constant (5%) in the strategic allocations obtained (for yield levels ranging from 4.5% to 5.75%). Similarly, we note a significant decline in this real estate class's share of the 6% return level in the constrained surplus optimization model. We also find that the constraints that impose minimum and maximum values on asset classes have a significant impact on the risk of surplus. Comparing the 'Asset-only' and the 'surplus optimization' model results, we can see that the risk is lower for surplus optimization results. Naturally, this drop is due to the nature of its optimal portfolios which are made up of less risky assets (bonds) and correlated with the liabilities of the fund.

In sum, we found that the surplus optimization model was more appropriate to effectively minimize the likelihood of a mismatch between assets and liabilities and, therefore, efforts to find ways to improve its practicality would be better for pension funds.

- **Fund Separation Model (LHP / PSP)**

We were unable to find a portfolio that verified the condition of surplus variance nullity in the case of our plan. Hence, the portfolio of variance of the global minimum surplus is the LHP does not exist for our LDI strategy, which is obvious since the civil pension system has been in deficit since 2014. As a result, we will not have wealth to invest in a PSP performance portfolio.

Thus, by applying the model of Keel and Müller (1995), we obtain the decomposition relating to the real estate, the equity and the bond as follows. For a yield level of 0.05, we find: $\mathrm{x}^{\min} = (0.6524, -0.335, 0.0867)$. That is to say to cover the liabilities of civil regime, it is necessary to build a LHP portfolio composed of 65.24% of the real estate of 8.67% in bond and a short sale of the shares of 33.5%, which is not consistent with the regulation of FDR regime management.

- **Stress test of LDI fund**

In order to test the resilience of our LDI fund design, we tested a stress scenario (down 30% from the MASI RB index and 20% from the real estate index).

These stress scenarios have kept the same character of our surplus optimization model with regulatory constraints and without economic shock. Thus, we note that with the increase in yield, the portfolio is moving more and more towards risk-free assets (bonds). On the other hand, the share of risky assets has decreased significantly and the proportion of real estate has remained almost constant.

Table 15. Scenario Stress on 'Optimization Allocations - Surplus' Tested.

Return	0.0425	0.045	0.0475	0.05	0.0525	0.055	0.0575	0.06
Risk	9.3161%	9.5346%	10.0415%	10.5672%	11.109%	11.6648%	12.2326%	12.8738%
Proportion								
IPAI	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.0103
MASI	0.3	0.2761	0.2223	0.1685	0.1147	0.0609	0.0071	0
MBI	0.65	0.6739	0.7277	0.7815	0.8353	0.8891	0.9429	0.9897

Source: Own elaboration.

5. Conclusions.

In this paper, we have attempted to develop a stochastic model for the generation of economic scenarios within the civil regime of the Moroccan pension fund and whose final purpose is that of risk management and the long-term projection of financial and macroeconomic indices. In this framework, we have adopted a model similar to that of Ahlgrim et al. (2005).

In this context, we generated economic scenarios to simulate the R_t reserve and the solvency ratio for a given allocation. To understand the behavior of these random variables, four stress scenarios are defined, and then the solvency ratio and the reserve are projected in each of these scenarios. Finally, their empirical distribution is plotted for the optimal strategic allocation obtained. It is then observed that the strategic allocation obtained makes it possible to improve the level of the reserve and the funding ratio without solving the problem of plan insolvency. Similarly, we observed that the more the allocation profile is risky and the thicker the distribution tail of the R_t.

We also calculated the Value-at-Risk to measure the insurer's exposure to the risk of ruin, depending on the allocation under consideration. Similarly, in order to determine the extreme failure, we applied the CVaR to our strategic allocation.

In this paper, we sought an asset allocation by using the model LDI that offers a better return and allows us to better protect our fund against "unfavorable" changes in its liabilities.

Several additional works could complement and improve the study initiated and relative to the construction of our ESG. In this sense, the addition of other distribution models for high-performance real estate and the bond index is important and the integration of a Merton jumping process to project the price of equities is essential in the context of rising risks due to the significant increase in the risk of major crises at the beginning of the 21st century. Similarly, conditional volatility can be taken into account by choosing the most reliable and relevant model between the GARCH model and the estimated historical volatility. Naturally, the use of a Poisson process, also called the rare event process, seems to be adapted to crisis modeling, a rare event by hypothesis.

The modeled dependence between the different risks is Gaussian, whereas the financial assets exhibit a stronger correlation in a situation of extreme stress (notion of tail-correlation). It is therefore possible to consider the use of copula to take this effect into account.

Some risks, such as the risk of longevity and stochastic mortality, are not modeled, but it is possible to extend the models used to incorporate these risk factors.

As for the other financial variables that can be defined, the 2007 crisis highlighted financial risks sometimes underestimated by the insurers: liquidity risk and credit risk. The integration of a liquidity premium as a ESG variable therefore seems relevant.

We recall that there are other candidate models in the literature of Ahlgrim et al. (2008) to generate reliable and realistic scenarios for our pension fund, in particular the model based on a schematic tree-based design, Kouwenberg (2001), which is more suited to a series of dynamic models of asset-liability management based on techniques of stochastic programming.

Introduce economic uncertainties in the development of a GSE such as a sharp decline in inflation and growth or an oil shock or a food shock involving inflation. We will replace in this case a single ESG by several weighted ESGs. The aim of this approach is to reduce the number of completely impossible scenarios by considering on the one hand that specialized ESG s produce scenarios more realistic than general ESG s and on the other hand that a single ESG with equally weighted scenarios leads to a too strong influence of the past. The ESGs created will be weighted using advanced indicators.

We will also mention the use of economic indicators whose evolution reflects a little in advance that of the real economy. In times of crisis, the leading indicators make it possible to anticipate economic downturns and to modify, for example, the composition of our portfolio. However, the number of months ahead depends on the chosen indicator and it is necessary to test its reliability in times of crisis.

Our context and the conclusions of our work lead us to propose a certain number of rules to which the technical follow-up of a pension system must comply. In the first place, it would seem delicate to pretend to set a priori realistic and perennial assumptions for the evaluation of the commitments of the regime, and it is therefore necessary to set up technical control; steering refers to the regular updating of assumptions, in particular mortality, in order to integrate its observed changes and to adjust the drifts modeled as a function of these observations.

The strategic allocation gives good long-term results in terms of asset allocation. Nevertheless, it is important to keep a place in the tactical allocation, i.e. the possibility of modifying this allocation in the short term depending on market conditions. Indeed, in the short term, the anticipated returns are different from what they are in the long term. Tactical allocation is important in market situations with uncertain mid-term trends and large market amplitudes over a few months' horizons. It requires either a delegation to a management company or a sufficiently reactive decision-making structure within the pension fund.

The longer the investor's horizon, the more the civil pension scheme can simulate the integration of other asset classes, in order to reinforce the allocation model by seeking "absolute return". We can cite in this framework many assets including alternative assets, derivatives, futures...

In addition, it is essential to define the "best" strategy for investing these reserves in the financial markets, particularly in a context of strong economic uncertainties, such as the recent financial crises.

The LDI model is capable, unlike traditional ALM techniques, of dealing with long-duration liabilities with such ease, because it implements a very fine analysis of the financial risks of liabilities, and employs a very diversified range of financial products, namely bonds of different maturities as well as real estate which is a low-risk asset with significant returns, but this expertise remains limited in the case of Morocco since it does not have derivatives, swaps and inflation-linked bonds that are used to better manage interest rate and inflation risks.

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