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Broadcasting La Liga

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Broadcasting La Liga*

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Abstract

We study the allocation of revenues raised from the collective sale of broadcasting rights for the Spanish Football League (La Liga), which is strongly regulated by the Spanish government since 2015. The allocation process is decomposed in four dimensions: lower bounds, sport performance, economic performance, and broadcasting performance. For each dimension, we compare the allocation suggested by La Liga with other alternatives grounded on the fair allocation literature. Based on our analysis, we argue that the allocation process implemented by La Liga could be modified in meaningful ways, while still obeying regulation measures.

Keywords: *La Liga, broadcasting rights, sport leagues, resource allocation.*

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1 Introduction

For sports clubs, the sale of broadcasting and media rights is currently their biggest source of revenue. There were huge (and somewhat controversial) efforts to resume competitions worldwide in the aftermath of the COVID-19 pandemic's first wave, in order to secure broadcasting contracts (in spite of having empty stadiums). This was, for instance, the case of the Spanish Football League (La Liga), a multi-million euro business with an increasing trend in the last decades (at least, until the pandemic hit), which will be the focus of this paper.¹

According to official barometers in 2014, 48% of Spanish citizens were interested in football and (funnily enough) 67.4% declared themselves followers of a certain professional club. Three quarters of those acknowledged to watch their club's games on TV, whenever they could. Back then, FC Barcelona and Real Madrid CF, the two Spanish giant football clubs, earned each more than 20% of the revenues generated from broadcasting La Liga. This was in stark contrast with North American sport leagues, where contracts essentially involve equal sharing (e.g., Fort and Quirk, 1995). But also with other major European football leagues. For instance, in England, back then, the two clubs earning more only made together 13% of the revenues generated by the Premier League.² This aspect outraged the remaining Spanish football clubs, to the extent that it became a political issue, which prompted the Spanish government to regulate the business of broadcasting games on TV.³

Regulation was, by no means, minor. A detailed (18-page long) Royal Decree appeared in the Official Bulletin of the Spanish State on May 1st, 2015, stating the urgent measures to be implemented. The main aspect was to impose a collective sale of broadcasting rights. Thus, an ensuing key problem arose in which the revenues collected from the sale (which we call the endowment and denote E) had to be shared among the clubs. To solve this problem, strict guidelines were also enforced by the Spanish government. As explicitly stated in the

¹The reader is referred to García and Rodríguez (2002, 2003, 2006, 2013) or Ascari and Gagnepain (2006), among others, for alternative analyses of different aspects of Spanish football.

²Some might argue this was the main reason why in the last 18 editions of the Spanish Football League only twice the champion was neither FC Barcelona nor Real Madrid CF (it was both times Atlético de Madrid), whereas the Premier League witnessed 5 different champions in its last 10 editions (actually, 4 different ones in the 4 consecutive seasons from 2013 to 2017).

³In the era of streaming, sports still seem to be mostly consumed via television programming (e.g., Lee, 2019). This might, nevertheless, change soon. For instance, Netflix announced in the Summer of 2020 that games from Ligue 1 would soon be available in their platform.

corresponding Royal Decree, the aim was to “limit differences among participating entities” by means of an “equitative distribution” according to sport outcomes, ticket sales and the capability to generate resources from selling broadcasting rights.⁴

More precisely, the Royal Decree decomposes the endowment E in four parts, each reflecting a different dimension. The amount received by each club is the sum of the amounts received in each dimension. The four dimensions are described next:

1. **Lower bounds.** Half of the total endowment is devoted to this (first) dimension. It is divided equally among all clubs, hence guaranteeing a specific *lower bound* to each: $\frac{E}{40}$.
2. **Sport performance.** One quarter of the total endowment is devoted to this (second) dimension. It is divided among clubs taking into account the sport performance during last 5 seasons. More precisely, the champion gets a score of 17%. The second one gets 15%. It continues down the line with 13%, 11%, 9%, 7%, 5%, 3.5%, 3%, 2.75%, 2.5%, 2.25%, 2%, 1.75%, 1.5%, 1.25%, 1%, 0.75%, 0.5% and 0.25%. The allocation (of $\frac{E}{4}$) is proportional to the resulting weighted average of those 5-season scores, with a weight of 35% for the last season, 20% for the previous to last season and 15% for each of the other three.⁵
3. **Economic performance.** One twelfth of the total endowment is devoted to this (third) dimension. It is divided among clubs proportionally to ticket sales in the last five seasons.
4. **Broadcasting performance.** One sixth of the total endowment is devoted to this (last) dimension. The Royal Decree does not specify the way in which this amount should be divided among clubs. La Liga decides the amount received by each club, but it does not specify how such amounts are computed.

In this paper, we first analyze each of the above dimensions separately and then the problem as a whole. For each dimension, we propose alternative ways of dividing the amount assigned among clubs.⁶ For the first, third and fourth dimensions, we use models of economics and

⁴Carreras and García (2018) argue that the new system has reduced relative differences among clubs, but not the absolute ones.

⁵Those that played in the second division, or below, in one of those seasons, thus get zero in the corresponding part of the weighted average.

⁶Unfortunately, La Liga is not providing the figures of the decomposition of the amount received by each club in terms of the four dimensions.

management. For the second dimension, we use scores inspired in other competitions. More precisely, we proceed as follows:

1. **Lower bounds.** As mentioned above, dividing equally half of the endowment is a specific way of guaranteeing *lower bounds*. They are formalized and widely studied in the sizable literature on claims problems, a canonical model of distributive justice with a long tradition in economic research (e.g., O'Neill, 1982; Thomson, 2019). Thus, we propose to use other lower bounds from that literature. One of them happens to coincide (for the data analyzed) with the one used by La Liga. As we shall show, the other lower bound assigns different amounts to the clubs.
2. **Sport performance.** We consider two alternatives for this dimension. In the first one, in the same vein as the Premier League, we consider that, in each season, the score of the champion is 20, the score of the second is 19, and so on until the last one gets 1. The overall score is obtained after a weighted aggregation of the last 5 seasons, with weights being as in La Liga. In the second alternative, each club would get, each season, a score equal to the points obtained. Again, the overall score is computed as in the case of La Liga (by means of a weighted average).
3. **Economic performance.** As in the first dimension, we associate to this one a claims problem. More precisely, we interpret the amount of ticket sales as the claim of each club in each dimension and consider the four classical rules for claims problems to allocate the corresponding budget therein. One of the four rules (the proportional rule) gives rise to the allocation implemented by La Liga. The other three alternatives, given by the remaining classical rules (constrained equal awards, constrained equal losses, and Talmud) provide different allocations.
4. **Broadcasting performance.** For the fourth dimension, we consider the formal model we introduced at Bergantiños and Moreno-Tertero (2020a) to divide an endowment based on the data on audiences.⁷ Two polar and somewhat focal rules are salient in such a model: the equal-split rule (which allocates the revenues from each game equally among two clubs playing, and aggregates across games) and concede-and-divide (which concedes each club the audience from its fan base and divides equally the residual). We use both rules, as

⁷See also Bergantiños and Moreno-Tertero (2020b, 2021, 2022a,b,c; 2023).

well as compromises among them, to suggest possible allocations for this dimension. As we shall show, these allocations differ from the one used by La Liga for this dimension.

Clubs are heterogenous with respect to the above dimensions (standings, ticket sales, or audiences). Powerhouses (such as Barcelona and Real Madrid) have high values in each input, whereas others have all of them low. We shall informally refer to them as big and small clubs, respectively. Small clubs would prefer allocations with low variance, whereas big clubs would prefer allocations with high variance. It turns out that, in the first dimension, the allocation proposed by La Liga is the one with the lowest variance, whereas, in the second dimension, it is the one with highest variance. In the other two dimensions, it takes intermediate values.

Rather than focussing on each dimension in isolation, we consider full-fledged allocations obtained by combining some of the suggestions mentioned above for each of the dimensions. We then compare them with the allocation proposed by La Liga. As the full combination of each of the suggestions mentioned above gives rise to a large number of possible hybrid allocations of the whole endowment, we focus on some basic ones:

- The one induced by enforcing in each dimension the lowest variance among clubs.
- The one induced by enforcing in each dimension the highest variance among clubs.
- The one induced by taking in each dimension the average of the proposed allocations.

As expected, small clubs are favored in the first, while big clubs are favored in the second and none of them prefers the third. Nevertheless, when we compare the allocation from La Liga with the first two (extreme) allocations, we discover that two clubs obtain more with it.

When we compare the allocation from La Liga with the third listed above, two clubs are clearly favored by the former (they receive at least 14 millions more with La Liga). One club receives around 31 millions less. Two other clubs receive around 9 millions less.

The previous analysis is assuming that the amount devoted to each dimension is the one suggested by La Liga. But other plausible ways can be endorsed too. For instance, considering only the broadcasting performance dimension (and, thus, dismissing the others). A rationale for this is that if revenues are raised from selling broadcasting rights, then broadcasting performance is the dimension that really matters. Alternatively, we could assign half of the endowment to the broadcasting performance, and divide the other half equally among the remaining three

dimensions. When we compare the allocation of La Liga with those new schemes, big clubs are favored with them, whereas the opposite occurs for small clubs.

This is a new stage in our research agenda on sharing the revenues raised from the collective sale of broadcasting sports (e.g., Bergantiños and Moreno-Ternero, 2020a,b; 2021; 2022a,b,c; 2023). As such, it connects to a literature dealing with broadcasting sports (e.g., Cave and Crandall, 2001; Szymanski and Késenne, 2004; García and Rodríguez, 2006; Buraimo, 2008; Buraimo and Simmons, 2009; Peeters, 2011, 2012; Hansen and Tvede, 2016). It also touches the sizable literatures on fair allocation (e.g., Thomson, 2017, 2019) and operations research in the sports industry (e.g., Wright, 2009; Csató, 2021).

What is the value added by this paper with respect to our previous work on this area? First, this paper is empirical, whereas our earlier papers are strongly theoretical, focussing on studying several rules mathematically. Nevertheless, in two of our earlier papers we also have a partial empirical analysis of La Liga. More precisely, in Bergantiños and Moreno-Ternero (2020a), we consider the season 2016-2017 (the previous to the one considered in this paper). We then explore two specific issues for it: the allocation of the whole endowment according to the broadcasting performance, and a hybrid allocation in which the first three dimensions are shared as in La Liga whereas in the fourth one is carried with the two polar rules mentioned above (the equal-split rule and concede-and-divide). Thus, we only consider 2 of the hybrid schemes (among the 72 existing ones) arising from the combinations of all the possible alternatives outlined above. In this paper, we consider data from the season 2017-2018 and analyze other hybrid options. In Bergantiños and Moreno-Ternero (2021), we consider data from the season 2017-2018 (as in this paper) and also analyze two specific issues. The first one is also the allocation of the whole endowment according to the broadcasting performance. The second one is to check whether the allocation implemented by La Liga could be rationalized by the family of compromise rules introduced therein. In this paper, we explore other hybrid schemes arising from the options outlined above to address the four dimensions of this problem.

The rest of the paper is organized as follows. The technical aspects of our analysis are introduced in Section 2. In Section 3, we analyze La Liga's allocation. Section 4 is devoted to the analysis of the lower bounds, Section 5 to sport performance, Section 6 to economic performance and Section 7 to broadcasting performance. In Section 8, we study the allocations obtained from several combinations of the four dimensions. Finally, we conclude in Section 9.

2 Preliminaries

In this section, we briefly introduce some mathematical models, from the literature on economics and management, which will be used in this paper.

Let N describe a finite set of clubs playing in a certain league with a double round-robin format. Its cardinality is denoted by n . We assume $n \geq 3$. Let E denote the endowment (coming from the collective sale of broadcasting rights) to be allocated among clubs in N . We assume that E is divided in four parts $\{E_1, E_2, E_3, E_4\}$ such that $E = \sum_{k=1}^4 E_k$. Besides, E_1 is shared in order to guarantee to each club a lower bound. E_2 is shared according to sport performance. E_3 is shared according to economic performance. E_4 is shared according to broadcasting audiences. For each $k = 1, 2, 3, 4$, we denote by $R^k(E_k) \in \mathbb{R}^N$ the way in which E_k is shared among clubs. Then,

$$R(E) = \sum_{k=1}^4 R^k(E_k) \in \mathbb{R}^N$$

is the way in which E is shared among clubs.

A *claims problem* considers a situation in which an insufficient amount of a perfectly divisible good (the endowment) has to be allocated among some agents who hold claims against it. This simple and useful model to study distributive justice, introduced by O'Neill (1982), is enough to accommodate many situations that trace back to ancient sources such as Aristotle's essays and the Talmud. Thomson (2019) surveys the literature dealing with this model.⁸

Formally, a *claims problem* is a triple (N, c, E) consisting of a population N , a claims profile $c \in \mathbb{R}_+^N$, and an *endowment* $E \in \mathbb{R}_+$ such that $\sum_{i \in N} c_i \geq E$.

A *claims rule* is a mapping R that associates with each problem (N, c, E) an allocation $R(N, c, E) \in \mathbb{R}^N$ such that $\sum_{i \in N} R_i(N, c, E) = E$ and for each $i \in N$, $R_i(N, c, E)$ specifies the amount received by agent i , which should satisfy the condition $0 \leq R_i(N, c, E) \leq c_i$.

The four most popular claims rules are the following. The proportional rule (P) distributes the endowment proportionally to the claims. The constrained equal awards rule (CEA) distributes the endowment equally among all agents, subject to no agent receiving more than she claims. The constrained equal losses rule (CEL) imposes that losses are as equal as possible

⁸Krumer and Moreno-Ternero (2022) have recently applied this model to the allocation of additional slots for the FIFA World Cup.

subject to no one receiving a negative amount. Finally, the Talmud rule (T) behaves like CEA or CEL , depending on whether the endowment falls short or exceeds one half of the aggregate claim, using half-claims instead of claims.

Formally, for each (N, c, E) and each $i \in N$,

Proportional rule:

$$P_i(N, c, E) = \frac{c_i}{\sum_{j \in N} c_j} E.$$

Constrained equal awards rule:

$$CEA_i(N, c, E) = \min\{c_i, \lambda\},$$

where $\lambda \geq 0$ is chosen so that $\sum_{j \in N} \min\{c_j, \lambda\} = E$.

Constrained equal losses rule:

$$CEL_i(N, c, E) = \max\{0, c_i - \lambda\},$$

where $\lambda \geq 0$ is chosen so that $\sum_{j \in N} \max\{0, c_j - \lambda\} = E$.

Talmud rule:

$$T_i(N, c, E) = \begin{cases} CEA_i(N, \frac{c}{2}, E) & \text{if } E \leq \frac{1}{2} \sum_{j \in N} c_j, \\ \frac{c_i}{2} + CEL_i\left(N, \frac{c}{2}, E - \frac{1}{2} \sum_{j \in N} c_j\right) & \text{if } E \geq \frac{1}{2} \sum_{j \in N} c_j. \end{cases}$$

We now consider the model introduced by Bergantiños and Moreno-Ternero (2020a) and explored later in Bergantiños and Moreno-Ternero (2020b, 2021, 2022a,b,c; 2023) to study broadcasting problems. For each pair of clubs $i, j \in N$, we denote by a_{ij} the broadcasting audience (number of viewers) for the game played by i and j at i 's stadium. By convention $a_{ii} = 0$, for each $i \in N$. Let A denote the resulting matrix of broadcasting audiences generated in the whole tournament involving the clubs within N . A *broadcasting problem* is a pair (N, A) where N is the set of clubs and A is the audience matrix.

Let $\alpha_i(N, A)$ denote the total audience achieved by club i , i.e.,

$$\alpha_i(N, A) = \sum_{j \in N} (a_{ij} + a_{ji}).$$

Without loss of generality, we normalize the revenue generated from each viewer to 1 (to be interpreted as the “pay per view” fee). Thus, we sometimes refer to $\alpha_i(N, A)$ as the *claim* of club i . When no confusion arises, we write α_i instead of $\alpha_i(N, A)$.

For each matrix A , let $\|A\|$ denote the total audience of the tournament. Namely,

$$\|A\| = \sum_{i,j \in N} a_{ij} = \frac{1}{2} \sum_{i \in N} \alpha_i.$$

As mentioned in Bergantiños and Moreno-Ternerero (2020a), each broadcasting problem (N, A) can be associated with a claims problem (N, c^A, E^A) where $c_i^A = \alpha_i$, for each $i \in N$, and $E^A = \|A\|$. This would allow to import claims rules in order to solve broadcasting problems. We can, nevertheless, avoid such a detour to solve broadcasting problems directly. More precisely, a *broadcasting rule* is a mapping that associates with each (audience) matrix the list of the amounts the clubs get from the total (broadcasting) revenue. Formally, for each problem (N, A) , $R(N, A) \in \mathbb{R}^N$ and

$$\sum_{i \in N} R_i(N, A) = \|A\|.$$

Two rules stand out as focal for this problem (e.g., Bergantiños and Moreno-Ternerero, 2020a). First, the so-called *equal-split rule*, which splits equally the audience of each game a_{ij} among two clubs, thus ignoring the existence of fans for each club. The total audience assigned to each club is computed as the sum, over all games played by such club, of the audiences assigned to each game. Formally,

Equal-split rule, ES: for each problem (N, A) , and each $i \in N$,

$$ES_i(N, A) = \frac{\alpha_i}{2}.$$

Second, the so-called *concede-and-divide*, which concedes each club its number of fans and divides equally the rest. For each club i we estimate f_i , the number of fans of club i . Now, for each game, a_{ij} is divided as follows: i receives $f_i + \frac{a_{ij} - f_i - f_j}{2}$ and j receives $f_j + \frac{a_{ij} - f_i - f_j}{2}$. Again, the total audience assigned to each club is computed as the sum over all games played by such club. As shown in Bergantiños and Moreno-Ternerero (2020a), this rule could be computed by means of an easier formula, without needing to estimate the number of fans of each club. More precisely,

Concede-and-divide, CD: for each problem (N, A) and each $i \in N$,

$$CD_i(N, A) = \frac{(n-1)\alpha_i - \|A\|}{n-2}.$$

Both the *equal split rule* and *concede-and-divide* are backed up by the axiomatic approach. As a matter of fact, they share a solid common ground as they are just characterized by three

properties (Theorem 1 in Bergantiños and Moreno-Tertero, 2020a). Two properties (*equal treatment of equals* and *additivity*) are common in both characterizations. One states that two clubs with the same audiences should receive the same amount;⁹ another that revenues should be additive on the audience matrix. The third property in each characterization comes from a pair of polar properties modeling the effect of *null* or *essential* clubs, respectively. The two rules are, nevertheless, somewhat extreme (and polar) in their treatment of (neutral versus hard-core) fans. That is why in Bergantiños and Moreno-Tertero (2021) we consider a family of rules that offer a compromise between both of them. They are defined as convex combinations of the two rules. Formally,

Compromise rules, $\{C^\lambda\}_{\lambda \in [0,1]}$: for each $\lambda \in [0, 1]$, each problem (N, A) , and each $i \in N$,

$$C_i^\lambda(N, A) = (1 - \lambda)ES_i(N, A) + \lambda CD_i(N, A).$$

It turns out that the family is characterized by three basic (and normatively solid) axioms: *equal treatment of equals* and *additivity* (already mentioned above), and *maximum aspirations*, which states that no club can obtain an amount above its total audience.

3 The real life case: La Liga

The 20 clubs playing the Spanish Football League (La Liga) during the season 2017-18, and the allocation of the revenues raised from selling the broadcasting rights for that season (in millions of euros and in percentage terms) are in Table 1.¹⁰

Insert Table 1 here

As we can see, two clubs (Barcelona and Real Madrid) dominated the sharing, collecting (when combined) almost 23% of the pie. This is, nevertheless, a considerable reduction (in relative terms) from previous years in which they collected together almost one half of the pie (as mentioned at the introduction).

According to the guidelines in the Royal Decree setting the grounds to regulate broadcasting football in Spain, half of the overall revenue had to be shared equally. This means 33.14 million euros for each club. This appears in Column 4 (Lower bound) of Table 1.

⁹This is the standard axiom pursuing impartiality, a principle with a long tradition of use in the theory of justice (e.g., Moreno-Tertero and Roemer, 2006).

¹⁰The source is La Liga's website. See, for instance, <http://www.laliga.es/lfp/reparto-ingresos-audiovisuales>

One quarter of the overall revenue had to be shared according to league performance. By league performance, La Liga refers to a weighting system considering the standings at the end of the previous five seasons.¹¹ More precisely, the champion in a given season gets a score of 17%. The second one gets 15%. It continues down the line with 13%, 11%, 9%, 7%, 5%, 3.5%, 3%, 2.75%, 2.5%, 2.25%, 2%, 1.75%, 1.5%, 1.25%, 1%, 0.75%, 0.5% and 0.25%, respectively. A zero score is given to those clubs that played in the second division, or below, in one of those years. One quarter of the budget is then allocated proportionally to the resulting weighted average of those 5-year scores, with a weight of 35% for the last season, 20% for the previous to last season and 15% for each of the other three. This appears in Column 5 (Sport performance) of Table 1.

One twelfth of the overall revenue is shared proportionally to economic performance, to be understood as the average amount raised by each club in ticketing during the last 5 seasons. This appears in Column 6 (Economic performance) of Table 1.¹²

The last column of Table 1 gathers the residual amounts for each club, to be interpreted as the amounts associated to the capability of each club to generate resources from selling broadcasting rights.¹³ We must mention that La Liga does not specify how such numbers are computed. We shall elaborate further on this aspect later in the text.

¹¹As La Liga unveiled the allocation in December 2018, we considered 2017-2018 as the last of the previous five seasons (and 2013-2014 as the first one). An alternative interpretation of the Royal Decree would be that this allocation excludes the 2017-2018 season (thus considering 2012-2013 as the first one). We find that interpretation less natural, given the timeline just mentioned to unveil the overall allocation. It is nevertheless fair to argue that it is not clear from the information contained in La Liga's website which interpretation was indeed considered.

¹²For this, we consider data on season tickets for the previous three seasons, which are the only ones available, obtained from Palco 23, the leading newspaper in economic information of the sport business in Spain. See, for instance, <https://www.palco23.com/clubes/los-clubes-arrancan-la-liga-santander-concerca-de-600-000-abonados.html> and <https://www.palco23.com/clubes/los-clubes-de-primera-y-segunda-rozan-los-800000-abonados-en-2017-2018.html>.

¹³Obviously, the amounts in the previous two dimensions depend on the data and the above-mentioned considerations about them we made. Consequently, they also affect the residuals we consider for this fourth dimension.

4 First dimension: Lower bounds

In this section, we discuss several ways to divide E_1 among clubs, with the objective that each club must receive, at least, a certain amount (which is called a lower bound). In the case of La Liga, the Royal Decree states that half of the total endowment is divided equally among clubs. Namely, $E_1 = \frac{E}{2}$ and

$$R^1(E_1) = \left(\frac{E_1}{n}, \dots, \frac{E_1}{n} \right).$$

We may consider alternative (unequal) lower bounds. We do it by considering the claims problem $(N, (\alpha_i)_{i \in N}, E_1)$. Two lower bounds are normally highlighted in the literature on claims problems (e.g., Thomson, 2019).

The first one is the average-truncated-claims lower bound on awards (*TLB*), introduced by Moreno-Tertero and Villar (2004), and denoted by $b = (b_i)_{i \in N}$. Formally, for each $i \in N$,

$$b_i = \frac{1}{n} \min\{\alpha_i, E_1\}.$$

It seems natural that each club receives at most α_i . Likewise, E_1 (the endowment to divide in this dimension) should be an upper bound. Thus, each club could receive, at most, $\min\{\alpha_i, E_1\}$. As we have n clubs, we construct as a lower bound for each club $\frac{1}{n}$ of the previous amount. In doing so, we construct a *maximal* lower bound.

The other lower bound is the conditional equal division lower bound on awards (*CLB*), introduced by Moulin (2000), and denoted by $\beta = (\beta_i)_{i \in N}$. Formally, for each $i \in N$,

$$\beta_i = \min \left\{ \alpha_i, \frac{E_1}{n} \right\}.$$

Note that the above guarantees each club an equal sharing of the endowment $\left(\frac{E_1}{n}\right)$ except for those with a claim α_i below such an amount.

As mentioned above, both *CLB* and *TLB* have been highlighted in the literature on claims problems. The first one is obviously stronger than the second one (if a rule satisfies *CLB* then it also satisfies *TLB*). It is actually deeply associated to the *constrained equal awards rule* mentioned above. As a matter of fact, that is the only *consistent* rule that satisfies *CLB* (e.g., Thomson, 2019). On the other hand, *TLB* is satisfied by many more rules. When combined with other axioms, it characterizes the *constrained equal awards rule* and the *Talmud rule*, among others.

We now apply the previous lower bounds to La Liga. In the theoretical model considered at Bergantiños and Moreno-Ternero (2020a), we normalize the revenue generated from each viewer to 1. In the case of La Liga, the revenue generated by each viewer is 6.73.¹⁴ In what follows, our computations reflect that number.

Insert Table 2 here

In Column 3 of Table 2, the normalized *TLBs* are listed. For instance, we explain the case of Real Madrid:

$$\begin{aligned} E_1 &= 662.8, \\ \alpha_{RM} &= 47.1 * 6.73 = 316.98, \text{ and} \\ TLB_{RM} &= \frac{1}{20} \min\{316.98, 662.8\} = 15.84. \end{aligned}$$

Now, given the configuration of the problems we are considering, these reasonable lower bounds (when aggregated) amount only to 132.56. If, following the suggestion in the Royal Decree, we want to allocate one half of the overall amount in lower bounds, we can simply scale up all those amounts (multiplying by $\frac{662.80}{132.56} = 5$ each). This is what appears in Column 4 at Table 2.

In Column 5 of Table 2, the normalized *CLBs* are listed. For instance, we explain the case of Real Madrid:

$$CLB_{RM} = \min \left\{ 316.98, \frac{662.8}{20} \right\} = 33.14.$$

Notice that the allocation of *CLBs* coincides with the allocation used by La Liga for this dimension. This is a consequence of the specific numbers we have (with all claims being sufficiently high with respect to an equal sharing of the available amount).

As expected, clubs with high audiences obtain more with $5 * TLB$ than with La Liga (or, equivalently, *CLB*). In this case, only four clubs belong to that category.

To conclude with this section, note that the bounds obtained from $5 * TLB$ have some clubs above the (equal-sharing) amount 33.14 (which is the amount suggested by *CLB* or LaLiga). This is a consequence of the fact that some clubs have α_i above E_1 , but not others, whereas all

¹⁴It is computed as the total revenue (1325.60) divided by the total number of viewers. Audiences come from Havas Sports and Entertainment. See, for instance, <https://www.palco23.com/media/a-la-caza-de-la-audiencia-futbolera-que-club-logro-mas-visibilidad-en-tv>.

of them have it above $\frac{E_1}{n}$. If, for some reason, we would want to impose that no clubs receive more than 33.14, the allocation given by $5 * TLB$ could be redistributed (with progressive transfers) so that, in the end, each club would receive 33.14.

5 Second dimension: Sport performance

As mentioned above, the Royal Decree requires to share a quarter of the overall endowment based on sport performance in the last 5 seasons. But this is done according to a specific (and somewhat arbitrary) weighting scheme for the ordinal standings in each season.¹⁵ We consider two plausible alternatives here.

We first provide some notation to compare the allocation implemented by La Liga and the two alternatives we introduce here. Let $\rho = (\rho_1, \dots, \rho_n) \in \mathbb{R}_+^N$ denote an index of sport performance for all clubs in the league. We divide E_2 among clubs proportionally to the index ρ . Namely,

$$R^2(E_2) = \frac{E_2}{\sum_{i=1}^n \rho_i} (\rho_1, \dots, \rho_n).$$

In the case of La Liga, $E_2 = \frac{E}{4}$ and the index ρ is a weighted average of the table standings in the last 5 seasons, as explained above. We gather the resulting allocation at Column 2 of Table 3.

Insert Table 3 here

The motivation that underlies La Liga's proposal to endorse a heterogeneous scheme is to reward further top standings. This is somewhat in line with other instances of sports and cultural economics. For instance, Formula One endorses now a scoring scheme in which the first (in a race) gets 25 points, the second 18, the third 15 and the followers 12, 10, 8, 6, 4, 2 and 1, respectively. Similarly, in MotoGP the first (in a race) gets 25 points, the second $20 = 25 - 5$, the third $16 = 20 - 4$, the fourth $13 = 16 - 3$, the fifth $11 = 13 - 2$, and from the sixth to the 15th one point less each (that is, from 10 to 1). At the Olympic Games, the

¹⁵It might be argued that the scheme is trying to take into account the number of points associated with each standing, as it can be seen that if one represents the average number of points for the standings in last seasons, it shows different slopes for the clubs in the first 7-8 positions and the rest of clubs (although the bottom three clubs also show a different slope). We thank a reviewer for raising this issue.

podium height difference between first and second is greater than the difference between second and third. Finally, in the Eurovision Song Contest 12 points are awarded for the top song, 10 for the second, 8 to 1 for the eight remaining songs; all other songs get zero (e.g., Ginsburgh and Moreno-Ternero, 2022).

In contrast with the motivations described above, we may also want to explore the linear case, which is actually in place at the English Premier League. More precisely, in the first alternative we propose, which we call *homogeneous*, we modify the weighting scheme in each season to consider a homogeneous one in which each step is equally distant from each other. That is, we consider that, in each season, the champion gets 20, whereas the second gets 19, the third gets 18, and so on until the last one gets 1. Then, the overall score is obtained after a weighted aggregation of the last five seasons, with weights being as in the case of La Liga. The results appear in Column 3 of Table 4 (Homogeneous).

The second alternative we propose considers a scheme determined by the actual cardinal information offered by the scoring at the end of the season (and not just the ranking). One might argue that sporting performance is more a qualitative than a quantitative variable. Thus, standings matter largely. But so do points (and not just as means to obtain standings). For instance, Atlético de Madrid's performance in the season 2018/2019 (second place with 76 points) seems to be poorer than Real Madrid's performance in the season 2014/2015 (second place with 92 points) and, therefore, we might want to reward them differently. The underlying logic is that fans enjoyed more in the first *path* than in the second (although the finish line was the same in terms of ranking). Likewise, this is also reflected in the reward scheme implemented by the UEFA Champions League, where every victory lands a club €2.8m and a draw €930,000. And also in some other instances of real life beyond sports. For instance, as of today, one can hardly find wine reviews without numerical ratings (e.g., Gergaud et al., 2021). These ratings are sometimes used for rankings, which are indeed important. But, most of the times, wine stores and consumers simply face wine ratings, on which they base their decisions, without looking further to the rankings they might generate.

In the second alternative just described, each club would get, each season, a score equal to the points obtained and, again, the allocation is made proportionally to the overall score

obtained after a weighted aggregation of the last five seasons, with weights as in La Liga.¹⁶ The results appear in Column 4 of Table 3 (Points).

We can observe from Table 3 that the two alternatives (columns 3 and 4) yield quite similar results. Nevertheless, the allocation suggested by La Liga is much more different, specially for the clubs with high sport performance. The two powerhouses (Real Madrid and Barcelona), as well as Atlético, are largely favored by the current scheme in practice for this dimension. On the other edge, the majority of the remaining clubs are worse treated with that scheme. Based on this, one is tempted to say that the egalitarian desideratum in the first dimension is not only forgotten with this one, but, actually, somewhat swallowed.

6 Third dimension: Economic performance

The Royal Decree also requires to share one twelfth of the overall endowment proportionally to economic performance in the last 5 seasons.¹⁷ By economic performance, La Liga refers to the average of the revenues generated from ticket sales in the last five seasons.¹⁸ And it divides the endowment for this dimension ($\frac{E}{12}$) proportionally. We consider three alternatives here.

Again, we introduce some notation first. Let $\xi = (\xi_1, \dots, \xi_n)$ denote the index of economic performance for all clubs in the league and E_3 the amount devoted to this dimension. La Liga takes $E_3 = \frac{E}{12}$ and, as the index, the average of the (game and season) ticket sales in the last 5 seasons. The allocation implemented is, again, proportional according to the index, *i.e.*,

$$R^3(E_3) = \frac{E_3}{\sum_{i=1}^n \xi_i} (\xi_1, \dots, \xi_n).$$

As in the case of the first dimension, we consider a claims problem and propose the alternative allocations using such a claims problem. Let (N, c, E_3) be such that N is the set of clubs, $c = \xi$ and $E_3 = \frac{E}{12}$. Notice that the allocation implemented by La Liga is just $P(N, c, E_3)$. We then consider as alternatives the allocations obtained by applying the other three rules on claims problems to (N, c, E_3) . Namely, $CEA(N, c, E_3)$, $CEL(N, c, E_3)$ and $T(N, c, E_3)$.

¹⁶The points obtained by the clubs in each season are available from Google, Wikipedia or many Spanish newspapers, such as Marca or As.

¹⁷This is part of what La Liga dubbed *social relevance*, together with the broadcasting dimension.

¹⁸Other relevant economic aspects (such as merchandising, sponsorships and transfer fees) which are becoming more important than ticketing for top clubs are ignored. So are TV revenues, which are even more important than the above-mentioned aspects. But they will be treated as an independent dimension in the next section.

We were unable to obtain fully appropriate data for this dimension. As a matter of fact, we only have data on season tickets (which are less expensive, per capita, than individual game tickets) for three seasons (instead of five). This is an obvious limitation of our analysis. For instance, some clubs cap the number of season ticket holders, whereas some others have a relatively low number of season ticket holders. Nevertheless, there is typically a correlation between demand for game tickets and demand for season tickets. Popular clubs are so in both dimensions. Not so popular clubs fail in both dimensions.¹⁹

Table 4 reports the ticket sales (Column 2), the allocation of La Liga (Column 3) and the three alternative allocations (Columns 4, 5, and 6).

Insert Table 4 here

When the endowment is almost one half of the aggregate claim, the proportional and Talmud outcomes are quite similar. Thus, the allocation of La Liga and the allocation obtained via the Talmud rule are quite similar. The constrained equal-awards allocation favors clubs with low sales whereas the constrained equal-losses allocation favors clubs with large sales.

7 Fourth dimension: Broadcasting performance

As mentioned above, one sixth of the overall amount (i.e., 220.93 millions) is allocated according to the capability of each club to generate resources from selling broadcasting rights. In this dimension, La Liga is rather silent. In contrast with the other dimensions, for which we can compute the allocation to each club (for instance, in the second dimension, taking into account the position of the club in the last five seasons), in this dimension we do not know exactly the way in which the allocation is computed. This is surprising because it refers to the crucial aspect ultimately driving revenues. TV platforms will be eager to broadcast highly demanded

¹⁹Furthermore, popular clubs have similar strategies aiming to maximize attendance each game. For instance, Betis decided to implement (for the first time) during the season 2022/2023 a punishment for season ticket holders that did not use them at least 85 percent of the games. By “using” the season tickets, it was allowed to concede them to the club before a game, so that the club could sell a regular ticket associated to that seat for that game to another person (returning half of the amount raised to the season ticket owner). Also, in the case of Barcelona, there was uproar in May 2022, when the majority of attendants to the Europa League game in the quarterfinals at Nou Camp were Eintracht supporters, who apparently got tickets for the game thanks to Barcelona season ticket holders (who preferred to make money rather than supporting their club that day).

games to attract more consumers and sponsors. But the Royal Decree only mentions explicitly “participation in generating resources for the sale of TV rights”. As we shall argue later, we shall identify participation as audience. This seems to be in line with La Liga too (although they do not specify further beyond this identification).

We can at least compute the allocation received by each club in this dimension as the difference between the whole allocation and the amount computed in the three previous dimensions.²⁰ Namely,

$$R^4(E_4) = R(E) - \sum_{k=1}^3 R^k(E_k).$$

We then compare such a *residual* allocation with some direct proposals we make for this dimension.

Viewers of each game can essentially be divided in two categories: those watching the game because they are fans of one of the clubs playing, and those watching the game because they thought that the specific combination of clubs rendered the game interesting. As mentioned above, and in line with Szymanski (2001), we refer to them as hard-core (club) fans and neutral (football) fans, respectively. We argue that the revenue generated by the first category should be allocated to the corresponding club, whereas the revenue generated by the second category should be divided equally between both clubs.²¹

As we mentioned in Section 2, we consider the model introduced in Bergantiños and Moreno-Ternero (2020a) to study this dimension. Two somewhat polar rules are salient in that model: the equal-split rule and concede-and-divide. They are actually somewhat extreme from the point of view of treating fans. The former assumes that only neutral fans exist. The latter assumes that there are as many hard-core fans as possible (compatible with the audiences).

As mentioned above, the theoretical model in Bergantiños and Moreno-Ternero (2020a) normalizes the revenue generated from each viewer to 1. As in the case of the first dimension, we need to adapt this assumption to the data. More precisely, the total amount to be divided in this dimension is 220.93 and the total audience is 197.05 (as we have argued above). Thus,

²⁰As mentioned above, the amounts in the previous dimensions depend on the data and some considerations about them we made. Consequently, they also affect the amounts corresponding to this fourth dimension.

²¹Forrest et al., (2005) empirically identify that neutral fans are more likely to create increased demand for televised matches than they are to increase demand for stadium seating.

we can interpret that the revenue generated by each viewer in this dimension is $\frac{220.93}{197.05} = 1.12$.

Insert Table 5 here

In Columns 4 and 5 of Table 5 we provide the values obtained from the equal-split rule (*ES*) and concede-and-divide (*CD*), making use of 1.12 to normalize. For instance, in the case of Real Madrid:

$$ES_{RM} = \frac{47.1 * 1.12}{2} = 26.376, \text{ and}$$

$$CD_{RM} = \frac{19 * (47.1 * 1.12) - 220.93}{18} = 43.47.$$

As we can see, the allocation implemented by La Liga is quite different from the allocations proposed by *ES* and *CD*. Some clubs (for instance, Sevilla) receive an amount similar to the one proposed by *ES*. Some clubs (for instance, Barcelona) receive an amount similar to the one proposed by *CD*. Some clubs (for instance, Betis) receive an amount lower than the ones proposed by *ES* or *CD*. Some clubs (for instance, Real Sociedad) receive an amount larger than the ones proposed by *ES* or *CD*.

The family of compromise rules, which are a convex combination of the equal-split rule and concede-and-divide, is considered (and characterized by means of three basic principles with strong normative appeal) in Bergantiños and Moreno-Ternero (2021). As argued therein, the allocation obtained by a club should be somewhat in between the allocations proposed by the equal-split rule and concede-and-divide to such a club. In other words, there should be a rule within the compromise rules explaining the outcome for each club. In the last column of Table 5, we explore whether the amount obtained by each club in the allocation used by La Liga corresponds to some compromise rule. For instance, Real Madrid receives the amount that the rule $C^{0.85}$ would yield for this setting. In contrast, Betis receives less than the amount proposed by any rule within the family because $5.27 < \min \{15.70, 20.86\}$. On the other hand, Barcelona receives more than the amount proposed by any rule within the family because $41.51 > \max \{25.28, 41.10\}$.

Our analysis casts serious doubts about the allocation implemented by La Liga in this dimension. To wit, we can infer from Table 5 that less than half of the clubs obtain amounts that can be rationalized by some compromise rule, i.e., within the amounts suggested by the equal-split rule and concede-and-divide. More precisely, six clubs are favored, as they obtain

more than any amount suggested by a compromise rule. Five clubs obtain amounts below those suggested by the compromise rules. The remaining nine clubs obtain amounts suggested by the compromise rule whose parameter is given by the corresponding cell in the last column. Note that one cannot infer from here that the allocation implemented by La Liga favors clubs with lower or higher audiences. The “above” category includes clubs such as Barcelona (the second most watched club) and Leganés (the least watched club). The “below” category includes clubs such as Betis (the third most watched club) and Villareal (the fifth least watched club). Clubs with intermediate audiences (such as Athletic Bilbao or Español) might belong to one or the other category.

It is also remarkable to notice that two clubs (Real Sociedad and Girona) have equal audiences but obtain (quite) different amounts. As a matter of fact, we have several related disturbing features too. For instance, Real Madrid has a higher audience than Barcelona but receives a lower amount. The case of Betis is even more remarkable, as it has the third largest audience but it actually receives the fourth lowest amount.²²

We conclude with a final question: should we pick a specific rule within the family of compromise rules? As mentioned above, the equal-split rule panders neutral fans, whereas concede-and-divide does so with hard-core fans. In practice, we know the total number of viewers of each game, but not the partition in those two categories. Nevertheless, it is possible to estimate the average number of hard-core fans and neutral fans watching the games. For instance, we can take a sample of viewers and ask them to report the games they have watched, and if they are hard-core fans of some club. Let p_h denote the percentage of viewers who have watched a game being a hard-core fan of some of the clubs. Then, $1 - p_h$ denotes the percentage of viewers who have watched a game without being a hard-core fan of some of the clubs. As argued in Bergantiños and Moreno-Ternero (2021), C^{p_h} could be a salient rule among those within the family of compromise rules. Unfortunately, we do not know p_h . But, according to official barometers in 2014, 67.4% declared themselves followers of a certain professional club. Thus, we have considered the compromise rule corresponding to $\lambda = 0.674$. Furthermore, it turns out that the compromise rule that yields a closer allocation to the allocation used by La Liga (according to the Euclidean distance) is the rule corresponding to $\lambda = 0.71$, which

²²A caveat though is that Betis had more games broadcasted in non-subscription TV than the other clubs. We elaborate further on this issue at the Discussion section.

reinforces 0.674 as a reasonable estimation of p_h .

We compare the allocation implemented by La Liga with the allocation provided by the rule $C^{0.674}$ (second and third column in Table 6, respectively). We observe that one club (Betis) would obtain 13.91 millions of euros more with $C^{0.674}$. Other five clubs (Valencia, Celta, Español, Villarreal and Getafe) would also obtain more (with surpluses ranging from 0.36 millions to 7.67). The remaining fourteen clubs (including Barcelona and Real Madrid) would obtain less (with deficits ranging from 0.05 millions to 5.64 millions).²³

Insert Table 6 here

8 Combining the four dimensions

We now move the analysis from considering each dimension separately to account for all of them combined. More precisely, in this section, we compare the complete allocation implemented by La Liga, with other complete allocations, obtained by combining some of the proposals presented above (at each dimension).²⁴ As we have considered 2 possibilities in the first dimension (including the one used by La Liga), 3 in the second dimension, 4 in the third dimension, and 3 in the fourth dimension, the total number of combinations to obtain (hybrid) complete allocations is 72. Then, we have decided to select some of them, based on the average and the standard deviation.

In Table 7 below, we summarize the standard deviation (σ) of the different allocations in each dimension.

Insert Table 7 here

We can see that, for dimension 1, the allocation suggested by La Liga obviously has a lower standard deviation than the alternatives. For dimension 2, the opposite occurs, whereas for dimensions 3 and 4, it lies somewhat in between.

In Table 8, we compare the allocation of La Liga with other three allocations obtained by combining allocations of the four dimensions, as follows:

- In Column 3 (Low SD), we select for each dimension the allocation with the lowest σ .

²³Similar features occur with $C^{0.71}$ instead of $C^{0.674}$.

²⁴At the risk of stressing the obvious, we reiterate that this comparison relies on the analysis we made for each of the approaches individually (and, in particular, the data issues that arose in each of them).

Namely, La Liga in dimension 1, Points in dimension 2, *CEA* in dimension 3, and *ES* in dimension 4. The complete allocation is the sum over the allocations in each dimension.

- In Column 4 (High SD), we select for each dimension the allocation with the highest σ . Namely, $5 * TLB$ in dimension 1, La Liga in dimension 2, *CEL* in dimension 3, and *CD* in dimension 4. The complete allocation is the sum over the allocations in each dimension.
- In Column 5 (Average), we select the average of the allocations considered in such dimension. For instance, in dimension 1, we compute the average between La Liga and $5 * TLB$. The complete allocation is the sum over the allocations in each dimension.

Insert Table 8 here

As the allocations with the lowest σ are more similar to the average, we should expect that big clubs obtain more with the allocation High SD whereas small clubs obtain more with Low SD. Besides, no club should obtain more with the average, or any allocation between Low SD and High SD.

The two powerhouses (Barcelona and Real Madrid) obtain more with High SD, as expected. It is remarkable that two other clubs (Betis and Celta), which are not among the big ones (in the first dimensions), also obtain more with High SD. Sixteen clubs (including the last fifteen from the list) obtain more with Low SD.

As we can see, no club obtains more with the average than with the maximum between Low SD and High SD. Nevertheless, Athletic Bilbao obtains more with La Liga than with the maximum between Low SD and High SD. This fact is remarkable because Low SD and High SD are some kind of extreme allocations.

In column 6 of Table 8 (Difference) we compute the difference between La Liga and the Average columns. The club more favored by La Liga is Atlético Madrid obtaining 8.58 more with La Liga. The worst treated club (by far) is Betis, receiving 18.44 less.

The computations of Table 8 are made assuming that $\frac{1}{2}$ of the endowment is devoted to the first dimension, $\frac{1}{4}$ to the second dimension, $\frac{1}{12}$ to the third, and $\frac{1}{6}$ to the fourth. But other ways of dividing the endowment between the dimensions could be applied. For instance in Bergantiños and Moreno-Ternero (2020a) it is considered the case in which the endowment is divided considering only the fourth dimension. As argued therein, from the point of view of the

broadcasting firms, the fourth dimension is the one that really matters. We are not so extreme in this paper and we consider the case where the fourth dimension is the most important one (awarded $\frac{1}{2}$) and the other dimensions are equally important (awarded $\frac{1}{6}$ each one).

Columns 3 and 4 of Table 9 are computed similarly to columns 2 and 5 of Table 8 changing the weight of each dimension; namely $(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{2})$ instead of $(\frac{1}{2}, \frac{1}{4}, \frac{1}{12}, \frac{1}{6})$.

Insert Table 9 here

As the weight of the lower bound has decreased from La Liga to La Liga 2 and the weight of the broadcasting performance has increased, we should expect that big clubs obtain more with La Liga 2 whereas small clubs obtain less. Actually, this is the generally the case. Notice that the three biggest clubs (Barcelona, Real Madrid, and Atlético de Madrid) obtain more with La Liga 2 and the eleven smallest clubs obtain more with La Liga.

In the last column of Table 9 (Difference 2) we compute the difference between La Liga and Average 2. The results are qualitatively different from the ones obtained in Table 8 with Difference. In this case, the most favored clubs are Villareal, Leganés, Eibar, Getafe, and Alavés (all of them with a surplus larger than 10). The worst treated clubs are Real Madrid, Barcelona and Betis. For the rest of the clubs, the differences lie in between -1.88 and 9.13 . The general trend is that the two biggest clubs are worse treated with La Liga and the smallest clubs are better treated (again, Betis is a remarkable exception).

We end this section by comparing our earlier partial empirical analysis on this issue with the one carried out in this paper.

In Bergantiños and Moreno-Ternero (2020a), there are two analyses for the season 2016-2017 (the previous to the one we analyze here). In the first one (Table 3) the whole budget is divided following the equal-split rule and concede-and-divide. We can see it as a particular case of the model of this paper, where the weights of the dimensions are $(0, 0, 0, 1)$. In the second one (Table 4), the first three dimensions are shared as in La Liga and for the fourth one, the equal-split rule and concede-and-divide are used. Then, two cases (from the 72 mentioned at the beginning of this section) are considered. In this paper, we analyze other cases.

In Bergantiños and Moreno-Ternero (2021), there are two analyses for the season 2017-2018 (the same we analyze here). The first one (Table 2) is similar to Table 3 of Bergantiños and Moreno-Ternero (2020a) but for a different season. In the second one (also Table 2) it is

checked whether the whole allocation of La Liga could be rationalized by a compromise rules. For some clubs, it is possible, but for other clubs it is not possible. This analysis is similar to the one considered in Table 5 of this paper. The difference is that here we only consider the fourth dimension, whereas in Bergantiños and Moreno-Ternero (2021) the whole allocation is considered.

9 Discussion

In this paper, we have expanded our research agenda on the problem of sharing the revenues from the collective sale of broadcasting rights for sports leagues, to analyze in detail the specific case of La Liga, the Spanish Football League, which was highly regulated by the Spanish government in 2015. Our aim was to provide alternatives to La Liga's choice, showing how the distribution would change in each case. As mentioned below, our analysis is based on some specific assumptions on how some variables (in particular, audience) are measured.

We believe that some interesting lessons can be obtained from the hybrid schemes suggested by the Spanish Football League Association, as a result of regulation. On the positive side, they guarantee all participating clubs lower bounds, which have a long tradition in normative work (e.g., the conflicting claims literature, or the fair allocation literature). They also compromise between the “needs-blind” view carried by performance pay and the “incentives-blind” view carried by an equal sharing of the whole pie, which seems to be another reasonable desideratum. In other words, they compromise balance concerns alongside efficiency concerns in resource allocation, a long-standing goal within economics and operations research (e.g., Karsu and Morton, 2014). On the negative side, a key aspect of hybrid schemes is to decide how to share a portion of the pie based on audiences. And this aspect is not sufficiently justified in the allocation implemented by the Spanish Football League.

In Bergantiños and Moreno-Ternero (2020a), we introduced a theoretical model to analyze this problem. Given that the second and third dimension (sport and economic performance) require enriching the informational basis of the model (bringing a new prior into the problem), one might consider rules in which only the first and fourth dimensions are combined. For instance, consider the rule in which one half of the overall amount is equally shared whereas the other half is shared according to the equal-split rule. Namely, use the weights $(\frac{1}{2}, 0, 0, \frac{1}{2})$

instead of $(\frac{1}{2}, \frac{1}{4}, \frac{1}{12}, \frac{1}{6})$. It turns out this rule is precisely the intermediate member of the UE -family of rules we study elsewhere (e.g., Bergantiños and Moreno-Ternero, 2022b). If instead of considering equal weights for the first and the fourth dimension, we consider all possible convex combinations, we obtain the whole family. Likewise, we could consider concede-and-divide instead of the equal-split rule in the above definition. Then, the rule is precisely the intermediate member of the UC -family of rules we also study elsewhere (e.g., Bergantiños and Moreno-Ternero, 2022b). If instead of considering equal weights for the first and fourth dimension, we consider all possible convex combinations, we obtain the whole family.

To conclude, we should stress some data aspects that were crucial in our analysis. The key variable in our exercise is that corresponding to the total audience of each club (α_i in our model). This variable plays a crucial role in the discussion on the lower bounds (second dimension) and the broadcasting performance (fourth dimension). We acknowledge that the variable we use does not distinguish between the audience corresponding to subscription channels and that corresponding to free-to-air television. This is relevant because the size of both types of audiences is different and the choice of the games for each type of broadcasting is not random.²⁵ In the case of La Liga, each week comprises 9 games broadcasted in subscription channels and only 1 in free-to-air television, and the latter can never involve clubs playing European competitions at the time. In the season we analyze in this paper, most of the games broadcasted via free-to-air television involved Betis.²⁶ That is reflected in its α_i value. If all games had been broadcasted in subscription channels, Betis would still obtain a very high value of α_i , but certainly below the one of Atlético de Madrid. Thus, although we acknowledge this caveat has to me made regarding the implications of our analysis (on which clubs are benefited or not by LaLiga scheme), it does not render them invalid. Nevertheless, one might then argue that we would need to augment the α_i values we consider to obtain a more accurate picture of the overall audiences associated to clubs. For instance, in the case mentioned above, this might involve comparing (for each club) the average audience of games broadcasted in subscription channels with the average audience of games broadcasted in free-to-air television, and using the ratio of both means as some sort of *scaling coefficient* for games broadcasted in free-to-air television, in order to obtain the augmented α_i value. But, if so, we would need to do the same

²⁵There is evidence on this point for La Liga and also for the Premier League (e.g., García and Rodríguez, 2006; Buraimo, 2008; Buraimo and Simmons, 2009; Artero et al., 2019).

²⁶11 times in the first 29 weeks, offering on average a 30-percent larger audience for that channel.

with other similarly important aspects that are not captured either in our analysis, such as broadcasting time windows or audiences from abroad (where not all games are broadcasted). If it is fair to argue that the audience of games broadcasted via subscription channels is larger than the audience of games broadcasted via free-to-air television, so is to argue that the audiences of games in prime time is larger than those with respect to those in different time windows. Thus, another scaling factor associated to prime time would then also be needed to properly account for the augmented α_i value. A proper econometric analysis accounting for all these scaling factors is beyond the scope of this paper, but we acknowledge the outcomes of our analysis should be considered having these caveats in mind. Our methodology remains valid with scaled audiences or with non-scaled audiences (which is the case we show in our paper).

References

- [1] Artero, I., Bandrés, E., García, J., Rodríguez, P., 2019. Demand of Professional Sports: Attendance and Audience, in J. García (ed.), Sports (and) Economics, FUNCAS Social and Economic Studies 7, 183-218.
- [2] Ascari, G., Gagnepain, P., 2006. Spanish Football. Journal of Sports Economics 7, 76-89.
- [3] Bergantiños, G., Moreno-Tertero, J.D., 2020a. Sharing the revenues from broadcasting sport events. Management Science 66 (6), 2417-2431.
- [4] Bergantiños, G., Moreno-Tertero, J.D., 2020b. Allocating extra revenues from broadcasting sports leagues. Journal of Mathematical Economics 90, 65-73.
- [5] Bergantiños, G., Moreno-Tertero, J.D., 2021. Compromising to share the revenues from broadcasting sports leagues. Journal of Economic Behavior and Organization 183, 57-74.
- [6] Bergantiños, G., Moreno-Tertero, J.D., 2022a. Monotonicity in sharing the revenues from broadcasting sports leagues. European Journal of Operational Research 297, 338-346.
- [7] Bergantiños, G., Moreno-Tertero, J.D., 2022b. On the axiomatic approach to sharing the revenues from broadcasting sports leagues. Social Choice and Welfare 58, 321-347.

- [8] Bergantiños, G., Moreno-Tertero, J.D., 2022c. Separable rules to share the revenues from broadcasting sports leagues. *Economics Letters* 211, 110233.
- [9] Bergantiños, G., Moreno-Tertero, J.D., 2023. Decentralized revenue sharing from broadcasting sports. *Public Choice*. Forthcoming.
- [10] Buraimo, B., 2008. Stadium attendance and television audience demand in English league football, *Managerial and Decision Economics* 29, 513-523.
- [11] Buraimo, B., Simmons, R., 2005. A tale of two audiences: spectators, television viewers and outcome uncertainty in Spanish football, *Journal of Economics and Business* 61, 326-338.
- [12] Carreras, M. García, J. 2018. TV rights, financial inequality and competitive balance in European football: evidence from the English premier league and the Spanish La Liga, *International Journal of Sport Finance* 13, 201-224.
- [13] Cave, M., Crandall, R., 2001. Sports rights and the broadcast industry. *The Economic Journal* 111, F4-F26.
- [14] Csató, L., 2021. *Tournament Design: How Operations Research Can Improve Sports Rules*. Palgrave Pivots in Sports Economics. Palgrave Macmillan, Cham, Switzerland.
- [15] Forrest, D., Simmons, R., Buraimo, B., 2005. Outcome uncertainty and the couch potato audience. *Scottish Journal of Political Economy* 52, 641-661.
- [16] Fort, R., Quirk, J., 1995. Cross subsidization, incentives and outcome in professional team sports leagues. *Journal of Economic Literature* 33, 1265-1299.
- [17] García, J., Rodríguez, P., 2002. The Determinants of Football Match Attendance Revisited: Empirical Evidence from the Spanish Football League. *Journal of Sports Economics* 3, 18-38.
- [18] García, J., Rodríguez, P., 2003. From Sports Clubs to Stock Companies: Financial Structure in Spanish Football. *European Sports Management Quarterly* 3, 253-269.

- [19] García, J., Rodríguez, P., 2006. The determinants of audience for Spanish football: a first approach, in J. García, P. Rodríguez and S. Késenne (eds.), *Sport Economics after fifty years: essays in honour of Simon Rottenberg*, Universidad de Oviedo, 147-167.
- [20] García, J., Rodríguez, P., 2013. The determinants of football match attendance in Spanish football: an empirical analysis, in P. Rodríguez, S. Késenne and J. García (eds.), *The econometrics of sport*, Edward Elgar Publishing, 154-166.
- [21] Gergaud, O., V., Ginsburgh and J. Moreno-Terner, 2021. Wine ratings: seeking a consensus among tasters via normalization, approval and aggregation. *Journal of Wine Economics* 16, 321-342.
- [22] Ginsburgh, V., Moreno-Terner, J.D., 2022. The Eurovision Song Contest: Voting Rules, Biases and Rationality. *Journal of Cultural Economics*, Forthcoming.
- [23] Hansen, B.O., Tvede, M., 2016. Revenue Sharing in European Football Leagues: A Theoretical Analysis. Chapter 12 in A.A. Pinto et al., (eds.), *Trends in Mathematical Economics*. Springer.
- [24] Karsu, O., Morton, A., 2014. Incorporating balance concerns in resource allocation decisions: A bi-criteria modeling approach. *Omega* 44, 70-82.
- [25] Krumer, A., Moreno-Terner, J.D. 2022. The allocation of additional slots for the FIFA World Cup. Mimeo.
- [26] Moreno-Terner, J., Roemer, J., 2006. Impartiality, priority and solidarity in the theory of justice. *Econometrica* 74, 1419-1427.
- [27] Moreno-Terner J., Villar A., 2004. The Talmud rule and the securement of agents' awards. *Mathematical Social Sciences* 47, 245-257.
- [28] Moulin, H., 2000. Priority rules and other asymmetric rationing methods. *Econometrica* 68, 643-684.
- [29] O'Neill, B., 1982. A problem of rights arbitration from the Talmud. *Mathematical Social Sciences* 2, 345-371.

- [30] Peeters, T., 2011. Broadcast rights and competitive balance in European soccer. *International Journal of Sport Finance* 6, 23-39.
- [31] Peeters, T., 2012. Media revenue sharing as a coordination device in sports leagues. *International Journal of Industrial Organization* 30, 153-163.
- [32] Szymanski, S., 2001. Income inequality, competitive balance and the attractiveness of team sports: some evidence and a natural experiment from English soccer. *The Economic Journal* 111, F69-F84.
- [33] Szymanski, S., Késenne, S., 2004. Competitive balance and revenue sharing in team sports. *The Journal of Industrial Economics* 52, 165-177.
- [34] Thomson, W., 2017. *The Theory of Fair Allocation*, Princeton University Press. Forthcoming.
- [35] Thomson W., 2019. *How to divide when there isn't enough: from Aristotle, the Talmud, and Maimonides to the axiomatics of resource allocation*, Econometric Society Monograph. Cambridge University Press.
- [36] Wright, M., 2009. 50 years of OR in sport. *Journal of the Operational Research Society* 60(Supplement 1), S161-S168.

Table 1. Breakdown of La Liga's allocation across dimensions

Club	Alloc. 17-18	Alloc. 17-18 (%)	Lower bound	Sport Perf.	Econ. Perf.	Broadc. Perf.
Barcelona	154,00	11,62	33,14	55,56	23,79	41,51
Real Madrid	148,00	11,16	33,14	49,08	24,81	40,96
Atlético Madrid	110,60	8,34	33,14	48,74	11,57	17,15
Sevilla	74,00	5,58	33,14	25,22	5,72	9,92
Athletic Bilbao	73,20	5,52	33,14	17,68	12,15	10,23
Valencia	65,70	4,96	33,14	23,22	6,17	3,17
Villareal	65,50	4,94	33,14	29,65	1,73	0,98
Real Sociedad	61,50	4,64	33,14	12,70	3,68	11,98
Málaga	53,50	4,04	33,14	6,60	2,46	11,29
Betis	52,90	3,99	33,14	10,91	3,59	5,27
Celta	52,90	3,99	33,14	10,65	2,07	7,04
Español	52,40	3,95	33,14	8,69	3,69	6,88
Las Palmas	46,80	3,53	33,14	3,07	2,77	7,82
Eibar	46,30	3,49	33,14	6,73	0,38	6,05
Alavés	46,10	3,48	33,14	4,13	1,02	7,81
Deportivo Coruña	46,00	3,47	33,14	3,15	2,74	6,97
Levante	45,10	3,40	33,14	4,22	0,83	6,91
Getafe	44,50	3,36	33,14	6,22	0,54	4,60
Girona	43,30	3,27	33,14	3,28	0,34	6,53
Leganés	43,30	3,27	33,14	1,87	0,42	7,87
Sum	1325,60	100,00	662,80	331,40	110,47	220,93

Table 2. Lower bounds

Club	La Liga	TLB	5*TLB	CLB
Barcelona	33,14	15,17	75,85	33,14
Real Madrid	33,14	15,84	79,21	33,14
Atlético Madrid	33,14	8,58	42,89	33,14
Sevilla	33,14	6,22	31,11	33,14
Athletic Bilbao	33,14	5,79	28,93	33,14
Valencia	33,14	6,56	32,80	33,14
Villareal	33,14	4,64	23,21	33,14
Real Sociedad	33,14	5,01	25,06	33,14
Málaga	33,14	5,92	29,60	33,14
Betis	33,14	9,42	47,09	33,14
Celta	33,14	5,99	29,94	33,14
Español	33,14	5,62	28,09	33,14
Las Palmas	33,14	5,35	26,74	33,14
Eibar	33,14	4,41	22,03	33,14
Alavés	33,14	4,61	23,04	33,14
Deportivo Coruña	33,14	4,81	24,05	33,14
Levante	33,14	5,08	25,40	33,14
Getafe	33,14	4,54	22,70	33,14
Girona	33,14	5,01	25,06	33,14
Leganés	33,14	4,00	20,01	33,14
Sum	662,80	132,56	662,80	662,80

Table 3. Sport performance

Club	La Liga	Homogeneous	Points
Barcelona	55,56	32,47	31,80
Real Madrid	49,08	30,90	29,78
Atlético Madrid	48,74	30,82	28,42
Sevilla	25,22	25,12	22,05
Athletic Bilbao	17,68	19,17	19,39
Valencia	23,22	22,47	20,97
Villareal	29,65	26,19	21,65
Real Sociedad	12,70	18,84	18,41
Málaga	6,60	12,56	13,11
Betis	10,91	16,36	13,68
Celta	10,65	17,19	14,90
Español	8,69	16,28	17,23
Las Palmas	3,07	5,95	7,69
Eibar	6,73	13,30	14,05
Alavés	4,13	8,01	9,56
Deportivo Coruña	3,15	6,11	10,06
Levante	4,22	8,18	11,71
Getafe	6,22	11,48	10,77
Girona	3,28	6,36	6,21
Leganés	1,87	3,64	9,92
Sum	331,40	331,40	331,38

Table 4. Economic performance

Club	Ticket sales	La Liga	CEA	CEL	Talmud
Barcelona	48,36	23,79	9,51	38,24	23,79
Real Madrid	50,44	24,81	9,51	40,32	23,79
Atlético Madrid	23,52	11,57	9,51	13,40	11,76
Sevilla	11,63	5,72	9,51	1,51	5,81
Athletic Bilbao	24,69	12,14	9,51	14,57	12,35
Valencia	12,54	6,17	9,51	2,42	6,27
Villareal	3,51	1,73	3,51	0,00	1,76
Real Sociedad	7,49	3,68	7,49	0,00	3,74
Málaga	5,00	2,46	5,00	0,00	2,50
Betis	7,29	3,59	7,29	0,00	3,65
Celta	4,21	2,07	4,21	0,00	2,11
Español	7,49	3,69	7,49	0,00	3,75
Las Palmas	5,63	2,77	5,63	0,00	2,82
Eibar	0,77	0,38	0,77	0,00	0,39
Alavés	2,08	1,02	2,08	0,00	1,04
Deportivo Coruña	5,57	2,74	5,57	0,00	2,78
Levante	1,69	0,83	1,69	0,00	0,85
Getafe	1,10	0,54	1,10	0,00	0,55
Girona	0,70	0,34	0,70	0,00	0,35
Leganés	0,85	0,42	0,85	0,00	0,42
Sum	224,57	110,46	110,45	110,46	110,46

Table 5. Broadcasting performance

Club	Audiencias	La Liga	ES	CD	C^lambda
Barcelona	45,10	41,51	25,28	41,10	Above
Real Madrid	47,10	40,96	26,40	43,47	0,85
Atlético Madrid	25,50	17,15	14,30	17,90	0,79
Sevilla	18,50	9,92	10,37	9,62	0,60
Athletic Bilbao	17,20	10,23	9,64	8,08	Above
Valencia	19,50	3,17	10,93	10,80	Below
Villareal	13,80	0,98	7,74	4,06	Below
Real Sociedad	14,90	11,98	8,35	5,36	Above
Málaga	17,60	11,29	9,87	8,56	Above
Betis	28,00	5,27	15,70	20,86	Below
Celta	17,80	7,04	9,98	8,79	Below
Español	16,70	6,88	9,36	7,49	Below
Las Palmas	15,90	7,82	8,91	6,54	0,46
Eibar	13,10	6,05	7,34	3,23	0,32
Alavés	13,70	7,81	7,68	3,94	Above
Deportivo Coruña	14,30	6,97	8,02	4,65	0,31
Levante	15,10	6,91	8,46	5,60	0,54
Getafe	13,50	4,60	7,57	3,70	0,77
Girona	14,90	6,53	8,35	5,36	0,61
Leganés	11,90	7,87	6,67	1,81	Above
Sum	394,10	220,93	220,93	220,93	

Table 6. Choosing a compromise rule

Club	La Liga	Compromise	Difference
Barcelona	41,51	35,94	5,57
Real Madrid	40,96	37,91	3,06
Atlético Madrid	17,15	16,73	0,42
Sevilla	9,92	9,87	0,05
Athletic Bilbao	10,23	8,59	1,64
Valencia	3,17	10,85	-7,67
Villareal	0,98	5,26	-4,28
Real Sociedad	11,98	6,34	5,64
Málaga	11,29	8,98	2,31
Betis	5,27	19,18	-13,91
Celta	7,04	9,18	-2,14
Español	6,88	8,10	-1,22
Las Palmas	7,82	7,32	0,51
Eibar	6,05	4,57	1,48
Alavés	7,81	5,16	2,65
Deportivo Coruña	6,97	5,75	1,22
Levante	6,91	6,53	0,38
Getafe	4,60	4,96	-0,36
Girona	6,53	6,34	0,20
Leganés	7,87	3,39	4,47
Sum	220,93	220,93	0,00

Table 7. Standard deviations

Dimension	Allocations	Stand. Dev.
Lower bound	La Liga	0,00
	5*TLB	16,16
Sport performance	La Liga	16,45
	Homogeneous	8,84
	Points	7,22
Economic performance	La Liga	7,05
	CEA	3,36
	CEL	12,00
	Talmud	6,92
Broadcasting performance	La Liga	10,59
	ES	5,39
	CD	11,38

Table 8. Comparing several allocations

Club	La Liga	Small SD	Large SD	Average	Difference
Barcelona	154,00	99,73	210,75	154,24	-0,24
Real Madrid	148,00	98,83	212,09	154,32	-6,32
Atlético Madrid	110,60	85,37	122,93	102,02	8,58
Sevilla	74,00	75,07	67,46	71,86	2,14
Athletic Bilbao	73,20	71,68	69,26	71,24	1,96
Valencia	65,70	74,56	69,24	69,58	-3,88
Villareal	65,50	66,04	56,92	60,01	5,49
Real Sociedad	61,50	67,39	43,12	58,04	3,46
Málaga	53,50	61,12	44,76	54,52	-1,02
Betis	52,90	69,81	78,86	71,34	-18,44
Celta	52,90	62,23	49,38	56,48	-3,58
Español	52,40	67,23	44,27	56,32	-3,92
Las Palmas	46,80	55,38	36,35	46,07	0,73
Eibar	46,30	55,30	31,99	44,87	1,43
Alavés	46,10	52,45	31,11	42,83	3,27
Deportivo Coruña	46,00	56,78	31,85	44,35	1,65
Levante	45,10	55,01	35,21	45,14	-0,04
Getafe	44,50	52,58	32,63	43,25	1,25
Girona	43,30	48,41	33,70	41,48	1,82
Leganés	43,30	50,58	23,70	37,59	5,71
Sum	1325,60	1325,56	1325,59	1325,58	0,02

Table 9. Allocations with different weights

Club	La Liga	La Liga 2	Average 2	Difference 2
Barcelona	154,00	220,20	200,35	-46,35
Real Madrid	148,00	216,28	203,17	-55,17
Atlético Madrid	110,60	118,12	109,13	1,47
Sevilla	74,00	69,05	67,98	6,02
Athletic Bilbao	73,20	77,82	75,08	-1,88
Valencia	65,70	48,38	62,89	2,81
Villareal	65,50	37,20	42,88	22,62
Real Sociedad	61,50	62,82	53,95	7,55
Málaga	53,50	54,26	52,33	1,17
Betis	52,90	41,29	71,56	-18,66
Celta	52,90	43,40	50,01	2,89
Español	52,40	44,86	50,78	1,62
Las Palmas	46,80	42,10	42,58	4,22
Eibar	46,30	34,44	34,16	12,14
Alavés	46,10	39,26	35,68	10,42
Deportivo Coruña	46,00	39,53	39,01	6,99
Levante	45,10	36,25	37,77	7,33
Getafe	44,50	30,07	32,60	11,90
Girona	43,30	33,53	34,17	9,13
Leganés	43,30	36,74	29,48	13,82
Sum	1325,60	1325,60	1325,57	0,03