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A note on the measurement of poverty persistence

Antonio Villar
Universidad Pablo de Olavide

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Department of Economics

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Antonio Villar
Universidad Pablo de Olavide & ISEAK

Abstract

We propose in this paper a poverty index that considers poverty persistence as an integral part of poverty measurement. Poverty is regarded as a social welfare loss in a multiperiod scenario. Using familiar tools (logarithmic utilities and a utilitarian social welfare function) we obtain a poverty index which is mathematically simple, easy to interpret, that can be decomposed into incidence, intensity, and inequality, and is additively decomposable by population subgroups. It consists of the log of the geometric mean of individual intertemporal utility losses.

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1 Introduction

The cumulative effects of poverty spells on households are widely recognized (e.g., Bane & Ellwood, 1986). Poverty in the long run negatively affects basic aspects of personal and social life, including health, employment, education, the prospects of the household, and social integration. Yet, intertemporal poverty is less commonly studied, mostly due to data restrictions (Fusco & van Kerm, 2023). The situation is changing, though, and now there is panel data information regularly available in several countries (e.g., the EU-SILC contains a four-year panel), and administrative data are becoming increasingly accessible and tractable. As pointed out by Gradín, del Río & Cantó (2012), "The increasing availability of longitudinal data in a variety of countries has stimulated advances in undertaking a more dynamic view of the [poverty] issue".

We understand that poverty duration is a key element in the appraisal of the welfare cost of poverty, that "time is an important additional dimension for understanding poverty and informing policy design" (Foster & Santos, 2012). To stress this feature note that reductions in any conventional poverty index are compatible with increments in the average duration of poverty among the poor. This phenomenon would induce the consolidation of poverty among a part of the population, which will not be captured those measures. So, we might be happy with poverty reductions, while ignoring the progressive marginalization of the less favoured households. In short, Sen (1976) three I's of poverty (incidence, intensity, and inequality) need be complemented by a measure of persistence.

The literature has already dealt with computing past poverty spells from different perspectives, which can be roughly grouped into three lines of research. One refers to the dynamic analysis of transitions between poverty and non-poverty, focusing on the features that affect the probability of those transitions (e.g., Huff Stevens, 1998). A second line deals with chronic poverty (e.g., Foster 2009, Calvo & Dercon 2009, Hoy & Zheng, 2011, Foster & Santos 2012, Alkire et al 2017), which implies defining an additional poverty threshold to identify chronically poor as those who have been poor during a given number of periods. And the third one, focusing on poverty persistence, aims at incorporating duration into the standard poverty measurement (e.g., Bossert, Chakravarty & D'Ambrosio, 2008, Gradín, del Río & Cantó, 2012 -which includes a detailed discussion of the topic up to that date). See Nicholas & Ray (2023), Fusco & van Kerm (2023) for a recent review of that literature.[1]

[1] As discussed in Gradín, del Río & Cantó (2012) and commented in Fusco & Kerm (2023), one can distinguish another line of research that considers different income components, permanent and transitory, and provides measures of chronic versus transitory poverty (e.g., Jalan & Ravallion, 1998).

Our contribution lies within the third line of research. We present here an elementary way of approaching poverty measurement that considers duration as one of its arguments. To do so we adopt a social welfare approach and obtain the welfare cost of poverty by aggregating the disutility of poor individuals (in line with the proposal in Villar 2023). We conceive here poverty in its simplest formulation, namely, a person is poor when his/her income falls short of a given money threshold, the poverty line. The (socially inadmissible) utility loss of a poor individual, for a given time span, is given by the discounted cumulative effect of the differences between the utility of achieving the poverty line and the utility of his/her actual income within each period. Using a utilitarian social welfare function, we obtain a poverty index that has an elementary mathematical structure, it is easy to interpret, can be decomposed into the three I's of poverty, incorporates structurally poverty persistence, and it is additively decomposable by populations subgroups.

The social welfare approach to economic problems involves, implicitly or explicitly, the assumption of some kind of interpersonal comparability. There are different interpretations of what that means (or who does the interpersonal comparison), which are outside the scope of this paper. Interpersonal comparability is introduced here in a crude but transparent way: we assume that all agents have the same utility function, which is increasing and concave, and only depends on their individual incomes. In our interpretation the utility loss of a poor agent is a normative appraisal the contribution to the welfare cost of poverty, rather than a description of a subjective perception.

The paper is organized as follows. Section 2 presents the poverty index and Section 3 closes with some remarks. There is no characterization of our measure, as all its elements are very familiar beasts (logarithmic utilities and utilitarian social welfare).

2 The model

Consider a society with n members, $N = \{1,2, \dots, n\}$ at a given point in time, that we call *today*. Those members, typically households or individuals, will be referred to as *agents*. An income distribution today is a vector $\boldsymbol{x} \in \mathbb{R}^n_{++}$. We denote by a scalar $z > 0$ today's poverty threshold and by $Q \subset N$ the corresponding set of the poor in this society. That is, the set of $h \in N$ with $x_h < z$, where $x_h > 0$ is the income of agent h today. Without loss of generality, we assume that the income of the poor is described by a sub-vector of $\boldsymbol{x}$ denoted by $\boldsymbol{y} \in \mathbb{R}^q_{++}$, i.e., $\boldsymbol{x} = (\boldsymbol{y}, \boldsymbol{x}_{-\boldsymbol{y}})$, where q stands for the number of the poor (the cardinal of Q).

Let $T + 1$ the length of the time span we use in our analysis. We denote by $t = 0,1,2, \dots, T$ the different periods, where the number tells us the distance from today (that is, $t = 1$ stands for "yesterday", $t = 2$ for the previous day, and so forth up to $t = T$). We can think of a time span of four or five years, to fix ideas. To save notation we shall avoid subindices in today's values, rather than labelling them with zeroes. For each agent $h \in Q$ there is a vector $\boldsymbol{y}_h = (y_h, y_h(1), \dots, y_h(T))$ that describes the income obtained in each of the periods considered. That information can be summarized by a matrix $\boldsymbol{Y} = (\boldsymbol{y}_1, \dots, \boldsymbol{y}_q) \in \mathbb{R}_{++}^{q\times(T+1)}$. As poverty lines may change along the time span, let $\boldsymbol{z} = (z, z_1, \dots, z_T)$ denote the corresponding vector of poverty thresholds.

The key idea is to measure poverty as a social utility loss, by aggregating individual utility losses of those who are poor today.[2] To do so we adopt three conventional assumptions:

(i) All agents have the same utility function, which only depends on their own incomes within each period. That is, $u_h^t(\boldsymbol{y}(t)) = u^t(y_h(t)), t = 0,1, \dots, T$. The index t in the utility function indicates that utility may be time dependent.

(ii) $u^t(y_h(t)) = \delta^t u(y_h(t))$, where $\delta \in [0,1]$ is a discount factor that gives us today's value of the utility of $y_h(t)$ at time t (we can think of this parameter as a function of the "interest rate", that is, $\delta = \frac{1}{1+r}$).

(iii) The utility function is increasing and concave and, in particular, its changes are inversely proportional to its levels, that is: $u^t(y_h(t)) = \ln y_h(t)$.

Let us denote by $\boldsymbol{P} = (\boldsymbol{Y}, \boldsymbol{z}, \delta)$ a poverty evaluation problem, or simply a problem. Note that the length of the time span and the number of the poor are given implicitly by the dimension of matrix $\boldsymbol{Y}$.

The utility loss of individual h along the $(T + 1)$ periods, with a discount factor δ, will thus be given by:

$$d_h(\boldsymbol{P}) = \sum_{t=0}^{T} \max\{0, \delta^t(\ln z_t - \ln y_h(t))\}$$

Let $\tilde{\mu}(\boldsymbol{y}_h) = \prod_{t=0}^{T} \max\left\{1, \left(\frac{z_t}{y_h(t)}\right)^{\frac{1}{T}\delta^t}\right\}$ stand for the geometric mean of the agent's time-adjusted deviations, $\left(\frac{z_t}{y_h(t)}\right)^{\delta^t}$, during the entire time span. Then, we can rewrite the expression above as:

$$d_h(\boldsymbol{P}) = T \ln \tilde{\mu}(\boldsymbol{y}_h) \qquad [1]$$

That is, the utility loss of agent h corresponds to the log of the geometric mean of the time-adjusted deviations, times the length of the time span.

[2] Our aim is measuring the utility loss of those who are poor today, considering past periods of poverty, rather than measuring the overall poverty in the period. Hence, a poor a $t > 0$ that is not poor today does not enter into the computation. And, symmetrically, a new poor enters with all his/her past poverty history up to T.

We define the ***welfare cost*** of poverty as the average value of the aggregate utility loss due to poverty. That is, for a vector $\boldsymbol{z}$ of thresholds, an intertemporal income distribution of the poor $\boldsymbol{Y}$, and a discount factor δ, the welfare cost of poverty, $C(\boldsymbol{P})$, is given by:

$$C(\boldsymbol{P}) = \frac{1}{nT}\sum_{h\in Q} T \ln \tilde{\mu}(\boldsymbol{y}_h)$$
$$= \frac{q}{n} \times \ln \prod_{h\in Q} [\tilde{\mu}(\boldsymbol{y}_h)]^{\frac{1}{q}}$$

Making $\prod_{h\in Q}[\tilde{\mu}(\boldsymbol{y}_h)]^{\frac{1}{q}} = \tilde{\mu}(\boldsymbol{P})$, the geometric mean of the geometric means of the agent's utility losses, we have:

$$C(\boldsymbol{P}) = \frac{q}{n} \times \ln \tilde{\mu}(\boldsymbol{P}) \qquad [2]$$

The welfare cost of poverty is thus given by the product of the incidence of poverty and the log of the geometric mean of the agents' utility losses. The term $\tilde{\mu}(\boldsymbol{P})$ is a measure that combines both the intensity and the inequality of poverty, incorporating structurally the effect of the past poverty periods.

Note that the term $\ln \tilde{\mu}(\boldsymbol{y}_h)$ in equation [1] corresponds to equation [2] in a society in which all individuals are poor and identical to agent h. Hence, $\frac{1}{T} d_h(\boldsymbol{P})$ can be regarded as an individual poverty index and provides a consistent way of comparing poverty between individuals.

We can easily decompose that term $\tilde{\mu}(\boldsymbol{P})$ to obtain separate measures of the intensity and the inequality of poverty, by using Atkinson's inequality index for $\varepsilon = 1$. As $A_1(\boldsymbol{d}) = 1 - \frac{\tilde{\mu}(\boldsymbol{P})}{\mu(\boldsymbol{P})}$ (i.e., one minus the ratio between the geometric mean and the arithmetic mean of the poor' deviations), we have:

$$C(\boldsymbol{P}) = \underbrace{\frac{q}{n}}_{Incidence} \times \left(\underbrace{\ln \mu(\boldsymbol{P})}_{Intensity} + \underbrace{\ln[1 - A_1(\boldsymbol{P})]}_{Inequality} \right) \qquad [3]$$

This expression makes it clear that the welfare cost of poverty is an increasing function of each of its components, as $\ln[1 - A_1(\boldsymbol{P})] < 0$. And, that poverty reduction policies will be more effective when focussed on those agents with lower incomes and larger poverty spells.

Equation [2] is also additively decomposable by population subgroups. To see that, suppose that the population can be divided into G different population subgroups, according to socio-demographic characteristics, $g = 1,2, \dots, G$. The set of the poor can be described, consequently, as $Q = \bigcup_{g=1}^{G} Q_g$, where Q_g is the set of the poor in population subgroup g, with cardinal $q(g)$. We assume that the parameter δ is common for all population subgroups, for the sake of simplicity (introducing different values is trivial).

According to equation [2] the social cost of poverty for population subgroup g will be given by:

$$C^g(\boldsymbol{P}) = \frac{q(g)}{n} \times \ln \tilde{\mu}\big(\boldsymbol{P}(g)\big)$$

As, trivially, $C(\boldsymbol{P}) = \sum_{g=1}^{G} C^g(\boldsymbol{P})$, we can write:

$$C(\boldsymbol{P}) = \frac{q}{n} \times \sum_{g=1}^{G} \frac{q(g)}{q} \times \ln \tilde{\mu}\big(\boldsymbol{P}(g)\big) \qquad [4]$$

And, letting the geometric mean of population subgroup g be given by:

$$\tilde{\mu}(\boldsymbol{g}) = \prod_{\boldsymbol{g}=\boldsymbol{1}}^{\boldsymbol{G}} \left[\tilde{\mu}\big(\boldsymbol{P}(g)\big)\right]^{\frac{p(q)}{Gp}}$$

We can alternatively write:

$$C(\boldsymbol{P}) = \frac{p}{n} \times G \times \ln \tilde{\mu}(\boldsymbol{g}) \qquad [4']$$

3 Final remarks

We have presented a way of measuring poverty that considers persistence as an integral part of the index. The formula is obtained in two steps. In the first step, we evaluate each agent's welfare loss along the time span considered, as the aggregate intertemporal utility loss. Assuming logarithmic utilities[3] the resulting formula is a variant of Watts (1968) poverty index (an index already characterized in Zheng 1993). We introduce a discount factor to make utilities homogeneous so that they can be added. The individual poverty indicator can be interpreted as a multidimensional poverty index in which time periods play the role of dimensions. In the second step we obtain the social welfare loss due to poverty by recurring to the familiar utilitarian social welfare function (characterized in D'Aspremont & Gevers, 1977).

It is easy to check that the resulting measure satisfies all the usual properties, including the following: Normalization (the index is zero when there are no poor people), continuity (the index is a continuous function on its domain), focus (changes in the income of the non-poor do affect the index), symmetry (permuting individual income profiles does not alter the index), monotonicity (increasing the income of the poor reduces the index), scale invariance (multiplying incomes and thresholds by a positive constant does not alter the measurement), replication Invariance (combining two identical populations does not alter de index), principle of transfers (a transfer from a richer to a poorer person does not decrease the index), and decomposability by

[3] Note that this utility function can be regarded as a particular case of the family of constant relative risk aversion utility functions $u(x) = \frac{x^{1-\varepsilon}}{(1-\varepsilon)}$, for ε = 1, adopted by Atkinson (1970) in the realm of inequality measurement.

components and additive decomposability by population subgroups (properties already discusses in Section 2).[4]

What we call the welfare cost of poverty refers to the intertemporal cost of those who are poor today, expressed in per capita, per period terms. Trivially, $T \times C(\boldsymbol{P})$ is a measure of the per capita utility loss for the entire time span, and $n \times T \times C(\boldsymbol{P})$ gives us the total utility loss. We might have chosen a different modelling option, focusing on the aggregate utility loss of all those who have been poor for some period during the time span considered. The formula applies to this case, mutatis mutandis, even though the interpretation of the measure would be slightly different.

Let us conclude by emphasizing that considering duration as part of conventional poverty measurement enhances the relevance of those policies aimed at reducing poverty, which provide the less well-off with a minimum standard of living. Minimal income schemes may be reinforced from this perspective as each agent who leaves the set of the poor carries with him the basket of past poverty periods.

[4] Zheng (1993) shows that the key properties to characterize the Watts poverty index are scale invariance, continuity, monotonicity, and additive decomposability by population subgroups.

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