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The allocation of additional slots for the FIFA World Cup

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Abstract: How to select participants to a sports tournament when there are more applicants than the tournament can handle? We propose to address this question resorting to standard tools from the fair allocation literature. To frame our discussion, we focus on the increase in the number of participating teams in the FIFA World Cup. We explore the allocation of additional slots among continental confederations. We consider ten different allocations. Based on our analysis, we can argue that the European soccer confederation (UEFA) has a solid basis to claim for additional slots.

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1 Introduction

How to select participants to a sports tournament when there are more applicants than the tournament can handle? This is an instance of one of the core problems in the history of economic thought: how to divide when there is not enough? (e.g., Thomson, 2019). But it is also in itself a broad question with ramifications to a wide array of disciplines. In this paper, we address this question by focusing on the relevant example of the FIFA World Cup and its recently approved increase in the number of participating teams.

FIFA World Cup, whose 2018 edition was viewed by 3.57 billion viewers around the globe,[1] is undoubtedly one of a few mega-events that affects many aspects of everyday life. For example, the World Cup qualification game between El Salvador and Honduras was a build-up for the so-called "Football war" between the countries in 1969. However, FIFA World Cup can also unite people. As evidence, Depetris-Chauvin et al. (2020) showed that a win of an African national team in the FIFA World Cup qualifiers or finals increases its fans' self-identification with their country at the expense of identification with the fans' ethnic group. Interestingly, Berthier and Boulay (2003) found a significantly lower myocardial infarction mortality in the day the French national team won the 1998 World Cup, whereas Carroll et al. (2002) reported an opposite result after England's loss in the 1998 World Cup. Finally, Edmans, Garcia and Norli (2007) found that a loss in the World Cup leads to a next-day abnormal lower stock return in the losing country.

Given such a broad influence of the FIFA World Cup, every change of the tournament's format may affect fields that are beyond soccer. One such a change is FIFA's decision to increase the number of participating teams from 32 (used between 1998 and 2022) to 48 (starting from 2026). One of the main issues of this expansion was the allocation of the new

[1] From https://www.fifa.com/worldcup/news/more-than-half-the-world-watched-record-breaking-2018-world-cup. Last accessed on 14/10/2022.

slots between the continental confederations.[2] On May 9, 2017, two days before the 67th FIFA Congress, the FIFA Council approved the slot allocation for the 2026 FIFA World Cup appearing in the last column of Table 1.[3]

Table 1: Distribution of slots in the World Cups between 1998 and 2026 including intercontinental playoff slots

Confederation	Members	1998	2002	2006	2010	2014	2018	2022	2026
Asia (AFC)	46	3.5	2.5	4.5	4.5	4.5	4.5	4.5	8.33
Africa (CAF)	54	5	5	5	5	5	5	5	9.33
North America (CONCACAF)	35	3	3	3.5	3.5	3.5	3.5	3.5	6.67
South America (CONMEBOL)	10	5	4.5	4.5	4.5	4.5	4.5	4.5	6.33
Oceania (OFC)	11	0.5	0.5	0.5	0.5	0.5	0.5	0.5	1.33
Europe (UEFA)	55	14	14.5	13	13	13	13	13	16
Host (Confederation)		1 (UEFA)	2 (AFC)	1 (UEFA)	1 (CAF)	1 (CONMEBOL)	1 (UEFA)	1 (AFC)	3 (CONCACAF)
Total	**211**	**32**	**32**	**32**	**32**	**32**	**32**	**32**	**48**

Notes: The allocation of slots includes intercontinental playoffs slots that are represented by decimal parts. For the 2026 FIFA World Cup, we add to the FIFA's approved allocation the remaining two slots from intercontinental playoffs to different confederations (0.33 slots to Africa, Asia, Oceania, and South America; 0.66 slots to North America; zero slots to Europe).

The 2026 FIFA World Cup will be the first edition with three host countries (Canada, Mexico, and USA). The united bid anticipated that all three host countries would be awarded automatic places, but this has not yet been resolved and will be decided by the FIFA council. In addition, the 2026 FIFA World Cup will be the first tournament in which all six confederations have guaranteed slots, as the ratification gives Oceania a guaranteed slot in the final tournament for the first time in FIFA World Cup history. In Table 1, that presents the allocation of slots per confederation starting from 1998, we see that teams from Oceania always had to participate in the intercontinental playoff.

[2] Due to the expansion in the number of participants, the group allocation policy will also change in the 2026 FIFA World Cup, which will contain groups with three teams instead of the usual four. This reform has already been extensively studied in the literature, mostly focusing on minimizing the risk of collusion (e.g., Chater et al., 2021; Guyon, 2020; Palacios-Huerta, 2018).

[3] The abbreviations in the parentheses refer to the official names of each confederation. In 2026, a playoff tournament involving six teams will be held to decide the last two FIFA World Cup slots, consisting of one team per confederation (except for UEFA) and one additional team from the confederation of the host countries (i.e., CONCACAF). In the 2022 FIFA World Cup, the last two FIFA World Cup slots were decided by means of two intercontinental playoff ties, involving one team per confederation from the following confederations: AFC, CONMEBOL, OFC, and CONCACAF.

The aim of this paper is to scrutinize such a slot allocation resorting to well-known tools from the literature on fair allocation. It goes without saying that fairness is a complex notion and that, as such, there are many plausible ways to approach it. That is the reason why we shall be offering a menu of options, each reflecting different normative principles on which they are grounded, rather than just one. Depending on the normative principles one might want to endorse, we would back up one option or another.

Our benchmark setting is the so-called problem of adjudicating conflicting claims, formalized by O'Neill (1982) with a simple and elegant model that has generated a sizable literature ever since (e.g., Thomson, 2003, 2015, 2019). In such a model, a set of agents hold claims against an endowment that is insufficient to fully honor all the existing claims. Rules that can be traced back to Aristotle and the Talmud can be considered to solve those problems. Sometimes, these rules are extended to account for *baselines* that might complement claims to describe relevant aspects of agents involved in the problems (e.g., Hougaard et al., 2012).

In our setting, the agents will be the continental confederations, and the endowment will be the slots for the World Cup to be allocated among them. The baseline will refer to the slots obtained by each confederation in the status quo (with only 31 slots that do not include the host nation, which qualifies automatically to the finals). We may want to respect the status quo or to ignore it altogether.

The claims will be based on confederation strengths in terms of soccer ability. Such strengths are obtained from the FIFA ranking of countries and its alternative, the Elo ranking. Both rankings have been used in previous literature to predict the outcome of soccer games (e.g., Hvattum & Arntzen, 2010; Krumer & Lechner, 2017; Lasek et al., 2013; Peeters, 2018; Wunderlich & Memmert, 2016). Lasek et al. (2013) showed that the Elo ranking had a better

predictive power than the FIFA ranking.[4] Nevertheless, our goal is not to determine which ranking is more appropriate.[5] Instead, we use both (Elo and FIFA) rankings to explore how robust our slot allocation methods are. For instance, we find that the methods that respect the status quo as a tentative allocation of the first 31 slots (and thus focus on allocating the extra slots) minimize the gap between the allocations arising from Elo and FIFA.

A main effect of our research is to be able to compare the set of possible allocations we propose (which are grounded on the fair allocation literature) to FIFA's proposal. Our results convey that in only one of the ten different allocations, Europe received less than 16 slots, as approved by FIFA. This allocation happens to be the one with the largest gap between the FIFA and Elo methods, and it also conveys that some confederations receive less slots than in the status quo (before the increase of available slots). In other allocations, Europe receives between 16.96 and 27.43 slots. When looking only at the methods preserving the status quo, the range of Europe's slots varies between 17.60 and 23.47. Thus, we can safely conclude from our analysis that Europe has a solid basis to claim for additional slots. One possible solution is to assign to Europe the two remaining slots that are assigned via playoffs.

We conclude this introduction referring to the related literature. First, we consider two somewhat related papers to ours. Recently, Csató (2022b) has investigated the fairness of the 2018 FIFA World Cup qualifying competition via Monte Carlo simulations. He finds, for instance, that a South American team could have tripled its chances by playing in Asia. These results might also suggest possible reallocations of the qualifying slots. Similarly, Stone and Rod (2016) argue that the current system of the FIFA World Cup qualification does not ensure

[4] Note that the FIFA World Ranking has been seriously revised in 2018 after the 2018 FIFA World Cup (e.g., FIFA, 2018). Since then, it also uses the Elo method of calculation.

[5] Peeters (2018) showed that Transfermarkt values obtained from a popular website (www.transfermarkt.com) outperform both FIFA and Elo rankings in predicting results of international soccer games. Unfortunately, our dataset does not provide reliable information on these values for many countries and, thus, we shall not use them in our analysis.

the best 32 teams in the world make it and, perhaps more importantly, that the allocation is not grounded on fairness pillars.

Second, we refer to the larger body of literature referring to the economic design of sporting contests (e.g., Szymanski, 2003). In general, this literature is concerned with diverse issues, ranging from the optimal number of participants in a (sports) competition to the optimal structure of prizes, competitive balance, or the quotas of qualifying teams associated to international competitions (that we consider in this paper). It is well known that the design of a sporting contest bears a close relationship to the design of an auction (e.g., Hillman and Riley, 1989). As mentioned by Szymanski (2003), "*in both cases, the objective of the organizer is to elicit a contribution (a bid, an investment, or some effort) from contestants who may as a result win a prize*."

Third, we make a special emphasis on the fair allocation literature we appeal to throughout this paper. A variety of formal criteria of fair allocation have been introduced in economic theory (e.g., Thomson, 2011). These criteria have broad conceptual appeal, as well as significant operational power, and have contributed considerably to our understanding of normative issues concerning the allocation of goods and services. The pioneering criterion was envy-freeness (e.g., Foley, 1967), which simply says that no agent should prefer someone else's assignment to his own. Other criteria formalizing ethical principles such as impartiality, priority, or solidarity have also played an important role in deriving fair allocation rules (e.g., Moreno-Ternero and Roemer, 2006, 2012). As mentioned above, our benchmark model will be based on the claims problem formalized by O'Neill (1982). Among other things, he singles out the so-called proportional rule to solve these problems. Ju et al. (2007) went further showing that such a rule is, essentially, the only one that is immune to manipulations via merging or splitting agents' claims. Hougaard et al. (2012) extended O'Neill's model to account for baselines in a first stage and allocate the residual in a second stage.

The remainder of the paper is organized as follows. Section 2 presents the benchmark model along with the FIFA and Elo ranking methods (the measures of strength we consider in our analysis). The empirical application, which gives rise to the allocations we propose for the allocation of FIFA World Cup slots, is contained in Section 3. In Section 4, we offer discussion and concluding remarks.

2 The benchmark model

Let N describe a finite set of agents (confederations in our case). Let E describe an endowment (slots for the World Cup) to be allocated among agents in N. Each agent $i \in N$ is characterized by a duplet (b_i, c_i), dubbed the agent's *baseline* and *claim*, respectively. The baseline refers to the amount (slots) each agent had in the status quo (the allocation of a previous smaller endowment). The claim refers to a quantitative measure of the strength of each agent. We shall explore in our empirical application several ways to define claims. If b denotes the baseline profile and c the claim profile, the problem is fully described by (N, E, b, c).

A rule is a mapping that associates with each problem an allocation indicating the amount from the endowment each agent receives. We impose from the outset an efficiency condition indicating that the whole endowment is allocated, and a boundedness condition indicating that no agent can receive a negative amount or an amount above its members.

A focal rule, which can be traced back to Aristotle, awards agents proportionally to claims. Formally,

Proportional. $P(N, E, b, c) = \frac{E}{\sum_i c_i} c$.

An alternative rule that can be traced back to the Talmud suggests equalizing awards (resp. losses) as much as possible, provided the amount to divide falls short (resp. exceeds) one half of the aggregate claim. This is, essentially, the only consistent rule that guarantees meaningful lower and upper bounds for all agents (e.g., Moreno-Ternero & Villar, 2004). Nevertheless, it requires that claims and endowments refer to the same commodity (e.g.,

money), which would render its use controversial in our setting (in which claims refer to the strength of confederations and endowment to slots).

The previous rules dismiss baselines to solve problems. A natural way to account for them is proposed by Hougaard et al., (2013a, 2013b). Therein, it is suggested to first assign agents their baselines, and to allocate the resulting deficit, or surplus, using a standard rule for the standard problem that results after embedding baselines into claims. Specifically, a deficit is allocated according to the amounts already received by the agents, whereas a surplus is allocated according to the gap between their claims and what has already been allocated to them. This is reminiscent to the idea of composition, with a long tradition in axiomatic work (e.g., Young, 1988; Moulin, 2000; Moreno-Ternero & Roemer, 2012).[6] In our case, baselines refer to the status-quo allocation and, therefore, the issue is to allocate the surplus (extra slots). To do so, claims must be adjusted down to properly account for the allocation of baselines in the first step. This gives rise to a two-stage version of the proportional rule introduced above. Formally,

Two-stage proportional. $P(N, E, b, c) = b + \frac{E - \sum_i b_i}{\sum_i \hat{c}_i} \hat{c}$, where $\hat{c} = c - b$.

To define claims, we use quantitative measures of the strength of each confederation. For that, we use two different types of ranking systems as described below.

Our first measure of strength is based on the FIFA ranking of each national team, which is the official ranking FIFA considers for the seeding procedure in different tournaments. Over the years this ranking was based on several models. The most recent one was approved in 2018. It relies on adding/subtracting the points associated to a game to/from the previous point totals,

[6] Its relative in the theory of axiomatic bargaining is the so-called "step-by-step negotiations" axiom, which is the basis for the characterization of the egalitarian solution in such a context (e.g., Kalai, 1977).

rather than averaging game points over a given period (which was the case in the previous version of the ranking).[7]

The Elo ranking was developed by Dr. Arpad Elo, and is mainly used by FIDE, the international chess federation, to rate chess players.[8] In addition to the FIFA rankings, it also takes home advantage and goal difference into account.

The FIFA World Ranking and the World Football Elo rating are both standard measures of quantitative studies on the FIFA World Cup (e.g., Csató, 2022a).

3 Empirical application

Dataset

To rule out a situation where one specific ranking does not represent the real strengths, for both ranking methods, we use the last published annual rankings in the years between 1998 and 2021. The reason for choosing 1998 as the first year for our analysis is because it was the first year with 32 participants in the FIFA World Cup (prior to that year, the FIFA World Cup only had 24 participants). We started working on data analyses on November 13th, 2021, whose Elo rankings were used for the year of 2021. The last available FIFA rankings list is from October 21st, 2021.

Objective

The following expansion of the FIFA World Cup is scheduled for 2026, with 48 participating teams. The question is how to allocate those slots to the different confederations. We shall address this question in two different ways. In each case, $E=48$ is the endowment, and the set N refers to the set of six confederations: Europe (UEFA), Asia (AFC), North, Central

[7] For additional information, see https://digitalhub.fifa.com/m/f99da4f73212220/original/edbm045h0udbwkqew35a-pdf.pdf. Last accessed on 15/10/2022. The reader is also referred to Cea et al. (2020), Csató (2021, Chapter 1.4) and Csató (2022a) for further insights on the methodology and shortcomings of the FIFA ranking. Kaminski (2022) is another recent paper that analyses the flaws of the previous FIFA World Ranking with an emphasis on the so-called host paradox (the dramatic underrating of the host(s) of major tournaments).

[8] For additional information, see https://eloratings.net/about. Last accessed on 15/10/2022.

America and Caribbean (CONCACAF), Africa (CAF), South America (CONMEBOL), and Oceania (OFC).

Now, in the first case, we shall ignore the current allocation for the 2022 FIFA World Cup (which will have 32 participating teams), whereas in the second case we shall take it into account. That is, in the first case, we shall ignore the existence of baselines (and will therefore use the proportional rule formally described above), whereas in the second case we shall consider the allocation of slots for the 2022 FIFA World Cup as the baseline profile (and will therefore use the two-stage proportional rule formally described above). In other words, in the second case, we keep baselines as a tentative allocation and add the allocation of the extra slots to it once claims are revised down accordingly (as explained above).

Allocation of slots without baselines

As for claims, we shall consider several options, gathered in Table 2. To begin with, we shall distinguish between FIFA ranking and Elo ranking (Panels A and B, respectively). Column (1) collects the sum of coefficients of all countries in each confederation across years in our database.[9] Column (2) truncates this sum to consider only the top 48 countries at the end of each year. Both columns can thus be considered as alternative definitions of claims. Now, Column (3) collects the average annual number of teams each confederation has in the top 31 (we exclude the slot assigned to the host in the status quo, before the World Cup extensions). And Column (4) collects the average annual number of teams each confederation has ranked between 32 and 48. Thus, these two last columns can be interpreted as possible allocations for the first 31 slots and the last 17 slots, respectively.

[9] Note that we have also considered for our computations countries that appeared after 1998. As the decision to include these countries was ultimately made by FIFA (mostly based on geopolitical reasons), we did not consider the possible manipulation that might emerge from this move benefitting some confederations. We should also mention that these new countries are rather weak (in terms of football outcomes) and that, therefore, adding their coefficients to a confederation might add little to make a difference.

Table 2: Coefficients and the number of top teams per continent

Panel A: Based on FIFA rankings				
	(1)	(2)	(3)	(4)
Confederation	Sum of coefficients of all countries	Sum of coefficients of the top 48 countries	Average annual number of teams in the top 31	Average annual number of teams ranked 32-48
Africa	621551	136829	3.17	4.13
Asia	437023	66492	1.42	2.25
Europe	918872	596794	18.54	7.88
North America	335670	72297	2.38	1.13
Oceania	76759	1246	0.00	0.13
South America	211248	170664	5.50	1.54
Total	**2601123**	**1044322**	**31**	**17**
Panel B: Based on Elo rankings				
Confederation	Sum of coefficients of all countries	Sum of coefficients of the top 48 countries	Average annual number of teams in the top 31	Average annual number of teams ranked 32-48
Africa	1747405	186800	1.17	3.38
Asia	1302340	177436	2.13	2.13
Europe	2126070	1191449	19.21	7.96
North America	1113595	152713	2.08	1.50
Oceania	355810	14354	0.33	0.00
South America	366210	362124	6.17	2.08
Total	**7011430**	**2084876**	**31**	**17**

Allocation of slots with baselines

Table 3 gathers numbers to construct another overall allocation of the 48 slots. As with Table 2, Panels A and B refer, respectively, to the FIFA ranking and the Elo ranking. Column (1) reflects the allocation of the 2022 FIFA World Cup (without the host nation), which we take as the status quo. Column (2) yields an additional allocation based on Column (3) of Table 2, i.e., the number of teams per confederation in the top 31. In other words, a confederation gets an extra allocation only if its average annual number of countries within the top 31 is larger than the corresponding number in the status quo. Column (3) is the result from aggregating the previous two columns. Column (4) yields the proportional allocation of the remaining slots (10.46) based on Column (4) of Table 2, i.e., the number of teams per confederation ranked between 32 and 48. Finally, Column (5) is the result from aggregating the third and the fourth columns, to construct a final allocation of the 48 slots.

This allocation in Column (5) at Table 3 is not made of integer numbers. The decimal parts should be interpreted as slots in a further qualifying round.[10] For instance, in the case of the FIFA rankings, for which the suggested allocation is 7.53 for Africa, 5.88 for Asia, 23.37 for Europe, 4.19 for North America, 0.58 for Oceania and 6.45 for South America, we could consider a final play-off round in which the last two slots would be awarded and in which the participants would be 1 team from North America, 2 teams from Europe and South America, and 3 teams from Africa and Oceania.

Table 3: Status Quo + allocation of extra slots based on the number of teams ranked 32-48

Panel A: Based on FIFA rankings

	(1)	(2)	(3)	(4)	(5)
Confederation	A Status Quo in 2022	B Extra allocation	A+B=C Intermediate allocation	D Allocation of the remaining slots	C+D Status Quo + new allocation
Africa	5	0	5	2.53	7.53
Asia	4.5	0	4.5	1.38	5.88
Europe	13	5.54	18.54	4.83	23.37
North America	3.5	0	3.5	0.69	4.19
Oceania	0.5	0	0.5	0.08	0.58
South America	4.5	1.00	5.50	0.95	6.45
Total	**31**	**6.54**	**37.54**	**10.46**	**48**

Panel B: Based on Elo rankings

Confederation	A Status Quo in 2022	B Extra allocation	A+B=C Intermediate allocation	D Allocation of the remaining slots	C+D Status Quo + new allocation
Africa	5	0	5	1.81	6.81
Asia	4.5	0	4.5	1.14	5.64
Europe	13	6.21	19.21	4.26	23.47
North America	3.5	0	3.5	0.80	4.30
Oceania	0.5	0	0.5	0	0.50
South America	4.5	1.67	6.17	1.12	7.28
Total	**31**	**7.88**	**38.88**	**9.13**	**48**

Notes: Column (1) presents the 2022 FIFA World Cup allocation (without host). Colum (2) presents the additional slots based on the number of teams per confederation in the top 31 (only if larger than the status quo). Column (3) sums up the slots in Columns (1) and (2). Column (4) allocates the remaining slots based on the number of teams per continent ranked between 32 and 48. Column (5) presents the final allocation.

Table 4 gathers numbers to construct another (two-stage) allocation of the 48 slots. Again, Panels A and B refer, respectively, to the FIFA ranking and the Elo ranking. As with Table 3, Column (1) reflects the allocation of the 2022 FIFA World Cup (without the host nation), which

[10] This also applies for the proposed allocations with decimals in the remaining tables.

we take as the status quo. Column (2) gathers the adjusted claims. That is, we reduce the claims from Column (1) of Table 2, by presenting the sum of coefficients of all the teams per confederation, after subtracting the top number (according to the status quo) of teams. Column (3) yields the proportional allocation of the remaining slots (17) according to the numbers in the previous column. Finally, Column (4) is the result from aggregating the first and third columns, to construct a final allocation of the 48 slots. That is, the allocation obtained from the two-stage proportional rule formally defined in the previous section.

Table 4: Status Quo + allocation of extra slots based on coefficients of teams ranked below status quo

Panel A: Based on FIFA rankings

	(1)	(2)	(3)	(4)
Confederation	A Status Quo in 2022	B Sum of coefficients of ranked below status quo	C Allocation of the remaining slots based on B	A+C Status Quo + new allocation
Africa	5	526119	4.66	9.66
Asia	4.5	359547	3.19	7.69
Europe	13	599720	5.31	18.31
North America	3.5	265344	2.35	5.85
Oceania	0.5	70905.5	0.63	1.13
South America	4.5	96915.5	0.86	5.36
Total	**31**	**1918551**	**17**	**48**

Panel B: Based on Elo rankings

Confederation	A Status Quo in 2022	B Sum of coefficients of ranked below status quo	C Allocation of the remaining slots based on B	A+C Status Quo + new allocation
Africa	5	1546790	4.65	9.65
Asia	4.5	1118772	3.36	7.86
Europe	13	1530888	4.60	17.60
North America	3.5	965514.5	2.90	6.40
Oceania	0.5	337427.5	1.01	1.51
South America	4.5	158446	0.48	4.98
Total	**31**	**5657838**	**17**	**48**

Notes: In Column (1), we present the allocation of the 2022 FIFA World Cup (without the host nation). In Colum (2), we present the sum of coefficients of all the teams per confederation except for the coefficients of the top number of teams according to the status quo (for half teams we reduce by half the sum of the coefficients of the respective team). In Column (3), we allocate the remaining slots based on the sum of coefficients presented in Column (2). In Column (4), we present the final allocation of teams.

Table 5: Summary of allocation of slots based on different methods

Panel A: Based on FIFA rankings

	(1)	(2)	(3)	(4)	(5)
Confederation	Simple proportion based on the sum of coefficients	Simple proportion based on coefficients in the top 48	Average annual number of teams in the top 48	Status Quo + new allocation of ranked 33-48	Status Quo + new allocation based on coefficients of teams below status quo
Africa	11.47	6.29	7.29	7.53	9.66
Asia	8.06	3.06	3.67	5.88	7.69
Europe	16.96	27.43	26.42	23.37	18.31
North America	6.19	3.32	3.50	4.19	5.85
Oceania	1.42	0.06	0.13	0.58	1.13
South America	3.90	7.84	7.04	6.45	5.36
Total	**48**	**48**	**48**	**48**	**48**

Panel B: Based on Elo rankings

Confederation	Simple proportion based on the sum of coefficients	Simple proportion based on coefficients in the top 48	Average annual number of teams in the top 48	Status Quo + new allocation of ranked 33-48	Status Quo + new allocation based on coefficients of teams below status quo
Africa	11.96	4.30	4.54	6.81	9.65
Asia	8.92	4.09	4.25	5.64	7.86
Europe	14.56	27.43	27.17	23.47	17.60
North America	7.62	3.52	3.58	4.30	6.40
Oceania	2.44	0.33	0.33	0.50	1.51
South America	2.51	8.34	8.28	7.28	4.98
Total	**48**	**48**	**48**	**48**	**48**

Notes: The allocation in Column (1) is based on proportions derived from the sum of coefficients of all the ranked countries from Column (1) of Table 2. The allocation in Column (2) is based on proportions derived from the sum of coefficients of the top 48 countries from Column (2) of Table 2. The allocation in Column (3) is based on the average number of teams each confederation had in the top 48, which is a sum of Columns (3) and (4) of Table 2. The allocation in Column (4) is the allocation presented in Column (5) of Table 3. The allocation in Column (5) is the allocation presented in Column (4) of Table 4.

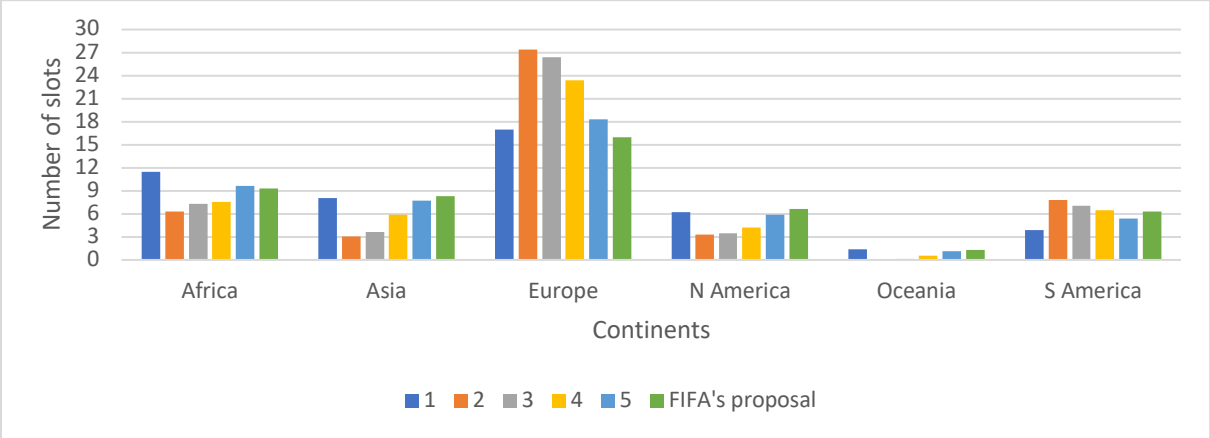
Comparison between different allocations

Table 5 summarizes all the allocations we have constructed. Column (1) yields the resulting proportional allocation, based on the sum of coefficients of all the countries within each confederation (thus, ignoring the status quo as a tentative allocation of the first 31 slots).[11] Column (2) yields the resulting proportional allocation, but now based on the sum of coefficients of only the top 48 countries (thus, also ignoring the status quo as a tentative allocation of the first 31 slots). Column (3) yields the resulting (two-stage) allocation obtained

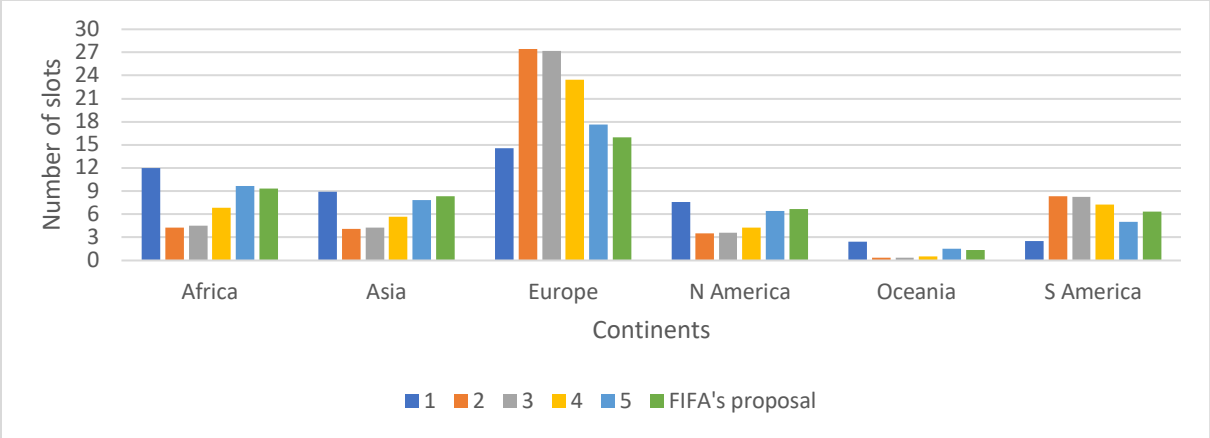
[11] Note that South America is punished in this column because it contains the fewest countries, although there are many strong teams among them.

from aggregating Columns (3) and (4) from Table 2. That is, the first 31 slots are allocated according to the average number of teams each confederation had in the top 31, and the next 17 slots are allocated according to the average number of teams each confederation had between the 32nd and 48th position. Finally, Columns (4) and (5) yield, respectively, the (two-stage) allocations presented in the last columns of Tables 4 and 5 (which were both applying the two-stage proportional rule, endorsing the status quo as a tentative allocation of the first 31 slots). We see that in Columns (1)-(3) of both Panels, there is at least one confederation whose number of slots is below the baseline, whereas in Columns (4)-(5), such a situation is not possible due to status quo allocation of the first 31 slots.

Figure 1: Allocation of slots according to our and FIFA's proposals
Panel A: Based on FIFA rankings



Panel B: Based on Elo rankings



Notes: Allocations 1-5 represent the allocations presented in Table 5. FIFA's proposal is the one presented in the last column of Table 2.

In Panels A and B of Figure 1, we visually summarize the five different allocations presented in Table 5 for FIFA and Elo rankings respectively.[12] In addition, for the convenience of comparison, we present the FIFA's proposed allocation appearing in the last column of Table 1. We see that according to 9 out of 10 of our allocations, Europe should have received more slots than the FIFA's proposal. On the contrary, in 9 out of 10 of our allocations, Asia should have received less slots than in FIFA's proposal. Similarly, in 8 out of 10 of our allocations, North America should have received less slots than in FIFA's proposal.

Table 6 collects the (absolute) differences each of the above five allocations yield for both rankings (FIFA and Elo). We observe from there that the last two allocations (namely, those that endorse the status quo as a tentative allocation of the first 31 slots) minimize such a gap. Thus, both allocations can be defended on the grounds of better compromising between both rankings. On the other hand, the first allocation (proportional to the sum of coefficients of all the countries within each confederation) is the one reflecting the largest difference between both rankings.

Table 6: Absolute differences between the methods

	(1)	(2)	(3)	(4)	(5)
Confederation	Simple proportion of all the countries	Simple proportion based on coefficients in the top 48	Average annual number of teams in the top 48	Status Quo + new allocation of ranked 33-48	Status Quo + new allocation based on coefficients of teams below status quo
Africa	0.49	1.99	2.75	0.72	0.01
Asia	0.85	1.03	0.58	0.24	0.18
Europe	2.40	0.00	0.79	0.10	0.71
North America	1.43	0.19	0.08	0.11	0.55
Oceania	1.02	0.27	0.21	0.08	0.39
South America	1.39	0.49	1.13	0.84	0.38
Total	**7.59**	**3.98**	**5.54**	**2.09**	**2.22**

Notes: In this table, we present the absolute difference between the allocations based on FIFA and Elo ranking systems.

[12] Note that the difference in the allocations is driven mostly by the allocation mechanism rather than the measure of strength (FIFA World Ranking or Elo).

Finally, Table 7 computes the gap between each of the ten allocations and FIFA's proposal. From this table, we see that the allocation that minimizes the gap with FIFA's allocation is the one presented in the last column of Panel B. It is based on the two-stage proportional rule, respecting the status quo as a tentative allocation of the first 31 slots, and allocating the additional 17 slots based on the coefficients of the remaining teams ranked below the status quo allocation of each confederation. According to this method, the continent with the largest deviation from FIFA's proposal is Europe whose deviation is 1.60 slots. Beyond the fact that this method is the closest to the actual FIFA's proposal and second-best in minimizing the gap between the FIFA and Elo rankings, it also assures that all six confederations have guaranteed slots in the World Cup, which is one of the goals FIFA was pursuing.

Table 7: Absolute differences between the methods and the FIFA's proposal

Panel A: Based on FIFA rankings					
	(1)	(2)	(3)	(4)	(5)
Confederation	Simple proportion based on the sum of coefficients	Simple proportion based on coefficients in the top 48	Average annual number of teams in the top 48	Status Quo + new allocation of ranked 33-48	Status Quo + new allocation based on coefficients of teams below status quo
Africa	+2.14	-3.04	-2.04	-1.80	+0.33
Asia	-0.27	-5.27	-4.66	-2.45	-0.64
Europe	+0.96	+11.43	+10.42	+7.37	+2.31
North America	-0.48	-3.35	-3.17	-2.48	-0.82
Oceania	+0.09	-1.27	-1.20	-0.75	-0.20
South America	-2.43	+1.51	+0.71	+0.12	-0.97
Abs Total	**6.37**	**25.87**	**22.2**	**14.97**	**5.27**
Panel B: Based on Elo rankings					
Continent	Simple proportion based on the sum of coefficients	Simple proportion based on coefficients in the top 48	Average annual number of teams in the top 48	Status Quo + new allocation of ranked 33-48	Status Quo + new allocation based on coefficients of teams below status quo
Africa	+2.63	-5.03	-4.79	-2.52	+0.32
Asia	+0.59	-4.24	-4.08	-2.69	-0.47
Europe	-1.44	+11.43	+11.17	+7.47	+1.60
North America	+0.95	-3.15	-3.09	-2.37	-0.27
Oceania	+1.11	-1	-1	-0.83	+0.18
South America	-3.82	+2.01	+1.95	+0.95	-1.35
Abs Total	**10.54**	**26.86**	**26.08**	**16.83**	**4.19**

Notes: In this table, we present the differences between the allocations presented in Table 5 and the FIFA's approval presented in the last column of Table 1.

4 Discussion and concluding remarks

We have studied in this paper the allocation of slots for the upcoming editions of the FIFA World Cup. We have focused on a variety of options with strong normative grounds. Common to all of them is the principle of proportionality, with a long tradition of use that can be traced back to Aristotle. In his maxim, sometimes called the formal principle of distributive justice (e.g., Young, 1994): "Equals should be treated equally, and unequals unequally, in proportion to the relevant similarities and differences". We have considered here alternative ways of defining relevant similarities and differences, hence giving rise to alternative allocations (once the principle of proportionality was appealed to).

Our work is motivated by FIFA's decision to increase the number of slots for upcoming editions of the FIFA World Cup. Thus, a relevant aspect of our analysis is the dichotomy arising from choosing to preserve the status quo (to be interpreted as the allocation of the actual number of slots) or not. If the choice is yes, then the problem amounts to allocate the extra slots, with the proviso that the worst that might happen to a confederation is to get zero extra slots. That is, no confederation can be worse off after the number of available slots increases.[13] If the choice is no, the status quo is ignored, and the problem boils down to allocate the overall number of available slots, which thus might make some confederations worse off than in the status quo.

In addition, we have seen that the allocation that gets closer to the actual proposal made by FIFA is the one that is based on teams' coefficients that are ranked below the status quo allocation (presented in Table 4 and Column 5 of Table 5). Moreover, it is possible to reduce this gap even more. This can be done by allocating two additional slots from the playoffs to Europe that has the highest gap between the allocation proposed by FIFA and our proposal.

[13] This is normally formalized by the axiom of resource monotonicity in normative work (e.g., Roemer, 1986). It is connected to the principle of solidarity (e.g., Moreno-Ternero & Roemer, 2006). In this setting, it renders the process immune to the so-called Alabama paradox in apportionment problems (e.g., Balinski and Young, 1982; Young, 1994).

Another possibility is to allocate one slot to Europe, and the remaining slot would be given to the winner of the playoffs, whose participants would be allocated according to the surplus each confederation has above the assigned slots. In such a case, this playoff would consist of two European and one African national team (note that all the rest, except for Oceania, which only has a surplus of 0.18, according to the Elo ranking, have more approved slots compared to the proposition presented in Table 4).

Another important dichotomy in our analysis is the use of FIFA or Elo ranking systems to construct the claims associated to confederations. We have studied how robust our allocations are to the choice of rating method. Those preserving the status quo are naturally more robust. Note that an alternative method for confederations' claims could be based on the number of active players or TV revenues generated by confederations. Unfortunately, these data are outside of our reach.

As mentioned above, our paper could be considered as a step towards the ambitious goal of deciding how to allocate slots to a sports tournament when there are more applicants than the tournament can handle. Another step would be dealing with the somewhat related problem of the allocation of slots among the European countries in the three European club football competitions (UEFA Champions League, UEFA Europe League, UEFA Europa Conference League).[14] Our paper offers a guideline to take that step too. First, one would need to determine the claims of the participating agents (European countries in that case) via the relevant UEFA teams' rankings or Elo rankings. Then, one would need to take a stance with respect to considering the current allocation as a status quo (baseline) or not. Finally, one would allocate (overall or residual) slots proportionally, converting decimals into slots for a prior play-off qualification stage.

[14] Csató (2022c) has recently discussed the effects of a reform in the Champions League qualification.

Finally, we acknowledge that the increase in the number of slots likely generates larger revenues. Much of the revenue raised in sporting contests (and the FIFA World Cup is not an exception) comes from broadcasting. This renders the ensuing revenue sharing process (e.g., Bergantiños & Moreno-Ternero, 2020) extremely relevant. We leave for further research to explore the interplay between such a revenue sharing process and the allocation of extra slots at the FIFA World Cup we have studied here.

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